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THE

NEW PACKARD COMMERCIAL ARITHMETIC.

BY

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NEW YORK:
S. S. PACKARD, Publisher.

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PREFACE.

THE PACKARD COMMERCIAL ARITHMETIC, of which this is a revision, has been before the public for a space of five years, during which time it has attained a large sale and has given satisfaction to its patrons. No special effort has been made to increase the sale by advertising or making strong statements on its behalf. The authors and publisher have well understood from the beginning that the success of a text-book to be used by intelligent teachers was not in any sense dependent upon promises made in advance, or claims of particular merit which might otherwise escape attention. The book grew naturally out of the practice of the school-room, and the manuscript lessons which were afterwards utilized as copy were submitted to the most exacting tests, and their value in building up the student in a sound knowledge of principles and in the details of practice thoroughly established. And the satisfaction expressed by those who have used the book, has amply sustained the judgment of the authors. The present revision has not been undertaken on account of any dissatisfaction expressed with the old edition, nor because it had not proved in the broadest sense effective, but because the authors have felt that some additions, and particularly in the direction of preparatory work, would make it more acceptable to a large number who now use it, while it would not in any way detract from its utility or symmetry as a complete text-book. The omission of the fundamental rules in the old edition emphasized the fact that it was an advanced commercial text-book, intended for schools not requiring primary instruction. It has come to our knowledge, however, [illegible] requiring the [illegible] an addition [illegible] in present- [illegible] en to direct [illegible] bring it into [illegible] directness and [illegible] presented, is [illegible] little that is [illegible] the learner in [illegible] large number of [illegible] departments. [illegible] satisfy teachers [illegible] addition the [illegible] recommend the [illegible] puzzles nor [illegible] by the busi- [illegible] or local laws [illegible] highest source [illegible]

[illegible] the authors [illegible] friends [illegible] times, and

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ARITHMETIC.

NOTATION AND NUMERATION.

1. Arithmetic is the science of numbers and the art of computation by them.

2. A **Unit,** or **Unity,** is one, or a single thing; as one, one foot, one dollar.

3. A **Number** is a unit, or a collection of units; as one, four, three feet, five dollars. Numbers are expressed by words, by letters, and by figures.

4. Notation is a system of representing numbers by symbols. There are two methods of notation in use, the *Roman* and the *Arabic.*

5. Numeration is a system of naming or reading numbers.

6. The Arabic method of notation employs ten characters or figures, viz.:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*
One,	***Two,***	***Three,***	***Four,***	***Five,***	***Six,***	***Seven,***	***Eight,***	***Nine,***	***Zero.***

The first nine of the above are called *significant figures,* because each, standing by itself, represents a value, or denotes some number. They are also called *digits,* from the Latin word *digitus,* which means *a finger.*

The last one is called *zero, naught,* or *cipher,* because when standing alone it has no value, or signifies nothing.

FRENCH AND AMERICAN NUMERATION TABLE

etc., etc.	Hundreds of Quintillions	Tens of Quintillions	*Quintillions.*	Hundreds of Quadrillions	Tens of Quadrillions	*Quadrillions.*	Hundreds of Trillions	Tens of Trillions	*Trillions.*	Hundreds of Billions	Tens of Billions	*Billions.*	Hundreds of Millions	Tens of Millions	*Millions.*	Hundreds of Thousands	Tens of Thousands	*Thousands.*	Hundreds	Tens	*Units.*
	5	0	8,	6	4	2,	1	9	5,	7	3	2,	4	3	6,	8	0	7,	5	9	3.
	7th period, Quintillions.			6th period, Quadrillions.			5th period, Trillions.			4th period, Billions.			3d period, Millions.			2d period. Thousands.			1st period. Units.		

ENGLISH NUMERATION TABLE.

etc., etc.	Hundreds of Trillions	Tens of Trillions	*Trillions.*	Hundreds of Thousands of Billions	Tens of Thousands of Billions	Thousands of Billions	Hundreds of Billions	Tens of Billions	*Billions.*	Hundreds of Thousands of Millions	Tens of Thousands of Millions	Thousands of Millions	Hundreds of Millions	Tens of Millions	*Millions.*	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	*Units.*
	5	0	8	6	4	2	1	9	5	7	3	2	4	3	6	8	0	7	5	9	3
				Billions.						Millions.						Units.					

NOTE.—It will be observed by a comparison of the French and English systems, that numbers consisting of nine figures or less are read the same.

7. Copy and read the following numbers:

73
69
48
90
102
333
570
895
616
348
222
843
1064
3604
4364
7208
8174
8006
7070
3300
12741
20809
47038
68605

8. Express by figures the following:

1. Nineteen.
2. Twenty-two.
3. Forty-six.
4. Sixty-eight.
5. Ninety-two.
6. Eighty-seven.
7. One hundred forty-four.
8. Three thousand sixteen.
9. Four thousand forty-four.
10. Six million two thousand six.
11. Sixteen million eight hundred two
12. Eighty-seven thousand sixty-two.

ROMAN NOTATION.

9. In the **Roman Notation,** seven capital letters are used to express numbers, as follows :

I	V	X	L	C	D	M
One,	*Five,*	*Ten,*	*Fifty,*	*One Hundred,*	*Five Hundred,*	*One Thousand.*

Other numbers are expressed by combining the letters according to the following principles :

1. If a letter is repeated, its value is repeated. Thus, III represents three ; XX, twenty ; CCC, three hundred.

2. If a letter of *less* value is placed *before* one of greater value, the less is taken from the greater. Thus, IV represents four ; IX, nine ; XL, forty.

3. If a letter of *less* value is placed *after* one of greater value, the less is added to the greater. Thus, VI represents six ; XI, eleven ; LX, sixty.

4. A bar (‾) placed over a letter increases its value a thousand times. Thus, $\overline{\text{X}}$ represents ten thousand ; $\overline{\text{M}}$, one million.

The Roman Notation is used for numbering dials, chapters, pages, etc.

10. TABLE OF ROMAN NOTATION.

Roman.	*Arabic.*	*Roman.*	*Arabic*	*Roman.*	*Arabic.*	*Roman.*	*Arabic.*
I,	1.	IX,	9.	XX,	20.	XC,	90.
II,	2.	X,	10.	XXI,	21.	C,	100.
III,	3.	XIII,	13.	XXX,	30.	CCC,	300.
IV,	4.	XIV,	14.	XL,	40.	D,	500.
V,	5.	XV,	15.	L,	50.	DCC,	700.
VI,	6.	XVIII,	18.	LX,	60.	M,	1000.
VIII,	8.	XIX,	19.	LXXX,	80.	MD,	1500.

11. Express by Roman notation :

1. Eighteen.
2. Thirty-six.
3. Forty-eight.
4. Seventy-six.
5. Sixty-four.
6. Eighty-seven.
7. Three hundred sixty.
8. Six hundred forty-nine.
9. Five hundred eighty-eight.
10. Two thousand sixty-two.
11. 584.
12. 777.
13. 1638.
14. 1886.
15. 80000.

12. Express by Arabic notation :

1. LXXVII.
2. CCXIX.
3. XCVIII.
4. CCCLIV.
5. DCXXVI.
6. DCCLXVI.
7. DCXLIV.
8. $\overline{\text{D}}$CXLIV.
9. MDCXLVI.
10. MCCLXXIX.
11. $\overline{\text{M}}$MCCCXXVII.
12. MMDCCXVIII.
13. MMCCXCIX.
14. MMDCCCLXXV.
15. MDCCCLXXXVII.

[illegible] or more num

[illegible] the num

[illegible] the sum

[illegible] read [illegible]

[illegible] equa

[illegible] equa [illegible] or the sum of [illegible]

[illegible] of cents, ct., or cts.

[illegible] the sum of two or more numbers.

[illegible]

[illegible] the numbers so that like units [illegible]

[illegible] to add at the right.

[illegible] 19 units, equa

[illegible] under the column of un

[illegible] of tens, obtaining for t

[illegible] 1 hundred 8 tens. Wr

[illegible] and add the 1 hundre

[illegible] 10, 16, 16 hundred

[illegible] and thousand

19. Rule.—*Write the numbers to be added so that like units stand in the same column.*

Commencing at the right, add each column separately, and if the sum is less than 10, write it under the column added.

If the sum of any column is 10 or more than 10, write the right-hand figure under the column added, and add the remaining figure or figures to the next column.

Proof.—*Find the sum by adding the columns in the opposite direction, thus forming new combinations of figures. If the results agree, the work is probably correct.*

EXAMPLES.

20. Copy or write from dictation and add the following:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)
789	682	1234	1357	7812	9876
123	109	5678	9135	3625	6789
456	375	9012	8642	4875	9787
246	468	3456	4109	9850	8678

(*7*)	(*8*)	(*9*)	(*10*)	(*11*)	(*12*)
568	431	9672	7812	8796	808
134	866	8738	1357	809	7612
680	219	4126	404	4205	37
419	581	1886	9686	6666	4123
723	49	7143	8072	7777	2264
842	376	8275	9706	8088	7714
906	408	9325	5555	4144	9008
294	792	4444	2009	9995	3348

21. There is nothing of more importance to the student than the ability to add a column of figures easily, accurately, and rapidly. In order that his labor may be lightened and much valuable time saved, not only in after-life but in his school work, he should have various kinds of daily drill exercises in addition, especially in the earlier part of his course of study. The following suggestions will be found valuable in securing accuracy and rapidity:

22. The 45 simple combinations should be used as an exercise in addition. They may be copied on the blackboard in the following or in irregular order, and the sum should be announced by the student at sight:

1	2	3	2	4	3	5	4	3	6	5	4	7	6	5
1	2	1	2	1	2	1	2	3	1	2	3	1	2	3

4	8	7	6	5	9	8	7	6	5	9	8	7	6	9
4	1	2	3	4	1	2	3	4	5	2	3	4	5	3

8	7	6	9	8	7	9	8	7	9	8	9	8	9	9
4	5	6	4	5	6	5	6	7	6	7	7	8	8	9

23. The above should be supplemented by exercises similar to the following:

74	64	44	94	34	24	14	54	84
8	8	8	8	8	8	8	8	8

45	75	35	15	55	65	95	25	85
9	9	9	9	9	9	9	9	9

It is just as easy to add 74 and 8 as 4 and 8. It should be impressed on the mind of the student that 4 and 8 when added always produce 2 in units' place, whatever the number of tens, and the tens are increased by 1. If the student is thoroughly drilled, he will not hesitate when near the end of the column or when the sum is above 20 or 30.

24. Make combinations of 10, 20, 30, or other numbers as often as possible, and add them as single numbers.

Thus, add 9 and 1, 8 and 2, 5 and 5, 4, 3, and 3, etc., as 10; 7 and 2, 6, 2, and 1, etc., as 9; 2 and 3, 4 and 1, 2, 2, and 1, as 5; 8, 7, and 5, 9, 7, and 4, etc., as 20; etc., etc.

In Example 1, Art. **27**, think only of the following results: 9, 19, 30, 50.

Drill on the following and similar combinations (10, 20, 30, etc.) until the student can announce the sums at sight:

3	2	1	4	5	3	8	2	2	6	1	2	2	1	3
4	3	4	1	1	1	1	7	6	2	8	1	4	3	3
3	5	5	5	4	6	1	1	2	2	1	7	4	6	4

7	6	6	5	4	8	9	4	3	2	5	8	9	7	9
6	8	5	4	7	3	2	7	8	9	7	7	8	7	7
7	6	9	8	9	9	9	9	9	9	8	5	3	6	4

1	1	1	3	3	6	8	4	7	7	7	6	9	8	9
2	4	3	3	3	6	4	4	7	4	7	7	6	8	4
3	2	2	1	3	6	1	4	4	4	7	8	7	9	8
4	3	4	3	1	2	7	8	2	5	9	9	8	5	9

When a figure or number comes between two numbers that make 10, think of the total at once; as 7 + 6 + 3 = 16, 6 + 7 + 4 = 17.

Notes.—1. When three figures are in regular order, the sum may be found by multiplying the middle figure by 3. Thus, 9 + 8 + 7 = 24 (3 × 8); 5 + 6 + 7 = 18; 6 + 7 + 8 = 21.

2. When five figures are in regular order, the sum may be found by multiplying the middle figure by 5. Thus, 1 + 2 + 3 + 4 + 5 = 15 (5 × 3); 5 + 6 + 7 + 8 + 9 = 35.

3. When a figure is repeated several times, multiply it, instead of adding.

4. Do not think of numbers between 10 and 20 as a certain number of units and 1 ten as they are named, but as 1 ten and a certain number of units. Thus, think of 14 as 1 ten and 4 units (onety-four, or one-four), not 4 units and 1 ten (fourteen).

5. Add downwards, for then the sum is found just where it should be placed—at the foot of the column. In proving results, add upwards.

25. Adding two columns at once.—Drill on the following or similar combinations of numbers of two figures each, until the student can announce the sums at sight:

12	24	45	24	37	41	37	62	27	57
16	33	33	26	42	58	45	34	48	27

When the above have been mastered, give exercises containing three or more numbers, as

12	16	22	24	19	42	56	37	51	27
31	29	33	36	31	24	21	33	25	34
42	38	56	22	44	17	24	28	38	45

26. The following "magic square" may be used as a drill exercise in addition. The sum downwards, from left to right, or diagonally is 54351. To vary the exercise, the teacher may dic-

[illegible]

[illegible]

EXAMPLES.

[illegible] from dictation and add the following:

(12)	(13)
514	376
827	499
842	678
462	437
460	245

(14)	(15)	(16)	(17)	(18)	(19)
1234	4121	1728	3416	17642	18114
5678	1865	5280	4725	176	285
9212	3760	2246	8850	20048	28510
3456	4825	4153	4975	248	30048
9753	7145	4839	2137	24800	400
8642	3333	2437	8910	1149	17512
7531	7163	4627	2048	1216	8
1594	4943	7342	175	385	14150
7777	7289	8916	1075	19175	30032

20. Find the sum of the following numbers: Forty-five thousand forty-five; sixteen thousand three hundred sixty; one hundred sixty-seven thousand; eight hundred fifty thousand ninety-two; nine million twenty-four.

21. 46 + 72 + 89 + 93 + 75 + 31 + 58 + 45 + 52 = ?

22. 376 + 416 + 287 + 123 + 456 + 789 + 916 + 328 = ?

23. 42 + 175 + 287 + 56 + 63 + 324 + 189 + 172 + 96 = ?

24. 365 + 1728 + 64 + 172 + 89 + 38 + 9 + 5280 + 176 = ?

25. A bushel of corn weighs 56 pounds, a bushel of rye 56 pounds, a bushel of wheat 60 pounds, a bushel of barley 45 pounds, a bushel of oats 32 pounds, and a bushel of buckwheat 48 pounds. What would be the total weight of one bushel of each of the above grains ?

26. A farmer raises 375 bushels corn, 419 bushels barley, 849 bushels wheat, 668 bushels oats, 957 bushels barley, and 389 bushels rye. Find how many bushels in all.

27. Find the total distance around a rectangular field 1728 feet long and 1683 feet wide.

28. An exporter of provisions buys 187 barrels hams, 428 barrels shoulders, 475 barrels pork, 229 barrels beef, and 392 barrels bacon. How many barrels in all ?

29. In an orchard there are 375 apple trees, 416 pear trees, 87 quince trees, 98 cherry trees, 238 peach trees, and 276 plum trees. How many trees in all ?

30. A man pays for a house and lot $6375. For repairs as follows: mason-work, $68; plumbing, $78; carpenter-work, $164; painting and decorating, $277. For how much must he sell it to gain $567 on the total cost ?

31. A manufacturer sells on Monday 255[illegible] barrels flour, on Tuesday 2618 bbls., on Wednesday 2178 bbls., on Thursday 2125 bbls., on Friday 2348 bbls., and on Saturday 2496 bbls. How many does he sell during the week?

32. Find the total number of pounds of tobacco produced in the following states in 1879: Kentucky, 171,121,134; Virginia, 80,099,838; Pennsylvania, 36,957,772; Ohio, 34,725,405; Tennessee, 29,365,052; North Carolina, 26,986,448; Maryland, 26,082,147; Connecticut, 14,044,652; Missouri, 11,994,077; Wisconsin, 10,878,463.

Add the following numbers as they stand, from left to right, and from right to left. [In making out bills and in other commercial operations, a great deal of time can be saved by adding in this manner, without re-arranging the numbers.]

33. 17, 27, 36, 14, 43, 42, 65, 73, 81, 35.
34. 176, 340, 203, 62, 177, 96, 398, 75, 148, 96.
35. 137, 414, 528, 345, 678, 975, 864, 357, 121, 234.
36. 6716, 512, 375, 475, 3842, 5927, 3875, 17525.
37. 2345, 16, 375, 4218, 376, 7, 8475, 247, 39.
38. 123427, 34825, 775, 716, 8976, 37412, 567356, 39723.

NOTE.—In tally-sheets of pounds, gallons, yards, feet, etc., for convenience in adding, place 10 (20 or 30) numbers in each column as in the following example. (See Note 3, Art. **24.**) Add the totals from left to right.

39. Find the total weight of the following 100 boxes of cheese:

67	64	62	61	66	68	64	62	61	60
61	67	60	64	67	60	63	61	68	64
60	63	63	64	66	65	67	61	60	63
63	60	62	61	68	64	65	66	63	67
62	62	61	65	61	66	63	67	62	65
64	61	60	68	66	64	63	67	69	66
66	65	62	61	68	67	64	63	66	66
65	66	65	64	67	67	68	66	63	61
61	68	64	62	65	60	61	60	62	65
62	64	66	66	63	61	62	67	68	69
***	***	***	***	***	***	***	***	***	***
									****

40. Find the total estimated value of the following crops for the year 1884: Corn, $640,735,589; wheat, $330,861,254; rye, $14,855,255; oats, $161,528,470; barley, $29,781,155.

41. Find the total number of bales of cotton produced in the following states in 1879: Mississippi, 955,808; Georgia, 814,441; Texas, 803,642; Alabama, 699,654; Arkansas, 608,256; South Carolina, 522,548; Louisiana, 508,569; North Carolina, 389,598.

42. Find the total number of bushels of wheat produced in the following states in 1879: Illinois, 51,136,455; Indiana, 47,288,989; Ohio, 46,014,869; Michigan, 35,537,097; Minnesota, 34,625,657; Iowa, 31,177,225; California, 28,787,132; Missouri, 24,971,727; Wisconsin, 24,884,689.

Complete the following statements by adding downwards and from left to right. The sums of the totals should be equal.

43. SALES FOR THE WEEK ENDING JAN. 21, 1887.

Days.	Dry Goods.	Shoes.	Furniture.	Books.	Crockery.	Millinery.	Totals.
Monday........	647.18	176.42	376.40	117.13	92.17	111.40	****.**
Tuesday........	356.25	97.20	184.00	64.72	38.15	57.65	***.**
Wednesday.....	716.40	200.48	417.65	156.25	127.16	328.40	****.**
Thursday.......	828.30	227.59	476.00	171.14	287.80	116.38	****.**
Friday.........	316.17	87.14	148.12	100.80	77.44	79.40	***.**
Saturday.......	929.12	249.67	417.13	426.00	225.98	429.75	****.**
Totals.........	****.**	****.**	****.**	****.**	***.**	****.**	****.**

44. SALES FOR 1887.

Months.	Domestics.	White Goods.	Notions.	Woolens.	Dress Goods.	Totals.
January........	12248.10	6478.10	3429.12	8117.40	5276.40	*****.**
February.......	13375.16	7149.37	4176.19	9190.56	6189.24	*****.**
March..........	17177.48	8214.92	4375.94	8237.41	3416.48	*****.**
April...........	15119.43	7175.12	4040.40	7116.40	5255.90	*****.**
May............	16284.19	8134.37	4287.38	3697.82	1716.32	*****.**
June...........	13464.25	6375.28	3343.72	2419.38	2100.58	*****.**
July...........	9119.47	4569.34	2417.75	3129.16	3269.43	*****.**
August.........	10219.36	5162.12	3112.96	4289.50	3716.96	*****.**
September......	14196.42	7289.40	4128.43	5391.42	4128.31	*****.**
October........	13184.16	6517.58	3692.29	7214.97	6247.39	*****.**
November.......	14174.12	7288.49	4140.50	8345.68	7116.40	*****.**
December.......	13184.96	6697.13	3990.82	8175.75	7348.19	*****.**
Totals.........	******.**	*****.**	*****.**	*****.**	*****.**	******.**

SUBTRACTION.

28. The **difference** between two numbers is a number which, added to the smaller, will produce a result equal to the greater.

29. Subtraction is the process of finding the *difference* between two numbers.

The greater of two numbers whose difference is required is called the *minuend*, and the smaller the *subtrahend.* The result is called the *remainder.*

30. The sign of subtraction is —, and is read *minus* or *less.*

Thus, 8 – 5 is read 8 minus 5, or 8 less 5, and means that 5 is to be taken from 8.

31. To find the difference between two numbers.

Ex. Find the difference between 967 and 384.

OPERATION.

967 Minuend.
384 Subtrahend.
583 Remainder.

ANALYSIS.—Write the smaller number under the greater so that units are under units, tens under tens, etc. Commence to subtract at the right. 4 units from 7 units are 3 units, which write below the line under the column of units. Since 8 tens cannot be taken from 6 tens, take 1 hundred from 9 hundreds, leaving 8 hundreds, and add it (1 hundred = 10 tens) to the 6 tens, making 16 tens. 8 tens from 16 tens are 8 tens, which write under the column of tens. 3 hundreds from 8 (9 – 1) hundreds are 5 hundreds. Hence the result is 583.

Instead of subtracting 1 from the figure of the *upper number* of the next higher order when it has been necessary to add 10 to the figure of the minuend, some persons add 1 to the figure of the *lower number* of the next higher order. This method depends on the principle that adding equivalent numbers to both minuend and subtrahend does not affect the remainder.

In practice, do not think of explanations, nor say 4 from 7 is 3, etc., but think only of results and write them at once. **Thus, in the above example, say or think only 3, 8, 5.**

32. **Rule.**—*Write the subtrahend under the minuend so that units of the same order stand in the same column.*

Commencing at the right, subtract each figure in the lower number from the one above it, and write the difference in the line below.

If any figure is greater than the one above it, add 10 to the latter, perform the subtraction, and then consider the next figure in the upper number decreased by 1 (or, consider the next figure in the lower number increased by 1).

EXAMPLES.

33. Find the difference between

1. 8716 and 4379.
2. 917642 and 9819.
3. 64321 and 23456.
4. 428165 and 317618.
5. 9371641 and 876543.
6. 7642878 and 6789119.
7. 8090403 and 7090508.
8. 6380912 and 5270937.
9. 7654321 and 1234567.
10. 7060509 and 6987969.
11. 80706040 and 23456789.
12. 76483672 and 87132191.
13. 123456789 and 9897960.
14. 72081099 and 87643229.
15. 16417528 and 90716801.
16. 43184296 and 37529510.
17. 100010001 and 9890978.
18. 30040050 and 29917168.
19. 20103040 and 19181746.
20. 40020003 and 20807064.

Find the difference between the numbers in each of the following groups. [In all of these cases the subtrahend is placed above the minuend, the purpose being to give the student practice in subtracting *downward* rather than upward, as the general custom is. It is often requisite in business to perform the work in this way, and the accountant should practice both methods.]

(21)	*(22)*	*(23)*	*(24)*	*(25)*	*(26)*
76534	19827	26347	72016	12345	81907
81279	84362	71356	99385	54321	94371

(27)	*(28)*	*(29)*	*(30)*	*(31)*	*(32)*
12467	31027	46789	24681	46897	36478
75112	42131	50000	30502	50901	41516

34. Short method of finding the balance of an account.

Ex. Find the balance of the following ledger account:

Dr. C. E. & W. T. PECK. *Cr.*

1882.					1882.				
Mar.	16	Merchandise.	1192	97	Apr.	22	Cash.	800	
"	30	Sundries.	567	40	"	22	Bills receivable.	1000	
"	31	Merchandise.	384	30	May	1	Merchandise.	317	28
Apr.	22	Interest.	16	48	"	17	Cash.	424	79
"	24	Merchandise.	846	51	July	1	Balance.	852	84
May	17	"	387	25					
			3394	91				3394	91
July	1	Balance.	852	84					

ANALYSIS.—It can readily be seen that the debit side is greater; therefore add that side first and write the sum as the total or footing of each side. Then pass to the other side of the account. The sum of the first column is 17, which subtracted from the next higher number, 21, ending with 1, the corresponding figure of the total, leaves 4, which write as the first figure of the balance, carrying the 2 to the next column. (If the right-hand figure of the sum of any column is the same as the corresponding figure of the total, subtract it from itself, and not from the next higher number ending with the same figure; or write 0 in the balance and carry the left-hand figure of the sum.) The sum of the figures in second column plus 2 carried is 11, which subtracted from 19 leaves 8, the second figure of the balance. Proceed in like manner until all the figures of the balance are obtained. Prove by adding all the numbers, including the balance.

EXAMPLES.

35. Find the balances of the following accounts:

(1.)		(2.)		(3.)	
Dr.	*Cr.*	*Dr.*	*Cr.*	*Dr.*	*Cr.*
817.20	812.20	237.25	112.27	1075.	375.60
222.22	214.13	900.	218.36	2318.42	218.24
427.30	375.	800.	717.49	812.10	717.37
810.75	412.	718.24	648.	938.40	244.45
416.30	717.	218.75	118.75	4312.	946.33
225.			538.98		222.48
719.46			203.13		**108.76**

MULTIPLICATION.

36. Multiplication is the operation of taking one number as many times as there are units in another.

The number taken or multiplied is called the *Multiplicand.* The number which indicates how many times the multiplicand is taken or multiplied, is called the *Multiplier.* The result obtained is called the *Product,*

37. The sign of multiplication is ×, and is read *times,* or *multiplied by.*

Thus, 5 × 4=20, is read 5 times 4 equals 20, or 5 multiplied by 4 equals 20.

38. Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

39. To find the product of two numbers when the multiplier does not exceed 12.

Ex. Multiply 456 by 7.

OPERATION.

$$\begin{array}{r} 456 \\ 7 \\ \hline 3192 \end{array}$$

ANALYSIS.—7 times 6 units are 42 units=4 tens and 2 units. Write the 2 units under the figure of the multiplier (the column of units) and add the 4 tens to the next product (the column of tens). 7 times 5 tens are 35 tens, plus 4 tens from the preceding product, are 39 tens = 3 hundreds and 9 tens. Write the 9 tens under the column of tens, and add the 3 hundreds to the next product (the column of hundreds). 7 times 4 hundreds are 28 hundreds, plus 3 hundreds from the preceding product are 31 hundreds = 3 thousands and 1 hundred. Write the 1 hundred under the column of hundreds and the 3 thousands in the column of thousands.

40. RULE.—*Commencing at the right, multiply each figure of the multiplicand by the multiplier, writing the result and carrying as in addition.*

EXAMPLES.

41. Multiply

1. 23456 by 7; by 8.
2. 37804 by 9; by 6.
3. 24687 by 2; by 4.
4. 36925 by 3; by 8.
5. 48716 by 5; by 9.
6. 90809 by 9; by 8.
7. 26048 by 5; by 7.

Multiply

8. 789123 by 2; by 3.
9. 123567 by 4; by 5.
10. 781693 by 6; by 9.
11. 417009 by 8; by 7.
12. 509048 by 8; by 7
13. 637485 by 6; by 8
14. 748596 by 7; by 5.

15. There are 5280 feet in one mile How many feet in 11 miles?

16. There are 4 gills in one pint, 2 pints in one quart, and 4 quarts in one gallon. How many gills in 63 gallons?

17. There are 2 pints in one quart, 8 quarts in one peck, and 4 pecks in one bushel. How many pints in 379 bushels?

18. There are 12 inches in one foot, and 3 feet in one yard. How many inches in 1760 yards?

19. In one gross there are 12 dozen, and in one dozen 12 units. Find the value of 45 gross lead-pencils at 3 cents each.

20. How many buttons on 6 dozen pair of shoes, if there are 9 buttons on each shoe?

the product of two numbers, when the
re than 12.

456 by 237.

Analysis.—Write the multiplier under the multipli-
d so that their right-hand figures are in the same ver-
l line. Since the multiplier consists of 7 units, 3 tens,
2 hundreds, the multiplicand is repeated or multiplied
7, by 30, and by 200. 7 times 456 is 3192, the first par-
product; 30 times 456 is 13680, the second partial
duct; 200 times 456 is 91200, the third partial product.
sum of these partial products is 108072, the entire
duct. In practice, the ciphers are omitted. In the
ration, observe that the first or right-hand figure of
s directly under the figure of the multiplier used.

*Write the multiplier under the multipli-
heir right-hand figures are in the same*

*multiplicand by each significant figure of
writing the first or right-hand figure of each
under the figure of the multiplier used.
ial products. The sum will be the desired*

EXAMPLES.

Multiply

; by 481. 9. 23456 by 294 ; by 3742.
; by 500. 10. 40607 by 144 ; by 4803.
; by 624. 11. 32738 by 176 ; by 5964.
; by 375. 12. 91609 by 201 ; by 6075.
; by 208. 13. 24135 by 345 ; by 7186.
; by 567. 14. 38246 by 678 ; by 8297.
; by 781. 15. 94538 by 987 ; by 9410.
; by 936. 16. 10908 by 406 ; by 2465.

hours in the month of January ?
minutes in the month of April ?
seconds in the month of February, 1899 ?
ost of 375 barrels pork at $14 per barrel.

22. How many pounds in 471 bushels corn, if there are 56 pounds in one bushel ?

23. In a bushel of timothy seed, there are 45 pounds. How many pounds in 2367 bushels ? In 3416 bushels ?

24. How many shoes in 24 boxes, if each box contains 12 pair ?

25. A certain building has 192 windows, and each window contains 24 panes of glass. How many panes in all ?

26. How many feet of wire will be required to fence a field 209 feet square, the fence being 6 wires high and on all sides of the field ?

45. To find the product of two numbers when there are ciphers at the right of the significant figures (6) of one or both.

Ex. Multiply 37600 by 47000.

OPERATION.

$$\begin{array}{r} 37600 \\ 47000 \\ \hline 2632 \\ 1504 \\ \hline 1767200000 \end{array}$$

ANALYSIS.—Write the numbers so that the right-hand significant figures are in the same vertical line. 37600 = 376 × 100, and 47000 = 47 × 1000. Since the product of two or more numbers is the same in whatever order they are multiplied, multiply 376 by 47, and their product by 100000 (100 × 1000), by annexing 5 (3 + 2) ciphers to the right.

46. RULE.—*Write the numbers so that their right-hand significant figures are in the same vertical line. Multiply the significant figures together as if there were no ciphers, and to their product annex as many ciphers as are found on the right of both numbers.*

EXAMPLES.

47. Multiply

1. 3600 by 40 ; by 300.
2. 1728 by 80 ; by 500.
3. 3456 by 70 ; by 420.
4. 3710 by 50 ; by 360.
5. 4000 by 30 ; by 800.
6. 2800 by 90 ; by 370.
7. 1360 by 60 ; by 200.
8. 4200 by 20 ; by 500.

Multiply

9. 48400 by 200 ; by 1400.
10. 37000 by 500 ; by 2500.
11. 12345 by 600 ; by 3600.
12. 28000 by 420 ; by 4700.
13. 19700 by 340 ; by 5800.
14. 14320 by 560 ; by 6900.
15. 84000 by 800 ; by 7320.
16. 96000 by 900 ; by 4800

42. To find the product of two numbers, when the multiplier is more than 12.

Ex. Multiply 456 by 237.

OPERATION.

$$\begin{array}{r} 456 \\ \underline{237} \\ 3192 \\ 1368 \\ \underline{912} \\ 108072 \end{array}$$

ANALYSIS.—Write the multiplier under the multiplicand so that their right-hand figures are in the same vertical line. Since the multiplier consists of 7 units, 3 tens, and 2 hundreds, the multiplicand is repeated or multiplied by 7, by 30, and by 200. 7 times 456 is 3192, the first partial product; 30 times 456 is 13680, the second partial product; 200 times 456 is 91200, the third partial product. The sum of these partial products is 108072, the entire product. In practice, the ciphers are omitted. In the operation, observe that the first or right-hand figure of each partial product is directly under the figure of the multiplier used.

43. RULE.—*Write the multiplier under the multiplicand so that their right-hand figures are in the same vertical line.*

Multiply the multiplicand by each significant figure of the multiplier, writing the first or right-hand figure of each partial product under the figure of the multiplier used.

Add the partial products. The sum will be the desired product.

EXAMPLES.

44. Multiply

1. 1728 by 37; by 481.
2. 2893 by 26; by 506.
3. 3904 by 18; by 624.
4. 5107 by 41; by 375.
5. 6079 by 59; by 208.
6. 8125 by 67; by 567.
7. 9236 by 78; by 781.
8. 7438 by 89; by 936.

Multiply

9. 23456 by 294; by 3742.
10. 40607 by 144; by 4803.
11. 32738 by 176; by 5964.
12. 91609 by 201; by 6075.
13. 24135 by 345; by 7186.
14. 38246 by 678; by 8297.
15. 94538 by 987; by 9410.
16. 10908 by 406; by 2465.

17. How many hours in the month of January?

18. How many minutes in the month of April?

19. How many seconds in the month of February, 1899?

20. Find the cost of 375 barrels pork at $14 per barrel.

21. There are 5280 feet in one mile. How many feet in 96 miles? In 208 miles?

22. How many pounds in 471 bushels corn, if there are 56 pounds in one bushel ?

23. In a bushel of timothy seed, there are 45 pounds. How many pounds in 2367 bushels ? In 3416 bushels ?

24. How many shoes in 24 boxes, if each box contains 12 pair ?

25. A certain building has 192 windows, and each window contains 24 panes of glass. How many panes in all ?

26. How many feet of wire will be required to fence a field 209 feet square, the fence being 6 wires high and on all sides of the field ?

45. To find the product of two numbers when there are ciphers at the right of the significant figures (6) of one or both.

Ex. Multiply 37600 by 47000.

OPERATION.

```
  37600
   47000
 --------
  2632
 1504
----------
1767200000
```

ANALYSIS.—Write the numbers so that the right-hand significant figures are in the same vertical line. 37600 = 376 × 100, and 47000 = 47 × 1000. Since the product of two or more numbers is the same in whatever order they are multiplied, multiply 376 by 47, and their product by 100000 (100 × 1000), by annexing 5 (3 + 2) ciphers to the right.

46. RULE.—*Write the numbers so that their right-hand significant figures are in the same vertical line. Multiply the significant figures together as if there were no ciphers, and to their product annex as many ciphers as are found on the right of both numbers.*

EXAMPLES.

47. Multiply

1. 3600 by 40 ; by 300.
2. 1728 by 80 ; by 500.
3. 3456 by 70 ; by 420.
4. 3710 by 50 ; by 360.
5. 4000 by 30 ; by 800.
6. 2800 by 90 ; by 370.
7. 1360 by 60 ; by 200.
8. 4200 by 20 ; by 500.

Multiply

9. 48400 by 200 ; by 1400.
10. 37000 by 500 ; by 2500.
11. 12345 by 600 ; by 3600.
12. 28000 by 420 ; by 4700.
13. 19700 by 340 ; by 5800.
14. 14320 by 560 ; by 6900.
15. 84000 by 800 ; by 7320.
16. 96000 by 900 ; by 4800

SHORT METHODS.*

48. To multiply any number of two figures by 11.

49. RULE.—*Place the sum of its digits between them when the sum is less than 10. When the sum is 10 or more than 10, write its right-hand figure in the second place and carry one to the left-hand figure of the multiplicand.*

EXAMPLES.

50. *1.* Multiply 34 by 11.

ANALYSIS.—3 + 4 = 7, which placed between 3 and 4 produces the product 374.

2. Multiply 68 by 11.

ANALYSIS.—6 + 8 = 14. Write 4 in the second place and carry 1 to the 6, the left-hand figure of the multiplicand producing the product 748.

3. Multiply the following numbers by 11: 24, 16, 18, 32, 43, 33, 72, 81, 37, 44, 92, 87, 93, 64, 35, 36, 47, 17, 19, 48, and 57.

51. To multiply any number by 11.

52. RULE.—*Write the 1st right-hand figure, add the 1st and 2nd, the 2nd and 3rd, and so on; finally write the left-hand figure, carrying as usual.*

EXAMPLES.

53. *1.* Multiply 783742 by 11. *Ans.* 8621162.

ANALYSIS.—Write the right-hand figure 2; for the remaining figures of the product, add 2 to 4, 4 to 7, 7 to 3, 3 to 8, 8 to 7, and write the left-hand figure, carrying when necessary.

2. Multiply the following numbers by 11: 245, 346, 325, 416, 784, 517, 875, 918, 4218, 7324, 7218, 1728, 4375, and 8376.

54. To multiply by any number of two figures ending with 1.

55. RULE.—*Multiply by the tens of the multiplier, writing the product under the multiplicand one place to the left, and add.* Or,

---

* It is suggested that these short methods be studied in connection with the more advanced work—one method with each lesson; or they may be presented to the student, one at a time, with the daily drill exercises on the fundamental rules.

Write as the first figure of the product the unit figure of the multiplicand; multiply each figure of the multiplicand by the tens of the multiplier, and at the same time, add mentally to each product the figure to the left of the one multiplied, carrying as usual.

EXAMPLES.

56. *1.* Multiply 456 by 61.

1ST OPERATION.	2ND OPERATION.
456 × 61	456
2736	61
27816	27816

ANALYSIS, 2ND METHOD.—Write 6 in the product. 6 × 6 + 5 = 41. Write 1 and carry 4. 6 × 5 + 4 (carried) + 4 = 38. Write 8 and carry 3. 6 × 4 + 3 (carried) = 27.

Multiply
2. 864 by 61; by 41.
3. 717 by 31; by 71.
4. 447 by 21; by 81.

Multiply
5. 2345 by 121; by 111.
6. 7416 by 51; by 81.
7. 8324 by 41; by 21.

NOTE.—The first method may be used with the following multipliers, by placing the products two places to the left.

Multiply
8. 375 by 301; by 401.
9. 425 by 201; by 101.
10. 469 by 601; by 501.

Multiply
11. 483 by 701; by 801.
12. 376 by 201; by 901.
13. 875 by 301; by 401.

57. To multiply by any number between 12 and 20.

58. RULE.—*Multiply by the units of the multiplier, writing the product under the multiplicand one place to the right, and add.* Or,

Multiply the units of the multiplicand by the units of the multiplier, write the units of the product, and carry the tens, if any, to the next product; multiply the remaining figures of the multiplicand by the units of the multiplier, and at the same time add mentally to each product the figure to the right of the one multiplied, carrying as usual; finally, to the left-hand figure of the multiplicand, add the number to be carried, if any, and write the result.

EXAMPLES.

ply 456 by 18.

PERATION.
456
18
208

ANALYSIS, 2ND METHOD. — 8 × 6 = 48. Write 8 and carry 4. 8 × 5 + 4 (carried) + 6 = 50. Write 0 and carry 5. 8 × 4 + 5 (carried) + 5 = 42. Write 2 and carry 4. 4 + 4 = 8.

Multiply

; by 17.
; by 16.
; by 19.
; by 16.
6. 1234 by 14 ; by 16.
7. 2345 by 16 ; by 18.
8. 3456 by 19 ; by 13.
9. 7891 by 17 ; by 15.

method may be used with the following multipliers by two places to the right.

Multiply

1 ; by 108.
2 ; by 103.
4 ; by 106.
5 ; by 109.
14. 147 by 108 ; by 101.
15. 385 by 104 ; by 107.
16. 783 by 105 ; by 103.
17. 546 by 107 ; by 106.

iply by any number ending with 0.

given multi-

multiplicand.

EXAMPLES.

63. To multiply by any multiple of 9 less than 90.

64. Rule.—*Multiply by the multiple of ten next higher than the given multiplier, and from the result subtract one-tenth of itself.*

EXAMPLES.

65. *1.* Multiply 785 by 63.

OPERATION.

785		
70		
54950	product by	70
5495	" "	7
49455		63

Analysis.—63 = 70−7. 785 × 70 = 54950. Divide 54950 by 10 by placing its digits one place to the right. 54950−5495 = 48455.

Multiply

2. 67 by 18 ; by 27.
3. 34 by 36 ; by 45.
4. 77 by 54 ; by 63.
5. 84 by 72 ; by 81.

Multiply

6. 345 by 36 ; by 45.
7. 567 by 18 ; by 72.
8. 518 by 27 ; by 63.
9. 724 by 54 ; by 81.

66. To multiply by 25.

67. Rule.—*Add two ciphers and divide the result by 4.* Or,

Divide the number by 4; if there is no remainder, add two ciphers; if there is a remainder of 1, add 25; of 2, add 50; of 3, add 75.

EXAMPLES.

68. *1.* Multiply 446 by 25.

OPERATION.

4) 44600
11150

Analysis.—Since 25 is equal to 100 divided by 4, multiplying by 100 and dividing the result by 4, is the same as multiplying by 25.

2. Multiply the following numbers by 25 :—24, 36, 37, 49, 62, 387, 448, 512, 746, 424, 817, 937, 544, 717 318, 324, 256, 556 9224, 8378, 5280, 1728, 5648.

69. **To multiply by any number one part of which is a factor of another part.**

EXAMPLES.

70. *1.* Multiply 576 by 287.

OPERATION.

576			
287			
4032	product by		7.
16128	"	"	28 (4 × 7).
530712	"	"	287.

2. Multiply 567 by 936.

OPERATION.

567			
936			
5103	product by		9.
20412	"	"	36 (4 × 9).
530712	"	"	936.

Multiply

3. 227 by 369 ; by 427.

4. 516 by 246 ; by 568.

5. 344 by 126 ; by 124.

6. 728 by 426 ; by 189.

7. 325 by 147 ; by 273.

Multiply

8. 932 by 183 ; by 927.

9. 718 by 284 ; by 832.

10. 529 by 546 ; by 756.

11. 638 by 217 ; by 618.

12. 435 by 248 ; by 428.

71. **To multiply by any number near and less than 100, 1000, etc.**

72. The Complement of a number is the difference between the number and the unit of the next higher order.

73. RULE.—*Add to the multiplicand as many ciphers as there are ciphers in the unit next higher than the multiplier, and from the result subtract the product obtained by multiplying the multiplicand by the complement of the multiplier.*

EXAMPLES.

74. *1.* Multiply 456 by 98.

OPERATION.

45600	product by		100.
912	"	"	2.
44688	"	"	98.

Multiply

2. 77 by 99 ; by 93.

3. 84 by 98 ; by 95.

4. 72 by 94 ; by 96.

Multiply

5. 387 by 93 ; by 999.

6. 416 by 95 ; by 994.

7. 528 by 93 ; by 992.

CROSS MULTIPLICATION.

75. Cross Multiplication depends upon the following rinciples:

Units	multiplied by	units	produce units
Tens	" "	units	" tens.
Units	" "	tens	
Hundreds	" "	units	" hundreds.
Tens	" "	tens	
Units	" "	hundreds	
Thousands	" "	units	" thousands.
Hundreds	" "	tens	
Tens	" "	hundreds	
Units	" "	thousands	
Ten-thousands	" "	units	" ten-thousands.
Thousands	" "	tens	
Hundreds	" "	hundreds	
Tens	" "	thousands	
Units	" "	ten-thousands	

Etc., etc.

Ex. Multiply 68 by 74. *Ans.* 5032.

OPERATION. ANALYSIS.

$$\begin{array}{r} 68 \\ \underline{74} \\ 5032 \end{array} \qquad \begin{array}{r|l} 4 \times 8 = 3 & 2 \\ 4 \times 6 + 3 \text{ (carried)} + 7 \times 8 = 8 & 3 \\ 7 \times 6 + 8 \text{ (carried)} = 5 & 0 \end{array}$$

Ex. Multiply 579 by 42. *Ans.* 24318.

OPERATION. ANALYSIS.

$$\begin{array}{r} 579 \\ \underline{42} \\ 24318 \end{array} \qquad \begin{array}{r|l} 2 \times 9 = 1 & 8 \\ 2 \times 7 + 1 \text{ (carried)} + 4 \times 9 = 5 & 1 \\ 2 \times 5 + 5 \text{ (carried)} + 4 \times 7 = 4 & 3 \\ 4 \times 5 + 4 \text{ (carried)} = 2 & 4 \end{array}$$

Ex. Multiply 567 by 348. *Ans.* 197316.

PERATION. ANALYSIS.

$$\begin{array}{r} 567 \\ \underline{348} \\ 197316 \end{array} \quad \begin{array}{ccccc} & & 8\times5=40 & 8\times6=48 & 8\times7=56 \\ & 4\times5=20 & 4\times6=24 & 4\times7=28 & \\ 3\times5=15 & 3\times6=18 & 3\times7=21 & & \\ \hline 19 & 7 & 3 & 1 & 6 \end{array}$$

76. To multiply together numbers of two figures each, whose units are alike.

Ex. Multiply 76 by 46. *Ans.* 3496.

OPERATION.

$$\begin{array}{r} 76 \\ 46 \\ \hline 3496 \end{array}$$

ANALYSIS.

$$\begin{array}{rr|l} & 6\times 6 = 3 & 6 \\ \left.\begin{array}{l} 6\times 7 \\ 6\times 4 \end{array}\right\} & 6\times 11+3\text{ (carried)} = 6 & 9 \\ & 4\times \ 7+6\text{ (carried)} = 3 & 4 \end{array}$$

Ex. Multiply 135 by 65. *Ans.* 8775.

OPERATION.

$$\begin{array}{r} 135 \\ 65 \\ \hline 8775 \end{array}$$

ANALYSIS.

$$\begin{array}{rr|l} & 5\times 5 = 2 & 5 \\ \left.\begin{array}{l} 5\times 13 \\ 5\times \ \ 6 \end{array}\right\} & \overline{5\times 19}+2\text{ (carried)} = 9 & 7 \\ & \overline{6\times 13}+9\text{ (carried)} = 8 & 7 \end{array}$$

77. RULE.—*Multiply units by units for the first figure of the product, the sum of the tens by units for the second figure, and tens by tens for the third figure, carrying when necessary.*

EXAMPLES.

78. Multiply

1. 56 by 56 ; 72 by 32 ; 94 by 44.
2. 65 by 75 ; 87 by 37 ; 46 by 36.
3. 99 by 49 ; 85 by 75 ; 34 by 24.
4. 47 by 37 ; 67 by 57 ; 85 by 45.
5. 125 by 65 ; 126 by 36 ; 154 by 84.
6. 76 by 76 ; 36 by 36 ; 114 by 114.

79. To multiply together numbers of two figures each, whose tens are alike.

Ex. Multiply 87 by 85. *Ans.* 7395.

OPERATION.

$$\begin{array}{r} 87 \\ 85 \\ \hline 7395 \end{array}$$

ANALYSIS.

$$\begin{array}{rr|l} & 5\times 7 = 3 & 5 \\ \left.\begin{array}{l} 8\times 5 \\ 8\times 7 \end{array}\right\} & 8\times 12+3 = 9 & 9 \\ & 8\times \ 8+9 = 7 & 3 \end{array}$$

Ex. Multiply 127 by 122. *Ans.* 15494.

OPERATION.

$$\begin{array}{r} 127 \\ \underline{122} \\ 15494 \end{array}$$

ANALYSIS.

$$\begin{array}{rrcl|l} & & 2 \times 7 = & 1 & 4 \\ \begin{array}{r} 12 \times 2 \\ 12 \times 7 \end{array} \Big\} & 12 \times & 9 + 1 = & 10 & 9 \\ & 12 \times & 12 + 10 = & 15 & 4 \end{array}$$

80. RULE.—*Multiply units by units for the first figure of the product, the sum of the units by tens for the second figure, and tens by tens for the remaining figures, carrying when necessary.*

EXAMPLES.

81. Multiply

1. 87 by 82 ; 81 by 87 ; 65 by 63.
2. 47 by 44 ; 56 by 52 ; 58 by 57.
3. 73 by 76 ; 79 by 75 ; 68 by 63.
4. 44 by 43 ; 52 by 55 ; 67 by 63.
5. 116 by 117 ; 107 by 105 ; 125 by 122.

82. To multiply together two numbers whose tens are alike, and the sum of whose units is ten.

83. RULE.—*Multiply the units together for the two right-hand figures of the product, one of the tens by 1 more than itself for the remaining figures.*

EXAMPLES.

84. *1.* Multiply 76 by 74. *Ans.* 5624.

ANALYSIS.—6 × 4 = 24, the two right-hand figures of the product. 6 × 7 (6 + 1) = 42, the remaining figures.

Multiply mentally

2. 24 by 26 ; 85 by 85 ; 128 by 122.
3. 17 by 13 ; 94 by 96 ; 112 by 118.
4. 34 by 36 ; 37 by 33 ; 104 by 106.
5. 25 by 25 ; 43 by 47 ; 143 by 147.
6. 35 by 35 ; 56 by 54 ; 152 by 158.

85. To multiply by means of complements (72).

Ex. Multiply 991 by 996.

OPERATION.

991..9
996..4
987036

ALGEBRAIC MULTIPLICATION.

$$\begin{array}{l} \left.\begin{array}{l} 991 = 1000 - 9 \\ 996 = 1000 - 4 \end{array}\right\} \text{sum} = 2000 - 13 \\ \hline (1000 \times 1000) - (9 \times 1000) \\ \qquad\qquad\qquad\quad - (4 \times 1000) + 36 \\ \hline (1000 - 13) \times 1000 \qquad\quad + 36 \end{array}$$

Analysis.—From the above algebraic multiplication, it is observed: 1st, that as many of the right-hand figures as there are ciphers in the unit of comparison may be obtained by multiplying the complements together; 2nd, that the second part of the result is equivalent to the sum of the numbers less the unit of comparison multiplied by that unit.

The sum of the numbers less the unit of comparison may be obtained by adding the numbers and omitting the 1 at the left-hand, or by subtracting either complement from the opposite number. Thus, 991 − 4 = 987.

86. Rule.—*From either number subtract the complement of the other, and to the right of the remainder write the product of the complements.*

Notes.—1. When there are less figures in the product of the complements than ciphers in the unit of comparison, write ciphers in the result to supply the deficiency.

2. When there are more figures in the product of the complements than ciphers in the unit of comparison, add the excess on the left-hand to the second part of the result.

3. After practice, the complements may be omitted in the operation.

EXAMPLES.

87. *1.* Multiply 88 by 95; 975 by 993; 9999 by 9999.

(*a.*)	(*b.*)	(*c.*)
88..12	775..225	9999...1
95...5	993....7	9999...1
8360	769575	99980001

Multiply

2. 97 by 99; by 94.
3. 88 by 91; by 95.
4. 89 by 93; by 96.
5. 75 by 97; by 98.
6. 92 by 98; by 93.
7. 86 by 94; by 95.

Multiply

8. 993 by 992; by 994.
9. 990 by 991; by 988.
10. 982 by 994; by 995.
11. 925 by 996; by 994.
12. 875 by 992; by 993.
13. 847 by 990; by 988.

88. To multiply together two numbers of the same number of figures over and near 100, 1000, etc.

Ex. Multiply 116 by 103.

OPERATION.

$$\begin{array}{r} \cancel{1}16 \\ 103 \\ \hline 11948 \end{array}$$

ALGEBRAIC MULTIPLICATION.

$$\begin{array}{l} \left.\begin{array}{r} 116 = 100 + 16 \\ 103 = 100 + 3 \end{array}\right\} \text{sum} = 200 + 19 \\ \hline 100 \times 100 + 16 \times 100 \\ \qquad\qquad\quad + 3 \times 100 + 48 \\ \hline (100 + 19) \times 100 \qquad\quad + 48 \end{array}$$

89. RULE.—*From the sum of the numbers subtract the unit of comparison, and to the right of the result write the product of the excesses.* (See Notes to Art. **86.**)

EXAMPLES.

90. Multiply

1. 112 by 106 ; by 111.

2. 102 by 103 ; by 104.

3. 122 by 108 ; by 105.

4. 116 by 107 ; by 112.

Multiply

5. 145 by 107 ; by 112.

6. 176 by 111 ; by 108.

7. 1004 by 1006 ; by 1007.

8. 1125 by 1008 ; by 1012.

91. To multiply together two numbers, one of which is more and the other less than 100, 1000, etc.

Ex. Multiply 109 by 97.

OPERATION.

$$\begin{array}{rl} \cancel{1}09 & 9 \text{ excess.} \\ 97 & 3 \text{ complement.} \\ \hline 10600 & \\ 27 & \left\{\begin{array}{l} \text{Product of excess} \\ \text{and complement.} \end{array}\right. \\ \hline 10573 & \end{array}$$

ALGEBRAIC MULTIPLICATION.

$$\begin{array}{l} \left.\begin{array}{r} 109 = 100 + 9 \\ 97 = 100 - 3 \end{array}\right\} 200 + 6 \\ \hline 100 \times 100 + 9 \times 100 \\ \qquad\qquad\quad - 3 \times 100 - 27 \\ \hline (100 + 6) \times 100 \qquad\quad - 27 \end{array}$$

92. RULE.—*Multiply the sum of the numbers less the unit of comparison by that unit, and from the product subtract the product of the excess and complement.*

EXAMPLES.

93. Multiply

1. 107 by 97 ; by 95.

2. 112 by 96 ; by 92

3. 116 by 94 ; by 98.

4. 108 by 91 ; by 99.

Multiply

5. 1005 by 91 ; by 93.

6. 1007 by 95 ; by 97.

7. 1012 by 99 ; by 92.

8. 1018 by 94 ; by 96.

DIVISION.

---

94. Division is the operation of finding how many times one number is contained in another.

The number divided is called the *dividend.* The number by which it is divided is called the *divisor.* The result obtained is called the *quotient.* The part of the dividend which remains after the operation is completed is called the *remainder.*

95. The sign of division is $\div$ and is read *divided by.*

Thus, $16 \div 2 = 8$ is read, sixteen divided by two equals eight.

96. To divide when the divisor does not exceed 12.

NOTE.—When the work is performed mentally, as in the following operation, the process is called *Short Division.*

Ex. Divide 1859 by 4.

OPERATION.

$$4\,\underline{)\,1859}$$
$$464\tfrac{3}{4}$$

ANALYSIS.—Write the divisor at the left of the dividend, as in the operation, and begin to divide at the left. 4 is not contained in 1 thousand, the highest order of the dividend, therefore, divide 18 hundreds by 4. 4 is contained in 18 (hundreds), 4 (hundred) times, and 2 hundreds remain. Write the 9 hundred under the line in hundreds' place, and reduce the 2 hundreds remaining to tens, making 20 tens, which added to the 5 tens of the dividend, make 25 tens. 4 is contained in 25 (tens), 6 (tens) times and 1 ten remains. Write the 6 tens under the line in tens' place, and reduce the 1 ten remaining to units, making 10 units, which added to the 9 units of the dividend, make 19 units. 4 is contained in 19 (units), 4 (units) times, and 3 units remain. Write the 4 units in units' place, and write the remainder over the divisor, with a line between them in the form of a fraction, thus, $\frac{3}{4}$ (three-fourths). The complete result is $464\frac{3}{4}$.

Observe that each quotient figure is placed directly under the last figure of the dividend used.

In practice, do not think of explanations, etc.; but, think only of the partial dividends and quotient figures. Thus, in the above example, say or think, 4 into 18 4 times, into 25 6 times, into 19 4 times, etc.

97. RULE.—*Write the divisor at the left of the dividend with a curved line between them.*

Beginning at the left, divide each figure of the dividend by the divisor, and place the quotient beneath the figure divided. Whenever a remainder occurs, prefix it to the following figure of the dividend, and divide as before.

Continue the operation until all the figures of the dividend have been divided, and place the remainder, if any, over the divisor at the right of the quotient.

98. PROOF.—*Multiply the quotient by the divisor, and to the product add the remainder. If the result equals the dividend, the work is probably correct.*

EXAMPLES.

99. Divide

1. 78912348 by 2; by 3.
2. 97652464 by 4; by 6.
3. 16327620 by 5; by 6.
4. 78070804 by 4; by 7.
5. 12345678 by 6; by 9.
6. 988054320 by 5; by 8.
7. 234568836 by 4; by 9.
8. 357212254 by 2; by 7.
9. 246886425 by 5; by 9.
10. 217181916 by 7; by 9.

Divide

11. 103050709 by 2; by 5.
12. 214161810 by 3; by 6.
13. 425262728 by 4; by 7.
14. 123456789 by 3; by 8.
15. 246801234 by 6; by 11.
16. 789123650 by 7; by 10.
17. 287236450 by 5; by 12.
18. 176111888 by 6; by 11.
19. 1010101010 by 7; by 9.
20. 200200200 by 8; by 12.

21. In one square yard there are 9 square feet. How many square yards in 41652 square feet?

22. There are 12 pence in one shilling. How many shillings in 124656 pence?

23. In a barrel containing 1068 eggs, how many dozen? What is their value at 23 cents per dozen?

24. In one foot there are 12 inches. How many feet in 63360 inches?

25. There are 2 pints in one quart, and 4 quarts in one gallon. How many gallons in 160048 pints?

26. There are 8 quarts in one peck, and 4 pecks in one bushel. How many bushels in 349056 quarts?

100. To divide by any divisor greater than 12.

NOTE.—When the work is all written, as in the following operation, the process is called *Long Division.*

Ex. Divide 13218 by 43.

OPERATION.
Divisor. Dividend. Quotient.
43) 13218 (307$\frac{17}{43}$
129
$\overline{318}$
301
$\overline{17}$ Remainder.

ANALYSIS.—Since 43 is not contained in 13 (thousands), we take 132 (hundreds) for the first partial dividend. 43 is contained in 132 (hundreds), 3 (hundred) times. 43 × 3 (hundreds) = 129 (hundreds), which write under the 132 (hundreds), and subtract. The remainder is 3 (hundreds), to which annex the 1 (ten) of the dividend, and the second partial dividend is 31 (tens). 43 is not contained in 31 (tens), therefore write 0 as the next figure of the quotient. Annex to the partial dividend, 31 (tens), the 8 (units) of the dividend, and the next partial dividend is 318 (units). 43 is contained in 318 (units), 7 (units) times. 43 × 7 (units) = 301 (units), which write under the 318 (units) and subtract. The remainder is 17 (units). Indicate the division of this remainder in the form of a fraction, thus: $\frac{17}{43}$, and annex it to the quotient, producing 307$\frac{17}{43}$ for the complete quotient.

101. RULE.—*Write the divisor at the left of the dividend, with a curved line between them.*

Take for the first partial dividend the least number of figures on the left that will contain the divisor, and write the quotient figures at the right.

Multiply the divisor by the quotient, write the product under the partial dividend, and subtract. To the remainder, annex the next figure of the dividend, for the second partial dividend.

Divide as before, and thus continue until all the figures of the dividend have been used.

Write the remainder, if any, over the divisor in the form of a fraction, and annex it to the quotient. The result will be the complete quotient.

102. PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder. If the sum equals the dividend, the work is probably correct.*

EXAMPLES.

103. Divide

1. 307845 by 26; by 143.

2. 248916 by 19; by 249.

3. 375428 by 38; by 375.

4. 481369 by 48; by 116.

5. 423706 by 25; by 208.

6. 3064028 by 18; by 429.

7. 1289434 by 64; by 567.

8. 7090805 by 73; by 432.

9. 6321457 by 87; by 618.

10. 2304802 by 92; by 729.

Divide

11. 8712460 by 73; by 817.

12. 1428716 by 84; by 365.

13. 2893429 by 69; by 144.

14. 7364128 by 14; by 128.

15. 2125639 by 70; by 320.

16. 3756425 by 64; by 231.

17. 4183691 by 36; by 365.

18. 3804072 by 96; by 729.

19. 1653891 by 33; by 640.

20. 2763940 by 95; by 160.

21. How many days in 8766 hours?

22. In 20000 pens, how many gross? (1 gross = 144.)

23. How many bushels in 21674 pounds of oats, if there are 32 pounds in one bushel?

24. There are 56 pounds in a bushel of rye. How many bushels in 19958 pounds?

25. There were 31392893 gallons of molasses imported into the United States in 1885. How manys hogsheads of 63 gallons each?

26. How many cords in 47164 cubic feet, if there are 128 cubic feet in one cord?

27. How many miles in 49164 rods, if there are 320 rods in one mile?

28. The expenditures of the United States for the year 1886 were $287,034,182. How much was that per day (365 days in the year)?

29. During the year 1882, 788992 immigrants arrived in the United States. What was the average number per day?

30. The population of the 38 States was 49,371,340 in 1880, and there are 325 members in the House of Representatives. What is the average population to each member?

31. The exports of cotton during the year 1885 were 1,889,514,368 pounds. How many bales averaging 476 pounds each?

32. How many rails 18 feet in length would be required for a railroad 51 miles long? (1 mile = 5280 feet.)

SHORT METHODS OF DIVISION.

104. Leaving out the Products.—In long division the process may be shortened by the following rule:

105. RULE.—*Subtract the several products from the next number greater ending with the corresponding figure in the dividend, and carry each time the left-hand figure of the minuend to the next product.*

NOTE.—If the right-hand figure of any product is the same as the corresponding figure of the dividend, subtract it from itself, and not from the next higher number ending with the same figure; or, write 0 in the remainder, carrying the left-hand figure of the product.

Ex. Divide 42343014 by 973.

```
42343014 | 973
 3423    |------
  5040   | 43518
   1751
    7784
     000
```

ANALYSIS.—The first quotient figure is 4, by which we multiply. 4 times 3 are 12, which subtracted from 14 (the next number greater ending with 4) leaves 2. Write 2 in the remainder and carry 1. 4 times 7 are 28, 1 carried makes 29, which subtracted from 33 (the next number greater ending with 3) leaves 4. Write 4 in the remainder and carry 3. 4 times 9 are 36, 3 carried makes 39, which subtracted from 42 leaves 3. Write 3 in the remainder and carry 4. 4 subtracted from 4 leaves 0. Bring down 3, the next figure of the dividend. So proceed until the division is finished.

NOTE.—Perform any of the examples in Art. **103** by this method.

106. To divide by 25.

107. RULE.—*Multiply the dividend by 4, and divide the product by 100 by cutting off two figures from the right.*

NOTE.—To divide by 125, multiply by 8 and divide the product by 1000 by cutting off three figures from the right.

Ex. Divide 11175 by 25.

OPERATION.

```
 11175
     4
------
447.00
```

ANALYSIS.—Since 25 is one-fourth of 100, multiplying by 4 and dividing by 100, is the same as dividing by 25.

EXAMPLES.

108. *1.* Divide the following numbers by 25: 1175, 1650, 1700, 2875, 3825, 4950, 3800, 1725, 1775, 1825, and 2000.

UNITED STATES MONEY.

109. United States Money is the legal currency of the United States. It consists of gold, silver, nickel, and copper coins, treasury and national bank notes, gold and silver certificates.

110. Legal Tender.—The term legal tender is applied to money which may be legally offered in the payment of debts.

111. The unit of value is the gold dollar of 25.8 grains.

TABLE.

10 Mills	=	1 Cent	*c., ct.*
10 Cents	=	1 Dime	*d.*
10 Dimes	=	1 Dollar	$
10 Dollars	=	1 Eagle	E.

NOTES.—1. In business operations, dollars and cents are principally used. Eagles and dimes are used only as the names of coins.

2. The currency of the Canadian Provinces is nominally the same as that of the United States.

112. The legal coins of the United States are as follows:

GOLD.

	Weight in grains.
1 dollar piece,	25.8
2½ dollar piece, or Quarter-eagle,	64.5
3 dollar piece,	77.4
5 dollar piece, or Half-eagle,	129.
10 dollar piece, or Eagle,	258.
20 dollar piece, or Double-eagle.	516.

SILVER.

	Weight.
Standard dollar,	412½ grains.
Half dollar, or 50 cent piece,	12½ grams, or 192.9 grains.
Quarter dollar, or 25 cent piece,	6¼ grams, or 96.45 grains.
Dime, or 10 cent piece;	2½ grams, or 38.58 grains.

COPPER AND NICKEL.

5 cent piece,	5 grams, or 77.16 grains.
3 cent piece,	30 grains.
1 cent piece,	48 grains.

113. The gold and silver coins of the United States contain 9 parts by weight of pure metal and 1 part alloy. The alloy of silver coins is copper; and of gold coins, copper, or copper and silver. (The silver in no case exceeds $\frac{1}{10}$ of the whole alloy.)

Coins are a *legal tender* in all payments at their
ations not below the standard weight* provided b
reduced in weight, below said standard, are
valuation in proportion to their actual weight.

ndard Silver Dollars are a *legal tender* at thei
for all debts except where otherwise express
he contract.

er Certificates.—Any holder of *standard silv*
posit the same with the Treasurer, or any Assistan
he United States, in sums not less than $10, and
e certificates, corresponding with the denomination
tes notes (**119**). These certificates are receivabl
axes, and all public dues.

sidiary Coins.—The present silver coins of the
of smaller denominations than $1 are a legal tender
t exceeding $10.

of any of the silver coins of the United States of smalle
an $1 may, on presentation of the same in sums of $20, o
reof, at the office of the Treasurer or any Assistant Treasur
ates, receive therefor lawful money of the United States.

cent pieces contain ¾
piece contains 95 per cent.
These coins are a legal
twenty-five cents.

States **Notes** ("Greenbacks") are a legal
ept duties on imports and interest on the

1879, they have been redeemable in coin at the office of the
ts of the United States in the Cities of New York and San
of not less than $50. They represent the values of $1, $2,
$100, $500, $1000, $5000, $10,000. The Act of May 31,
unt of $346,681,016, and forbade their further contraction.

---

of the United States, if reduced in weight by natural abrasion not
below the standard weight prescribed by law, after a

120. National Bank Notes (640) are not a *legal tender;* but, since they are secured by bonds of the United States deposited with the U. S. Treasurer at Washington, and are redeemed in lawful money by the national banks and the Treasurer of the United States, they are usually accepted in the payment of debts in any part of the United States.

They are receivable in all parts of the United States in payment of taxes and other dues to the United States except duties on imports, and for debts owing by the United States to individuals and corporations, within the United States except interest on the public debt.

They represent the values of $1, $2, $5, $10, $20, $50, $100, $500, and $1000. Since Jan. 1, 1879, no notes of the denomination of $1 and $2 have been issued to national banks. Nov. 1, 1886, their total circulation was $301,529,889.

121. To write United States money.

122. In writing U. S. money, the decimal notation is used. *Dollars* are written at the left of the decimal point and form the integral part. *Cents* are written as *hundredths* of a dollar, and occupy the first two places at the right of the decimal point. *Mills* are written as *thousandths* of a dollar, and occupy the third decimal place.

Thus, twelve dollars, forty-eight cents, and six mills, is written $12.486.

When the number of cents is less than ten, a cipher must be written in the first place at the right of the point. Thus, eight dollars and six cents is written, $8.06.

In the final results of business operations, if the *mills* are more than *five,* they are regarded as an additional cent; if less than *five,* they are rejected.

In checks, notes, drafts, etc., *cents* are usually written as hundredths of a dollar in the form of a *fraction.* Thus, twenty-five cents may be written, $\$\frac{25}{100}$.

123. Express the following amounts by figures:

1. Eighty-six dollars, nineteen cents, five mills.
2. Fourteen dollars, seventy-five cents, three mills.
3. Five hundred twenty-six dollars, seventy cents.
4. Two thousand dollars, thirty cents, two mills.
5. Seven hundred dollars, nine cents.
6. Fifty thousand dollars, seven mills.
7. Four hundred eight dollars, two cents, five mills.
8. Two hundred fifty dollars, sixty cents, three mills.

[illegible]

United States mon

EXAMPLES.

132. *1.* Add 5 dollars, 16 cents; 18 dollars, 5 cents; 404 dollars, 75 cents; 25 dollars, 8 cents; 2376 dollars, 40 cents; 8 dollars, 2 cents.

2. Add $170, $106.40, $240, $200.40, $70, $.70, $234.75.

3. Add $108.25, $2345, $6.04, $7.10, $192.43, $117.05.

4. Add $.06, $6, $108.16, $500.64, $564, $5.64, $117.10, $2081.48.

5. From $124.16 subtract $109.25.

6. From $117 subtract $98.49.

7. From $575 subtract 575 cents.

8. A merchant makes the following deposits in a bank: $1875.24, $416, $234.70, $558.96, and $437.10. He draws the following checks: $442.37, $120.92, $316.75, $242.71, $195, $716.32, $100.48, and $76.19. What is the balance of his bank account?

133. To multiply United States money.

Ex. Find the cost of 9 desks at $2.45 each.

OPERATION.

$$\begin{array}{r} 2.45 \\ \underline{9} \\ 22.05 \end{array}$$

ANALYSIS.—Disregarding the decimal point, multiply as in ordinary multiplication. 9 times 245 cents is 2205 cents = $22.05.

134. RULE.—*Multiply as in simple numbers, and from the right of the product point off as many figures as there are figures to the right of the decimal points in both numbers.*

NOTE.—If, as in Examples 6 and 7, the number expressing cents would make a convenient multiplier, use it as such, considered as an abstract number, and point off the result according to the rule.

135. Multiply

1. 12 dollars and 18 cents by 8.

2. 4 dollars and 25 cents by 12.

3. 16 dollars and 9 cents by 17.

4. 27 dollars and 8 cents by 25.

5. 43 dollars and 50 cents by 76.

6. 8 cents by 2345; by 3456.

Multiply

7. $.07 by 1239; by 13416

8. $20.04 by 20; by 108.

9. $176 by 18; by 144.

10. $36.25 by 36; by 117.

11. $48.19 by 48; by 288.

12. $50.08 by 75; by 192.

[illegible] United States money.

[illegible] what is one worth?

[illegible] worth 225 cents [illegible]

[illegible] cents or 243 cents [illegible]

[illegible] what is one worth?

[illegible] consists of dollars [illegible]

[illegible] number of times [illegible]

[illegible] can be bought for [illegible]

[illegible] $6.25 [illegible]

[illegible] $6.25 [illegible]

[illegible] $6.25 = [illegible]

[illegible] times. Hence th[illegible]

[illegible] division.

[illegible]

REVIEW EXAMPLES.

139. *1.* Find the sum of the following numbers: Twenty-six thousand forty-eight; twelve thousand four hundred eighty; one hundred thirty-six thousand; seven hundred ninety thousand forty-three; four million fifty-eight.

2. Subtract eight hundred fourteen thousand nine hundred sixteen from four million nineteen thousand.

3. Multiply five hundred sixty thousand seven hundred eight by eighteen hundred sixty.

4. A quantity of merchandise was bought for $27618.75, and sold for $32418.25. What was the gain?

5. Find the total length of the Brooklyn bridge, its measurements being as follows: Length of river span, 1596 feet; of each (2) land span, 930 feet; of New York approach, 1562 feet; of Brooklyn approach, 971 feet.

6. If I sell goods for $23876, and gain $5389, what did the goods cost me?

7. The estimated production of gold and silver of the world for 1884 was as follows: Gold, $98,990,772; silver, $116,525,949. For 1885, gold, $101,562,748; silver, $124,968,784. What was the total increase?

8. If the quotient is 375 and the divisor 246, what is the dividend?

9. If the product of two numbers is 450072, and one of the numbers is 987, what is the other number?

10. Divide 76432801 by 783. Prove that your solution is correct.

11. A clerk receiving a salary of $1256, pays $468 a year for board, $180 for clothing, and $150 for other expenses. What amount has he left?

12. If I take 24889 from the sum of 9872 and 24967, divide the remainder by 50, and multiply the quotient by 18, what is the product?

13. If 160 acres of land cost $10720, how many acres can be bought for $8844?

14. If 75 head of cattle cost $2550, what will 59 head cost?

15. Cash on hand at beginning of the day, $6492.75; cash received, $11456.75; cash paid out, $13285.26. Required the cash balance at the end of the day.

16. A merchant sold 426 barrels of flour for $2556, which was $639 more than he gave for it. What did it cost him a barrel ?

17. Mr. A has three farms, the first of which contains 158 acres, the second 32 acres less than the first, and the third as many as the other two. What is the value per acre, if all are worth $26128 ?

18. A merchant bought 387 yards of cloth at 79 cts. per yard ; he sold 298 yards at $1.16 per yard, and the remainder at 97 cts. per yard ; how much did he gain ?

19. The United States nickel and copper coinage for the year 1886 was 5,519 five-cent pieces, 4,519 three-cent pieces, and 1,696,613 one-cent pieces. Find total value of minor coinage.

20. The silver coinage for 1886 was as follows: 29,838,905 dollars, 6,105 half-dollars, 14,505 quarter-dollars, 1,767,642 dimes. What was the total value of the silver coinage ?

21. The gold coinage for 1886 was as follows : 243,584 double-eagles, 1,042,847 eagles, 3,751,629 half-eagles, 101 three-dollar pieces, 4,086 quarter-eagles, 8,567 one-dollar pieces. What was the value of the gold coinage ?

22. There are four bidders to supply the government with 800 tons Lehigh, 500 tons Cumberland, and 700 tons Baltimore coal. A offers Lehigh at $6.29, Cumberland at $4.38, and Baltimore at $7.23. B offers Lehigh at $6.80, Cumberland at $4.12, and Baltimore at $7.24. C offers Lehigh at $6.40, Cumberland at $4.45, and Baltimore at $7.18. D offers Lehigh at $6.17, Cumberland at $4.19, Baltimore at $7.20. Who is the lowest bidder for the whole amount, and how much does each bid amount to ?

23. A drover bought a number of cattle for $12204, and sold the same for $13560, by which he gained $4 per head. How many cattle were purchased ?

24. A farmer raised in one year 512 bushels of wheat, the next year twice as much as he raised the first year, and the third year four times as much as he did the second year. What was the value of the three crops at $1.65 per bushel ?

25. Bought 75 tons of hay at $16 per ton ; gave in payment 56 sheep at $3.75 each, and the remainder I paid in butter at 3 cts. per pound. How many pounds of butter were required ?

26. Bought 225 acres of land for $12,600, and sold 116 acres at $65 per acre, and the remainder at cost ; how much did I gain ?

27. A sold to B 175 acres of land at $135 an acre, and by so doing gained $1925; B sold the land at a loss of $1750. What did A pay per acre, and what was B's selling-price per acre?

28. A merchant sold 800 barrels of flour for $5867, 144 barrels of which he sold at $7 per barrel, and 225 barrels at $6.75. At how much per barrel did he sell the remainder?

29. According to the following table, what was the average immigration per year? What per month?

Years.	Number.	Years.	Number.	Years.	Number.
1875......	227,498	1879......	177,826	1883......	603,322
1876......	169,986	1880......	457,257	1884......	518,592
1877......	141,857	1881......	669,431	1885......	395,346
1878......	138,469	1882......	788,992	1886......	334,203

PROPERTIES OF NUMBERS.

140. A **Number** is a unit, or a collection of units; as *one, four, three* feet, *five* dollars.

141. All numbers are either *integral* or *fractional, abstract* or *concrete.*

142. An **Integral Number,** or **Integer** is a number which expresses whole things; as *two, four* gallons, *seven* dollars.

143. A **Fractional Number,** or **Fraction** is a number which expresses one or more equal parts of a unit; as *one-half, three-fourths.*

144. An **Abstract Number** is a number which does not refer to any particular object; as *one, six, ten.*

145. A **Concrete Number** is a number applied to an object, or quantity; as *three* apples, *five* pounds, *ten* dollars.

146. Integral numbers are either *odd* or *even, prime* or *composite.*

147. An **Odd Number** is a number whose unit figure is 1, 3, 5, 7, or 9; as 7, 21, 39.

148. An **Even Number** is a number whose unit figure is 0, 2, 4, 6, or 8; as 6, 40, 74.

149. A **Prime Number** is a number which can be exactly divided only by itself and unity; as 1, 7, 13, 29.

150. Numbers are **prime to each other** when no integral number greater than 1 will divide each without a remainder.

Numbers that are prime to each other are not necessarily prime numbers. Thus, 25 and 28 are prime to each other, but they are not prime numbers.

151. A **Composite Number** is a number which can be exactly divided by other integers besides itself and unity.

Thus 28, the product of 4 and 7, is a composite number. It is exactly divisible by 4 and 7.

DIVISIBILITY OF NUMBERS.

152. An **Exact Divisor** of a number is any number that will divide it without a remainder.

Thus 2, 3, 4, 6, 8, and 12 are exact divisors of 24.

153. A number is said to be *divisible* by another when the latter will divide the former without a remainder. Any number is divisible

1. By 2, if it is an even number; as 6, 28, and 32.

2. By 3, if the sum of its digits is divisible by 3; as 849 $(8+4+9=21$, 21 is divisible by 3), 7323, and 47892.

3. By 4, if the two right-hand figures are ciphers, or express a number divisible by 4; as 1100, 216, and 7328.

4. By 5, if the right-hand figure is 0 or 5; as 40 and 135.

5. By 6, if it is an even number and the sum of its digits is divisible by 3; as 216, 840, and 732.

6. By 8, if the three right-hand figures are ciphers, or express a number divisible by 8; as 3000 and 7168.

7. By 9, if the sum of its digits is divisible by 9; as 216, 783, and 12348.

PRIME FACTORS.

154. The **Factors** of a number are those numbers which when multiplied together will produce the number.

Thus 4 and 7; 2 and 14; 2, 2, and 7 are factors of 28. The number itself and unity are not regarded as factors.

The *factors* of a number are also the *exact divisors* of it.

155. A **Prime Factor** is a prime number used as a factor.

Thus, 2, 2, and 7 are the prime factors of 28. 4 is a factor of 28, but not a *prime* factor.

156. To find all the prime factors of a composite number.

Ex. What are the prime factors of 6930.

OPERATION.

$$\begin{array}{r|l} 2 & 6930 \\ \hline 3 & 3465 \\ \hline 3 & 1155 \\ \hline 5 & 385 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

ANALYSIS.—Any prime number that is an exact divisor of the given number is a prime factor of it. Divide the given number by 2 (**153**, 1), the least prime divisor of it, obtaining the quotient 3465. Next, divide this quotient successively by 3 (**153**, 2), 3, 5 (**153**, 4), and 7. The last quotient 11 is a prime number and therefore a prime factor. The several divisors 2, 3, 3, 5, 7 and the last quotient 11 are the prime factors required.

$$2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930.$$

157. RULE.—*Divide by the least prime number which will divide the given number without a remainder. In like manner divide the resulting quotient, and continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

EXAMPLES.

158. Resolve the following numbers into their prime factors :

1. 3465.
2. 3003.
3. 4158.
4. 3150.
5. 3675.
6. 2310.
7. 6552.
8. 7826.
9. 6006.
10. 5368.
11. 3825.
12. 5324.
13. 8192.
14. 6561.
15. 3125.
16. 1800.
17. 1935.
18. 2475.
19. 6630.
20. 2448.
21. 8525.
22. 9936.
23. 9576.
24. 5075.

COMMON MULTIPLES.*

159. A **Multiple** of a number is a number that is exactly divisible by it; or, it is any product of which the given number is a factor.

Thus, 12 is a multiple of 6; 15 of 5; etc.

160. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of them.

Thus, 12, 24, 36, and 48 are multiples of 4 and 6.

161. The **Least Common Multiple** of two or more numbers is the *least* number that is exactly divisible by each of them.

Thus, 12 is the least common multiple of 4 and 6.

162. PRINCIPLES.—1. *A multiple of a number contains all the prime factors of that number.*

2. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.*

3. *The least common multiple of two or more numbers contains all the prime factors of each of the numbers, and no other factors.*

163. To find the least common multiple of two or more numbers.

Ex. What is the least common multiple of 12, 18, 20, and 40?

FIRST OPERATION.

$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$40 = 2 \times 2 \times 2 \times 5$$
$$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

ANALYSIS.—Since 40, a multiple of 20, contains all the prime factors of 20, the number 20 may be omitted in the operation. Resolve the numbers into their prime factors. The least common multiple must contain 2 as a factor 3 times in order to be divisible by 40; it must contain 3 as a factor twice in order to be divisible by 18; and it must contain 5 as a factor, in order to be divisible by 40. 360, the product of the factors, 2, 2, 2, 3, 3, and 5, is the least common multiple of the given numbers, since it contains the different factors the greatest number of times that they occur in the given numbers, and no other factors (*Prin.* 3).

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* For Greatest Common Divisor, see Appendix, page 317.

SECOND OPERATION.

$$\begin{array}{r|lll} 2 & 12, & 18, & 40 \\ \hline 2 & 6, & 9, & 20 \\ \hline 3 & 3, & 9, & 10 \\ \hline & 1, & 3, & 10 \end{array}$$

$$2 \times 2 \times 3 \times 3 \times 10 = 360$$

ANALYSIS.—The factors of the required multiple may be found by the following process. Divide the given numbers by any prime number that will divide two or more of them, writing the quotients and the undivided numbers beneath. Treat the resulting numbers in like manner, and continue the process until no two of the numbers have a common factor or divisor. The product of the several divisors and the remaining quotients and undivided numbers will be the least common multiple.

164. RULE.—*Resolve the given numbers into their prime factors. The product of the different prime factors, taking each factor the greatest number of times it appears in any of the numbers, will be the least common multiple.* Or,

Divide the given numbers by any prime number (see Note 2) *that will exactly divide two or more of them, writing the quotients and undivided numbers beneath. Repeat the operation with the resulting numbers until there is no exact divisor of any two of them. The product of the several divisors and the last quotients and undivided numbers will be the least common multiple.*

NOTES.—1. In the operation, reject such of the smaller numbers as are divisors of the larger; also reject such of the quotients and undivided numbers as are divisors of the others.

2. Divide by composite numbers when they are exact divisors of all the numbers.

EXAMPLES.

165. Find the least common multiple of the following numbers.

1. 6, 10, 15, and 30.
2. 16, 24, and 48.
3. 30, 40, and 60.
4. 2, 4, 8, and 16.
5. 14, 21, and 28.
6. 5, 8, 15, and 18.
7. 6, 9, 21, and 24.
8. 12, 20, and 30.
9. 6, 10, 30, and 40.
10. 32, 48, and 60.
11. 24, 32, and 40.
12. 24, 36, and 40.
13. 32, 48, and 72.
14. 16, 22, 24, and 30.
15. 18, 28, 30, and 36.
16. 12, 16, 20, and 24.
17. 33, 44, 55, and 66.
18. 27, 36, 42, and 48.
19. 36, 45, 60, and 72.
20. 28, 35, 42, and 56.
21. 45, 55, 60, and 75.
22. 60, 72, 84, and 90.

CANCELLATION.

166. Cancellation is a method of shortening an operation by rejecting equal factors from both dividend and divisor.

167. PRINCIPLE.—*Dividing both dividend and divisor by the same number does not affect the value of the quotient.*

Ex. Divide 84 × 36 by 27 × 14.

OPERATIONS.

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{84}} \times \overset{4}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{27}} \times \cancel{14}} = 8$$

Or,

$$\begin{array}{r|l} \cancel{3}\ \cancel{27} & \cancel{84}\ \cancel{6}\ 2 \\ \cancel{14} & \cancel{36}\ 4 \\ \hline & 8 \end{array}$$

ANALYSIS.—Indicate the operations to be performed as in the margin. Since 36 and 27 contain the common factor 9, cancel or reject it from both, retaining the factors 4 and 3 respectively. 14 and 84 contain the common factor 14; therefore reject it, retaining the factor 6 in the dividend. [Since cancellation is a process of division, the rejecting of 14 does not destroy it, but divides it, leaving 1 as a quotient. It is unnecessary to write 1 as a quotient, except when there are no other factors in the dividend.] 3 is a common factor of 6 and 3; therefore reject it from both, retaining the factor 2 in the dividend. The product of the remaining factors, 2 and 4, is the required quotient.

168. RULE.—*Write the numbers denoting multiplication above a horizontal line, and the numbers denoting division below. The numbers above the line will form a dividend, and the numbers below, a divisor. Cancel the factors common to both dividend and divisor. The product of the remaining factors of the dividend divided by the product of the remaining factors of the divisor will be the required quotient.*

EXAMPLES.

169. Find the value of the following expressions:

1. $\frac{27 \times 48 \times 60}{54 \times 36 \times 40}$.

2. $\frac{1500 \times 144 \times 5}{365 \times 100}$.

3. $\frac{1760 \times 6 \times 145}{100 \times 365}$.

4. $\frac{1760 \times 175 \times 6}{36 \times 100 \times 10}$.

5. $\frac{40 \times 36 \times 42 \times 18}{9 \times 35 \times 30 \times 8}$.

6. $\frac{24 \times 30 \times 54 \times 35}{14 \times 15 \times 21 \times 64}$.

7. $\frac{360 \times 28 \times 27 \times 5}{25 \times 42 \times 18 \times 12}$.

8. $\frac{17 \times 36 \times 25 \times 144}{48 \times 60 \times 108 \times 51}$.

9. $\frac{144 \times 625 \times 37 \times 12}{288 \times 375 \times 185}$.

10. Multiply 72 by 3 × 18, divide the product by 8 times 9, multiply the quotient by 7 × 20, divide the product by 360, multiply the quotient by 6 times 8.

11. If 42 tons of coal cost $147, what will 16 tons cost?

12. A man gave 9 pounds of butter at 17 cents a pound for 3 gallons of molasses; how much was the molasses worth a gallon?

13. If 20 pounds of beef cost 250 cents, what cost 75 pounds?

14. How many potatoes at 65 cents per bushel will pay for 13 weeks' board at $7.50 per week?

15. A merchant bought 375 barrels of flour at $5.50 per barrel, and paid in cloth at $2.75 per yard; how many yards did it require?

16. How many pounds of coffee at 27 cents per pound should be given for 57 bushels of corn at 63 cents per bushel?

17. Sold 28 bushels of apples for $21; what should I receive for 42 bushels?

18. How many cows worth $35 each must be given in exchange for 84 tons of hay at $15 per ton?

19. How many bushels of corn at 52 cents a bushel must be exchanged for 324 bushels of oats at 39 cents per bushel?

20. If 430 bushels of wheat are obtained from sowing 7 bushels, how much would be obtained from sowing 21 bushels?

21. What should be paid for the transportation of 3600 pounds of cheese at the rate of 47 cents per 100 pounds?

22. What must be paid for transporting 31600 pounds of iron at $5 per ton of 2000 pounds?

23. What will 7840 pounds of coal cost, at $6 per ton of 2240 pounds?

24. If 3 men eat 7 pounds of meat in one week, how much would 6 men eat in 4 weeks?

25. How many canisters, each holding 40 ounces, can be filled from 3 chests of tea, each containing 55 pounds of 16 ounces?

26. How many times can 16 bottles, each holding 3 pints, be filled from 6 demijohns, each containing 10 gallons of 8 pints each?

27. A man exchanged 275 barrels of potatoes, each containing 3 bushels, at 54 cents per bushel, for a certain number of pieces of muslin each containing 45 yards, at 11 cents per yard. How many yards did he receive?

28. If a person travel 24 hours each day at the rate of 45 miles an hour, how many days would it require to pass around the globe, a distance of 25000 miles?

FRACTIONS.

170. A **Fraction** is one or more of the equal parts of a unit; as *one-half* ($\frac{1}{2}$), *two-thirds* ($\frac{2}{3}$), *one-fourth* ($\frac{1}{4}$), *seven-eighths* ($\frac{7}{8}$).

If a unit be divided into four equal parts, each part is called a fourth. If one of these parts be taken, the expression will be one-fourth ($\frac{1}{4}$); if three parts, three-fourths ($\frac{3}{4}$), etc.

171. The greater the number of equal parts into which a unit is divided, the less will be each part; the less the number of parts, the greater will be each part.

One-half ($\frac{1}{2}$) is greater than one-third ($\frac{1}{3}$); one-fourth ($\frac{1}{4}$) is less than one-third ($\frac{1}{3}$).

172. A fraction is usually expressed by two numbers, one written above the other, with a line between. Fractions written in this form are called **Common Fractions.**

173. The number below the line is called the **Denominator,** because while indicating the number of equal parts into which the unit is divided, it *denominates* or names those parts.

174. The number above the line is called the **Numerator,** because it shows how many of the parts are taken to form the fraction.

175. The numerator and denominator, taken together, are called the **Terms** of the fraction.

In the fraction $\frac{3}{4}$, 3 and 4 are the terms; 4 is the denominator, and shows that the unit is divided into four equal parts, called fourths; 3 is the numerator, and shows that three of these parts are taken to constitute the fraction.

176. A fraction is an expression of unperformed division. The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient.

177. A **Simple Fraction** is a single fraction, both of whose terms are integers.

178. Simple fractions are *proper* or *improper*.

179. A **Proper Fraction** is one that is less than a unit; the numerator being less than the denominator. Thus, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ are proper fractions.

180. An **Improper Fraction** is one that is equal to, or greater than a unit; hence the numerator must be equal to, or greater than the denominator. Thus, $\frac{3}{3}$, $\frac{5}{4}$, $\frac{8}{5}$, and $\frac{11}{8}$ are improper fractions.

181. A **Mixed Number** is an integer and a fraction united; as $2\frac{1}{2}$, $4\frac{3}{4}$, $18\frac{7}{8}$.

182. A **Complex Fraction** is one whose numerator is a fraction or a mixed number; as $\frac{\frac{3}{4}}{3}$, $\frac{105\frac{2}{3}}{12}$, $\frac{75\frac{3}{4}}{16}$, $\frac{3\frac{2}{3}}{5}$, $\frac{12\frac{1}{2}}{15}$, $\frac{16\frac{2}{3}}{100}$.

The expression $\frac{3\frac{3}{4}}{5\frac{1}{2}}$ indicates division, and is not properly a fraction. A unit cannot be divided into $5\frac{1}{2}$ equal parts.

183. PRINCIPLES.—1. *Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.*

2. *Dividing the numerator or multiplying the denominator by a number divides the fraction by that number.*

3. *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

EXERCISES.

184. *1.* Read the following fractions, and copy separately: 1, the simple fractions; 2, the proper fractions; 3, the improper fractions; 4, the mixed numbers; 5, the complex fractions:

$\frac{13}{16}$; $4\frac{1}{4}$; $\frac{17}{4}$; $\frac{5}{10}$; $\frac{7}{8}$; $\frac{11}{5}$; $\frac{10}{17}$; $175\frac{3}{4}$; $\frac{3\frac{3}{4}}{5}$; $\frac{7}{8}$; $3\frac{1}{5}$; $13\frac{2}{3}$;

$\frac{7}{9}$; $\frac{16}{15}$; $7\frac{3}{4}$; $8\frac{7}{9}$; $46\frac{3}{5}$; $\frac{141}{8}$; $\frac{18}{25}$; $\frac{14\frac{1}{2}}{20}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{8}{9}$.

2. Write the following fractions: three fourths; seven eighths; nineteen sixteenths; five, and one half; one hundred three thirty-seconds; one hundred, and three thirty-seconds; forty-eight, and five twelfths; eleven tenths; nine forty-fifths.

3. Write the following fractions: eight ninths; thirteen, and two-thirds; sixteen twenty-fourths; ten tenths; fourteen, and forty-six hundredths; sixty-five forty-eighths; nineteen one hundred nineteenths; thirty-six four hundred thirty-seconds.

REDUCTION OF FRACTIONS.

185. Reduction of Fractions is the changing their form without changing their value.

186. A fraction is reduced to *lower terms* when the numerator and denominator are expressed in smaller numbers.

187. A fraction is in its *lowest terms* when its numerator and denominator have no common divisor.

188. A fraction is reduced to *higher terms* when the numerator and denominator are expressed in larger numbers.

189. To reduce a fraction to its lowest terms.

Ex. Reduce $\frac{84}{126}$ to its lowest terms.

OPERATION.

$\frac{84}{126} = \frac{14}{21} = \frac{2}{3}$

ANALYSIS.—Dividing both terms of the fraction, $\frac{84}{126}$, by the common divisor, 6, the result is $\frac{14}{21}$; dividing both terms of $\frac{14}{21}$ by the common divisor, 7, the result is $\frac{2}{3}$. Since 2 and 3 have no common divisor, the fraction is reduced to its lowest terms (**187**).

The value of the fraction has not been changed, because both terms have been divided by the same number (**183**, 3).

190. RULE.—*Divide the terms of the fraction by any number that will divide both without a remainder, and continue the operation with the resulting fractions until they have no common divisor.*

EXAMPLES.

191. Reduce to their lowest terms,

1. $\frac{32}{48}$.
2. $\frac{45}{60}$.
3. $\frac{48}{64}$.
4. $\frac{63}{81}$.
5. $\frac{72}{128}$.
6. $\frac{85}{105}$.
7. $\frac{90}{135}$.
8. $\frac{18}{144}$.
9. $\frac{96}{108}$.
10. $\frac{132}{168}$.
11. $\frac{105}{120}$.
12. $\frac{144}{180}$.
13. $\frac{1296}{1416}$.
14. $\frac{275}{625}$.
15. $\frac{528}{1760}$.
16. $\frac{512}{728}$.
17. $\frac{648}{1024}$.
18. $\frac{325}{750}$.
19. $\frac{375}{1000}$.
20. $\frac{625}{2000}$.
21. $\frac{86}{946}$.
22. $\frac{125}{625}$.
23. $\frac{125}{1000}$.
24. $\frac{6125}{10000}$.
25. $\frac{288}{512}$.
26. $\frac{1056}{1584}$.
27. $\frac{1136}{3088}$.
28. $\frac{1278}{3474}$.
29. $\frac{2688}{3072}$.
30. $\frac{1024}{1728}$.
31. $\frac{1001}{1144}$.
32. $\frac{1440}{1728}$.

192. To reduce a fraction to higher terms.

Ex. Reduce $\frac{3}{4}$ to a fraction whose denominator is 32.

OPERATION.

$32 \div 4 = 8$

$\frac{3}{4} = \frac{24}{32}$

ANALYSIS.—The fraction $\frac{3}{4}$ is reduced to *thirty-seconds*, without changing its value, by multiplying the terms by the number that will cause its denominator 4 to become 32 (**183**, 3). By dividing the required denominator 32 by the given denominator 4, this number is found to be 8. Multiplying both terms of $\frac{3}{4}$ by 8, the result is $\frac{24}{32}$. In practice, say or think, 4 into 32 8 times. 8 times 3 are 24.

193. RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply the numerator of the given fraction by the quotient.*

EXAMPLES.

194. *1.* Reduce $\frac{3}{4}$ to 48ths.

2. Change $\frac{7}{12}$ to an equivalent fraction having 60 for its denominator.

3. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ each to 24ths.

4. Reduce $\frac{1}{2}$, $\frac{5}{9}$, $\frac{7}{18}$, $\frac{3}{4}$ each to 36ths.

5. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{9}{16}$ each to 48ths.

6. Reduce $\frac{5}{7}$, $\frac{3}{5}$, $\frac{2}{15}$ each to 105ths.

7. Reduce $\frac{11}{14}$, $\frac{5}{8}$, $\frac{1}{2}$ each to 56ths.

8. Reduce $\frac{7}{16}$, $\frac{11}{12}$, $\frac{19}{32}$ each to 96ths.

9. Reduce $\frac{7}{8}$, $\frac{8}{9}$, $\frac{3}{10}$ each to 360ths.

10. Reduce $\frac{17}{24}$, $\frac{5}{9}$, $\frac{11}{18}$ each to 72ds.

11. Reduce $\frac{4}{9}$, $\frac{20}{27}$, $\frac{17}{12}$ each to 108ths.

12. Reduce $\frac{5}{6}$, $\frac{3}{8}$, $\frac{11}{30}$ each to 360ths.

195. To reduce two or more fractions to equivalent fractions having their least common denominator.

196. **A Common Denominator** of two or more fractions is a denominator to which they can all be reduced, and is the common multiple of their denominators.

197. **The Least Common Denominator** of two or more fractions is the least denominator to which they can be reduced, and is the least common multiple of their denominators.

[illegible]

multiple of [illegible]

[illegible] denominator.

EXAMPLES.

[illegible] equivalent fractions [illegible]

[illegible]

[illegible] to an

[illegible]

[illegible] there are 8 eighths $(\frac{8}{8})$; and in 16, [illegible] eighths, or 128 eighths $(\frac{128}{8})$. [illegible]

201. Rule.—*Multiply the integer by the required denominator, and to the product add the numerator of the fraction, and under the result write the denominator.*

Note.—When the numerator of the fraction is a small number, add it mentally to the product of the integer and the denominator.

EXAMPLES.

202. *1.* In 27, how many ninths?

2. Reduce $46\frac{1}{2}$ to halves.

3. How many eighths of a peck in $37\frac{7}{8}$ pecks?

Reduce the following to improper fractions:

4. $37\frac{3}{4}$; $19\frac{7}{8}$; $208\frac{9}{16}$.
5. $56\frac{2}{3}$; $49\frac{5}{6}$; $182\frac{5}{7}$.
6. $375\frac{1}{2}$; $94\frac{9}{10}$; $46\frac{5}{8}$.
7. $44\frac{3}{4}$; $37\frac{5}{12}$; $19\frac{13}{16}$.
8. $12\frac{1}{2}$; $48\frac{7}{20}$; $45\frac{5}{18}$.
9. $81\frac{2}{3}$; $196\frac{1}{2}$; $375\frac{2}{4}$.
10. $116\frac{11}{25}$; $456\frac{4}{11}$; $87\frac{13}{36}$.
11. $24\frac{3}{14}$; $179\frac{13}{16}$; $1767\frac{3}{4}$.
12. $87\frac{3}{8}$; $490\frac{5}{12}$; $168\frac{11}{18}$.
13. $384\frac{5}{8}$; $161\frac{3}{4}$; $175\frac{25}{32}$.

203. To reduce an improper fraction to an integer or a mixed number.

Ex. Reduce $\frac{27}{4}$ to a mixed number.

Analysis.—$1 = \frac{4}{4}$; hence in $\frac{27}{4}$, there are as many units as 4 fourths are contained times in 27 fourths, or $6\frac{3}{4}$.

204. Rule.—*Divide the numerator by the denominator.*

EXAMPLES.

205. *1.* Change $\frac{317}{8}$ to a mixed number.

2. Reduce $\frac{64}{4}$ of a dollar to dollars.

Reduce to integers or mixed numbers:

3. $\frac{375}{4}$; $\frac{416}{8}$.
4. $\frac{138}{2}$; $\frac{376}{3}$.
5. $\frac{518}{9}$; $\frac{444}{5}$.
6. $\frac{303}{16}$; $\frac{427}{12}$.
7. $\frac{542}{15}$; $\frac{873}{4}$.
8. $\frac{3861}{12}$; $\frac{1248}{16}$.
9. $\frac{445}{9}$; $\frac{785}{48}$.
10. $\frac{387}{16}$; $\frac{441}{32}$.
11. $\frac{511}{25}$; $\frac{937}{60}$.
12. $\frac{411}{15}$; $\frac{1385}{36}$.
13. $\frac{387}{16}$; $\frac{1416}{32}$.
14. $\frac{517}{8}$; $\frac{2387}{49}$.
15. $\frac{373}{45}$; $\frac{449}{60}$.
16. $\frac{714}{32}$; $\frac{1000}{16}$.
17. $\frac{875}{48}$; $\frac{1276}{56}$.

…DITION OF FRACTIONS.

…dition of Fractions is the process of finding the sum of [illegible] fractions.

…CIPLE.—In order that fractions may be added, they must have [illegible] denominators and be parts of like units.

… the sum of [illegible] and [illegible]

…ERATION

[illegible] = [illegible] = [illegible] = [illegible]

ANALYSIS.—As these fractions have a common denominator, we add their numerators, [illegible] the common denominator, [illegible]

[illegible] and [illegible]

OPERATION

$$\ldots = \frac{9+8+10}{12} = \frac{27}{12} = 2\tfrac{3}{12} = 2\tfrac{1}{4}.$$

…ce the given fractions to equivalent fractions having the …nator, 12 (155). Then proceed as in previous example.

… sum of 29[illegible], 36[illegible], 17[illegible], and 42[illegible].

… The sum of the fractions is … added to the sum of the inte… … the required result.

… pieces containing 46[illegible] … 17[illegible] … 18[illegible] yards respect…

ANALYSIS.—The small figures represent … the quarters. The sum of the fourths is …

208. RULE.—*Reduce the given fractions to equivalent fractions having the least common denominator. Write the sum of the numerators over the common denominator, and reduce the resulting fraction to its simplest form.*

When there are mixed numbers or integers, add the integers and fractions separately, and then add the results.

NOTE.—Before adding, reduce all fractions to their lowest terms, and all improper fractions to mixed numbers.

EXAMPLES.

209. Add the following:

1. $\frac{5}{18}$, $\frac{11}{18}$, $\frac{7}{18}$, and $\frac{15}{18}$.

2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

3. $12\frac{1}{2}$, $7\frac{3}{8}$, $16\frac{9}{16}$, and $38\frac{3}{4}$.

4. $48\frac{4}{9}$, $46\frac{7}{18}$, $31\frac{2}{3}$, and $17\frac{1}{6}$.

5. $127\frac{7}{48}$, $\frac{23}{32}$, $175\frac{2}{3}$, and $\frac{5}{8}$.

6. $141\frac{9}{20}$, $197\frac{3}{4}$, and $43\frac{7}{12}$.

7. $75\frac{2}{5}$, $\frac{5}{9}$, $1028\frac{2}{3}$, and $\frac{13}{18}$.

8. $\frac{7}{8}$, $119\frac{1}{2}$, $240\frac{3}{4}$, and $17\frac{3}{16}$.

9. 46^1, 48^3, 40^2, 49, 47^3, and 46^2. (See Analysis opposite.)

10. 40^3, 41^1, 48^2, 44^1, 49^3, 48^2, 49^3, 49^1, 47^3, 48^3, 48^3, and 49^1.

11. $18\frac{8}{9}$, $27\frac{1}{4}$, $42\frac{2}{3}$, $51\frac{5}{8}$, and $14\frac{1}{6}$.

12. $146\frac{3}{4}$, $1\frac{4}{7}$, $53\frac{5}{14}$, and $68\frac{1}{2}$.

13. $1172\frac{5}{6}$, $19\frac{2}{3}$, $440\frac{7}{8}$, $6\frac{3}{4}$, and $10\frac{7}{12}$.

14. $\frac{7}{16}$, $106\frac{5}{12}$, $37\frac{8}{9}$, $7\frac{1}{6}$, and $176\frac{13}{18}$.

15. 175, $116\frac{7}{10}$, $143\frac{3}{5}$, and $27\frac{5}{7}$.

16. $20\frac{2}{3}$, $164\frac{3}{7}$, $\frac{11}{21}$, and $43\frac{5}{9}$.

17. $44\frac{1}{3}$, $16\frac{7}{8}$, $29\frac{7}{16}$, $13\frac{3}{4}$, and $44\frac{23}{24}$.

18. 31^1, 48^3, 62^1, 19^3, 27^2, 48^1, and 37^3.

19. 61^3, 48^1, 47^3, 48, 48^2, 49^1, and 45^3.

20. $19\frac{3}{8}$, $444\frac{5}{16}$, $737\frac{1}{4}$, and $385\frac{1}{2}$.

21. A farmer sold $317\frac{28}{60}$ bushels wheat, $176\frac{18}{45}$ bushels timothy seed, $202\frac{31}{48}$ bushels buckwheat, $526\frac{42}{56}$ bushels corn, $175\frac{24}{32}$ bushels oats, and $276\frac{45}{60}$ bushels clover seed. How many bushels did he sell altogether? (See Note, Art. **208**.)

22. A jeweler has nine diamonds whose respective weights expressed in carats are $\frac{31}{64}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{1}{2}$, $\frac{17}{32}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{47}{64}$, $\frac{19}{32}$. Find their total weight.

23. How many inches of moulding would be required for three frames whose dimensions are as follows: first, $11\frac{3}{8}$ inches wide, $17\frac{1}{2}$ inches long; second, $18\frac{1}{4}$ inches wide, $26\frac{5}{8}$ inches long; third $17\frac{7}{8}$ inches wide, $24\frac{3}{4}$ inches long?

SUBTRACTION OF FRACTIONS.

210. Subtraction of Fractions is the process of finding the difference between two fractions.

211. PRINCIPLE.—*In order that fractions may be subtracted, they must have like denominators and be parts of like units.*

Ex. From $\frac{8}{9}$ take $\frac{5}{9}$.

OPERATION.

$$\tfrac{8}{9} - \tfrac{5}{9} = \tfrac{3}{9} = \tfrac{1}{3}$$

ANALYSIS.—As these fractions have a common denominator, we take the difference between the numerators, and place it over the common denominator. $\frac{3}{9} = \frac{1}{3}$ is the result required.

Ex. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

OPERATION.

$$\tfrac{3}{4} - \tfrac{2}{3} = \frac{9-8}{12} = \tfrac{1}{12}$$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator (**197**). Then proceed as in the previous example.

Ex. From $176\frac{3}{8}$ subtract $89\frac{3}{4}$.

OPERATION.

$$\begin{array}{rr} 176\tfrac{3}{8} & \tfrac{3}{8} \\ \underline{89\tfrac{3}{4}} & \tfrac{6}{8} \\ 86\tfrac{5}{8} & \end{array}$$

ANALYSIS.—$\frac{6}{8}$ from $\frac{3}{8}$ we cannot take; we therefore take $1 = \frac{8}{8}$ from 176, leaving 175. $\frac{8}{8} + \frac{3}{8} = \frac{11}{8}$. $\frac{11}{8} - \frac{6}{8} = \frac{5}{8}$. $175 - 89 = 86$. $86 + \frac{5}{8} = 86\frac{5}{8}$.

212. RULE.—*Reduce the given fractions to equivalent fractions having the least common denominator. Write the difference between the numerators over the common denominator, and reduce the resulting fraction to its simplest form.*

When there are mixed numbers, subtract the integers and fractions separately, and add the results.

EXAMPLES.

213. Find the difference between

1. $\frac{3}{4}$ and $\frac{5}{9}$.	*4.* $2\frac{1}{2}$ and $1\frac{9}{16}$.	*7.* $1\frac{1}{6}$ and $\frac{5}{8}$.
2. $\frac{7}{8}$ and $\frac{5}{12}$.	*5.* $\frac{7}{10}$ and $\frac{8}{15}$.	*8.* $\frac{5}{9}$ and $\frac{4}{11}$.
3. $\frac{2}{3}$ and $\frac{11}{24}$.	*6.* $\frac{5}{7}$ and $\frac{3}{5}$.	*9.* 1 and $\frac{11}{16}$.

Find the difference between

10. $17\frac{1}{2}$ and $9\frac{1}{4}$.
11. $175\frac{1}{4}$ and $86\frac{1}{2}$.
12. $138\frac{3}{5}$ and $17\frac{1}{4}$.
13. $149\frac{1}{6}$ and $18\frac{3}{5}$.
14. $416\frac{7}{8}$ and $49\frac{3}{4}$.
15. $512\frac{3}{4}$ and $53\frac{7}{8}$.
16. 100 and $13\frac{3}{4}$.
17. $116\frac{5}{8}$ and $48\frac{2}{3}$.
18. $381\frac{2}{3}$ and $17\frac{3}{4}$.
19. $157\frac{5}{8}$ and $19\frac{3}{5}$.
20. 118^3 and 48^2.
21. $387\frac{3}{8}$ and $116\frac{3}{4}$.
22. $248\frac{5}{12}$ and $129\frac{1}{3}$.
23. $764\frac{3}{16}$ and $375\frac{1}{8}$.
24. $764\frac{1}{8}$ and $375\frac{3}{16}$.
25. $827\frac{1}{8}$ and $737\frac{3}{5}$.
26. $919\frac{3}{4}$ and $447\frac{5}{16}$.
27. 376^1 and 287^3.
28. 445^2 and 318^3.
29. 737^3 and 438^2.
30. 648^1 and 526^3.

MULTIPLICATION OF FRACTIONS.

214. To multiply a fraction by an integer.

215. Principle.—*Multiplying the numerator or dividing the denominator by a number multiplies the value of the fraction by that number* (**183**, 1).

Ex. What will 4 pounds of tea cost @ $\$\frac{7}{8}$ a pound?

OPERATIONS.

$$4 \times \frac{7}{8} = \frac{4 \times 7}{8} = \frac{28}{8} = 3\frac{1}{2}$$

Or,

$$4 \times \frac{7}{8} = \frac{7}{8 \div 4} = \frac{7}{2} = 3\frac{1}{2}$$

Or,

$$\frac{\cancel{4}}{1} \times \frac{7}{\underset{2}{\cancel{8}}} = \frac{7}{2} = 3\frac{1}{2}$$

Analysis.—If 1 pound costs $\$\frac{7}{8}$, 4 pounds will cost 4 times $\$\frac{7}{8}$, or $\$\frac{28}{8}$, equal to $\$3\frac{1}{2}$. Hence, 4 pounds of tea @ $\$\frac{7}{8}$ will cost $\$3\frac{1}{2}$.

To multiply $\frac{7}{8}$ by 4, multiply the numerator 7 by 4, or divide the denominator 8 by 4; either operation will give $3\frac{1}{2}$, the required product (*Prin.*).

By cancellation (**166**), the operation is shortened, and the result is obtained in its lowest terms.

Multiplying the numerator, as in the first operation, increases the number of parts, their size remaining the same: dividing the denominator multiplies the fraction by increasing the size of the parts, their number remaining the same.

Ex. Multiply $123\frac{3}{4}$ by 9.

OPERATION.

$$\begin{array}{r} 123\frac{3}{4} \\ \underline{9} \\ 6\frac{3}{4} \\ \underline{1107} \\ 1113\frac{3}{4} \end{array}$$

Analysis.—Multiply the fraction $\frac{3}{4}$ and the integer 123 separately, and add the products. In practice, when possible, add the products mentally; *e. g.*, 9 times $\frac{3}{4}$ are $\frac{27}{4}$, equal to $6\frac{3}{4}$. Write the $\frac{3}{4}$. 9 times 3 are 27, and 6 are 33. Write the 3, carry, and proceed as in simple numbers.

Ex. Multiply $227\frac{3}{4}$ by 175.

OPERATIONS.

	Or,
$227\frac{3}{4}$	$227\frac{3}{4}$
175	175
4) 525	$87\frac{1}{2}$
$131\frac{1}{4}$	$44\frac{3}{4}$
1135	1135
1589	1589
227	227
$39856\frac{1}{4}$	$39856\frac{1}{4}$

ANALYSIS.—As in preceding example.

Or, by aliquot parts, when the fractions are fourths, eighths, etc., the fractions generally used in commercial operations.

$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ ($\frac{1}{2}$ of $\frac{1}{2}$).

$\frac{1}{2}$ of $175 = 87\frac{1}{2}$.

$\frac{1}{4}$ of 175, or $\frac{1}{2}$ of $87\frac{1}{2} = 43\frac{3}{4}$.

216. RULE.—*Multiply the numerator or divide the denominator of the fraction by the integer.*

When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results.

EXAMPLES.

217. *1.* Find the cost of 20 yards of silk at $\$\frac{7}{8}$ a yard.

2. How much grain in 12 bins, each containing $76\frac{7}{8}$ bushels?

3. If 1 man earns $\$\frac{7}{8}$ in 1 day, how much will 16 men earn in 26 days?

4. If a ton of hay cost $\$16\frac{3}{4}$, how much will 22 tons cost?

5. Required the cost of 60 yards of muslin at $35\frac{3}{8}$ cents a yard.

Multiply

6. $\frac{9}{16}$ by 7.
7. $\frac{11}{16}$ by 8.
8. $\frac{20}{27}$ by 3.
9. $110\frac{1}{2}$ by 12.
10. $117\frac{3}{4}$ by 16.
11. $248\frac{5}{8}$ by 3.
12. $146\frac{2}{3}$ by 3.
13. $197\frac{1}{8}$ by 7.
14. $420\frac{9}{16}$ by 8.
15. $384\frac{5}{8}$ by 12.
16. $375\frac{1}{2}$ by 48.
17. $412\frac{3}{8}$ by 47.
18. $148\frac{7}{8}$ by 40.
19. $412\frac{3}{7}$ by 89.
20. $775\frac{1}{2}$ by 65.
21. $119\frac{9}{16}$ by 20.
22. $772\frac{3}{4}$ by 17.
23. $338\frac{5}{8}$ by 30.
24. $550\frac{5}{8}$ by 27.
25. $643\frac{3}{4}$ by 121
26. $875\frac{3}{8}$ by 234.
27. $916\frac{1}{2}$ by 275.
28. $234\frac{1}{2}$ by 318.
29. $678\frac{2}{3}$ by 427.
30. $625\frac{3}{4}$ by 516.
31. $718\frac{1}{4}$ by 542.
32. $275\frac{3}{8}$ by 287.
33. $813\frac{5}{8}$ by 319.
34. $444\frac{7}{8}$ by 412.
35. $555\frac{5}{6}$ by 875.
36. $817\frac{3}{4}$ by 416.
37. $913\frac{1}{4}$ by 375.
38. $787\frac{3}{4}$ by 525.

218. To multiply an integer by a fraction, or to find a fractional part of an integer.

219. PRINCIPLE.—*Multiplying by a fraction is taking such part of the multiplicand as the fraction is of a unit.*

Ex. If 1 ton of hay cost \$18, what will $\frac{3}{4}$ of a ton cost ?

OPERATIONS.

	Or,	Or,
4) 18	18	
$4\frac{1}{2}$	3	
3	4) 54	$\frac{3}{\underset{2}{\cancel{4}}}$ of $\frac{\overset{9}{\cancel{18}}}{1} = \frac{27}{2} = 13\frac{1}{2}$
$13\frac{1}{2}$	$13\frac{1}{2}$	

ANALYSIS.—If 1 ton cost \$18, $\frac{3}{4}$ of a ton will cost $\frac{3}{4}$ of \$18. $\frac{3}{4}$ of \$18 is 3 times $\frac{1}{4}$ of \$18. $\frac{1}{4}$ of \$18 is \$$4\frac{1}{2}$ (taking $\frac{1}{4}$ is the same as dividing by 4), and 3 times \$$4\frac{1}{2}$ is \$$13\frac{1}{2}$.

Or, $\frac{3}{4}$ of \$18 is $\frac{1}{4}$ of 3 times \$18. 3 times \$18 is \$54. $\frac{1}{4}$ of \$54 is \$$13\frac{1}{2}$.

Ex. Find the product of 175 and $8\frac{3}{4}$.

OPERATIONS.

	Or,
175	175
$8\frac{3}{4}$	$8\frac{3}{4}$
4) 525	$43\frac{3}{4}$
$131\frac{1}{4}$	3
1400	$131\frac{1}{4}$
$1531\frac{1}{4}$	1400
	$1531\frac{1}{4}$

ANALYSIS.—Multiply by the fraction $\frac{3}{4}$ and by the integer 8 separately, and add the products.

The first method is preferable, when the denominator of the fraction is not an exact divisor of the multiplicand.

Ex. Multiply 275 by $47\frac{3}{8}$.

FIRST OPERATION.	SECOND OPERATION.	THIRD OPERATION.
275	275	275
$47\frac{3}{8}$	$47\frac{3}{8}$	$47\frac{3}{8}$
8) 825	$34\frac{3}{8}$	$68\frac{3}{4}$
$103\frac{1}{8}$	3	$34\frac{3}{8}$
1925	$103\frac{1}{8}$	1925
1100	1925	1100
$13028\frac{1}{8}$	1100	$13028\frac{1}{8}$
	$13028\frac{1}{8}$	

ANALYSIS.—For the first and second operations, as in the preceding examples.

When the fractions are fourths, eighths, etc., multiply by means of aliquot parts.

$\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$ ($\frac{1}{2}$ of $\frac{1}{4}$).

$\frac{1}{4}$ of 275 = $68\frac{3}{4}$.

$\frac{1}{8}$ of 275, or $\frac{1}{2}$ of $68\frac{3}{4}$ = $34\frac{3}{8}$.

220. RULE.—*Multiply by the numerator of the fraction and divide the product by the denominator.* Or,

Divide by the denominator of the fraction and multiply the quotient by the numerator.

When the multiplier is a mixed number, multiply by the fraction and integer separately, and add the results.

EXAMPLES.

221. *1.* Find the cost of $8\frac{3}{4}$ yds. of ribbon at 25 cts. a yard.

2. What is the cost of $42\frac{7}{8}$ pounds of butter at 26 cts. a pound ?

3. Required the value of $48\frac{7}{8}$ yards of flannel at 75 cts. a yard.

Multiply

4. 84 by $\frac{3}{4}$.
5. 126 by $\frac{4}{7}$.
6. 49 by $\frac{5}{8}$.
7. 128 by $9\frac{1}{2}$.
8. 156 by $8\frac{1}{4}$.
9. 187 by $10\frac{3}{4}$.
10. 216 by $14\frac{3}{8}$.
11. 375 by $24\frac{7}{8}$.
12. 375 by $22\frac{2}{3}$.
13. 146 by $28\frac{3}{4}$.
14. 184 by $16\frac{4}{5}$.
15. 110 by $41\frac{7}{8}$.
16. 780 by $64\frac{5}{6}$.
17. 512 by $37\frac{1}{7}$.
18. 611 by $87\frac{1}{2}$.
19. 625 by $92\frac{3}{8}$.
20. 937 by $75\frac{3}{4}$.
21. 575 by $81\frac{7}{8}$.

222. To multiply a fraction by a fraction.

Ex. At $\$\frac{7}{9}$ a pound, what will $\frac{3}{4}$ of a pound of tea cost ?

OPERATION.

$$\frac{3}{4} \times \frac{7}{9} = \frac{21}{36} = \frac{7}{12}$$

$$\text{Or,}\quad \frac{\cancel{3}}{4} \times \frac{7}{\underset{3}{\cancel{9}}} = \frac{7}{12}$$

ANALYSIS.—If 1 pound cost $\$\frac{7}{9}$, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of $\$\frac{7}{9}$. $\frac{3}{4}$ of $\$\frac{7}{9}$ is 3 times $\frac{1}{4}$ of $\$\frac{7}{9}$. $\frac{1}{4}$ of $\$\frac{7}{9}$ is $\$\frac{7}{36}$, and 3 times $\$\frac{7}{36}$ is $\$\frac{21}{36}$, or $\$\frac{7}{12}$.

Ex. What is the value of $8 \times 8\frac{1}{3} \times \frac{7}{10} \times \frac{1}{14}$?

OPERATION.

$$\frac{\overset{\overset{2}{\cancel{4}}}{\cancel{8}}}{1} \times \frac{\overset{5}{\cancel{25}}}{3} \times \frac{\cancel{7}}{\underset{2}{\cancel{10}}} \times \frac{1}{\underset{2}{\cancel{14}}} = \frac{10}{3} = 3\frac{1}{3}$$

ANALYSIS.—Reduce the integer 8 and the mixed number $8\frac{1}{3}$ to improper fractions, and multiply as in the preceding example.

223. RULE.—*Reduce integers and mixed numbers to improper fractions.*

Cancel all factors common to the numerators and denominators.

Multiply the remaining numerators together for the numerator, and the remaining denominators for the denominator.

EXAMPLES.

224. Find the product of

1. $\frac{3}{8}$ and $\frac{2}{5}$.
2. $\frac{2}{3}$ and $\frac{2}{7}$.
3. $\frac{3}{4}$ and $\frac{5}{12}$.
4. $\frac{2}{5}$ and $\frac{10}{7}$.
5. $\frac{5}{8}$ and $\frac{12}{5}$.
6. 6, $3\frac{1}{3}$, and $\frac{4}{5}$.
7. $5\frac{2}{3}$, $\frac{6}{7}$, and $\frac{21}{34}$.
8. $12\frac{1}{2}$, $10\frac{2}{3}$, and $\frac{5}{12}$.
9. $\frac{1}{8}$, $13\frac{1}{3}$, and $\frac{1}{5}$.
10. $26\frac{1}{4}$, $\frac{4}{7}$, and $\frac{2}{3}$.
11. $\frac{3}{4}$, $\frac{8}{9}$, and $16\frac{4}{7}$.
12. $13\frac{1}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$.

Reduce the following compound fractions to simple ones.

A *Compound Fraction* is a fraction of a fraction.
The word "of" is equivalent to the sign ×.

13. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
14. $\frac{2}{3}$ of $3\frac{1}{7}$ of $\frac{4}{5}$.
15. $\frac{3}{8}$ of $\frac{5}{9}$ of $7\frac{1}{2}$.
16. $\frac{3}{4}$ of $\frac{7}{8}$ of $5\frac{1}{3}$.
17. $\frac{5}{6}$ of $\frac{8}{9}$ of 18.
18. $\frac{3}{8}$ of $11\frac{2}{3}$ of $\frac{5}{7}$.
19. $\frac{16}{21}$ of $\frac{14}{15}$.
20. $\frac{7}{12}$ of $\frac{8}{9}$ of $\frac{5}{72}$.
21. $\frac{5}{8}$ of $\frac{40}{49}$ of $\frac{14}{15}$.
22. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{8}{9}$.
23. $\frac{1}{4}$ of $12\frac{1}{2}$ of $6\frac{2}{5}$.
24. $\frac{5}{6}$ of $\frac{18}{25}$ of $4\frac{3}{4}$.

Find the value of the following expressions :

25. $\frac{2}{3}$ of 1728.
26. $\frac{3}{4} \times 375$.
27. $\frac{7}{8}$ times 864.
28. $\frac{2}{3}$ of $75 \times \frac{3}{4}$ of $16\frac{2}{3}$.
29. $\frac{7}{8} \times \frac{3}{4}$ of $\frac{5}{12} \times \frac{8}{9}$.
30. $(\frac{4}{5} + \frac{9}{10}) \times (\frac{3}{7} + \frac{5}{14})$.
31. $(\frac{3}{4} - \frac{2}{3}) \times (\frac{7}{8} + \frac{3}{4})$.
32. $(\frac{5}{12} + \frac{3}{4}) \times (\frac{5}{14} - \frac{1}{7})$.
33. $37\frac{1}{2}$ times $\frac{2}{3}$ of $\frac{9}{10}$.
34. $\frac{3}{8}$ of $\frac{5}{9} \times \frac{5}{6}$ of $\frac{2}{3}$.

225. To multiply mixed numbers together.

Ex. Multiply $147\frac{3}{4}$ by $41\frac{5}{8}$.

OPERATION

$$\begin{array}{lrl}
 & 147\frac{3}{4} & \\
 & \underline{41\frac{5}{8}} & \\
 & \frac{15}{32} & = \frac{5}{8} \times \frac{3}{4} \\
8\,)\,735 & 91\frac{7}{8} & = \frac{5}{8} \times 147 \\
4\,)\,123 & 30\frac{3}{4} & = 41 \times \frac{3}{4} \\
 & \left.\begin{array}{r}147\\ \underline{588}\end{array}\right\} & = 41 \times 147 \\
 & 6150\frac{3}{32} &
\end{array}$$

ANALYSIS.—Commencing at the right as in multiplication of integers, first multiply the fraction and integer of the multiplicand by the fraction of the multiplier; and then multiply the fraction and integer of the multiplicand by the integer of the multiplier. The separate steps are indicated at the right of the operation. The sum of the several partial products will be the required product.

226. Rule.—*Multiply the fractions together, each integer by the fraction of the other number, and the integers together. The sum of the partial products will be the product required.*

Note.—When both mixed numbers are small, reduce them to improper fractions and multiply as in multiplication of fractions (**223**).

EXAMPLES.

227. Multiply

1. $875\frac{1}{2}$ by $8\frac{1}{2}$; by $37\frac{1}{4}$.
2. $737\frac{1}{4}$ by $10\frac{1}{2}$; by $12\frac{1}{4}$.
3. $512\frac{3}{4}$ by $7\frac{1}{2}$; by $27\frac{1}{2}$.
4. $449\frac{3}{8}$ by $16\frac{1}{4}$; by $36\frac{1}{8}$.
5. $612\frac{1}{4}$ by $13\frac{1}{2}$; by $42\frac{5}{8}$.

Multiply

6. $716\frac{3}{4}$ by $27\frac{1}{4}$; by $58\frac{1}{2}$.
7. $447\frac{1}{3}$ by $45\frac{1}{3}$; by $64\frac{3}{8}$.
8. $459\frac{3}{4}$ by $37\frac{3}{4}$; by $39\frac{3}{4}$.
9. $378\frac{1}{2}$ by $43\frac{1}{3}$; by $40\frac{1}{4}$.
10. $479\frac{1}{4}$ by $56\frac{1}{4}$; by $27\frac{5}{8}$.

DIVISION OF FRACTIONS.

228. To divide a fraction by an integer.

229. Principle.—*Dividing the numerator or multiplying the denominator by a number, divides the value of the fraction by that number* (**183**, 2).

Ex. What cost 1 pound of tea, if 5 pounds cost $\$3\frac{1}{3}$?

OPERATIONS.

$$\frac{10}{3} \div 5 = \frac{10 \div 5}{3} = \frac{2}{3}$$

$$\text{Or, } \frac{10}{3} \div 5 = \frac{10}{3 \times 5} = \frac{10}{15} = \frac{2}{3}$$

$$\text{Or, } \frac{\overset{2}{\cancel{10}}}{3} \times \frac{1}{\cancel{5}} = \frac{2}{3}$$

Analysis.—If 5 pounds cost $\$3\frac{1}{3}$, 1 pound will cost $\frac{1}{5}$ of $\$3\frac{1}{3}$, or $\$\frac{2}{3}$.

To divide $\frac{10}{3}$ ($3\frac{1}{3}$) by 5, divide the numerator 10 by 5, or multiply the denominator 3 by 5; either operation will give $\frac{2}{3}$, the required quotient (*Prin.*).

Dividing the numerator, as in the first operation, decreases the number of parts, their size remaining the same; multiplying the denominator divides the fraction by decreasing the size of the parts, their number remaining the same.

Ex. Divide $867\frac{3}{4}$ by 4.

OPERATION.

$$4\,\underline{)\,867\frac{3}{4}} \qquad 3\frac{3}{4} = \frac{15}{4}$$

$$216\frac{15}{16} \qquad \frac{15}{4} \div 4 = \frac{15}{16}$$

Analysis.—Dividing as in simple numbers, 4 is contained in $867\frac{3}{4}$, 216 times and a remainder of $3\frac{3}{4}$. $3\frac{3}{4}$ equals $\frac{15}{4}$, which divided by 4 is $\frac{15}{16}$.

230. Rule.—*Divide the numerator or multiply the denominator of the fraction by the integer.*

When the dividend is a mixed number, divide the integer and the fraction separately, and add the results.

EXAMPLES.

231. Divide

1. $\frac{3}{4}$ by 6.
2. $\frac{5}{6}$ by 3.
3. $\frac{8}{9}$ by 6.
4. $\frac{5}{12}$ by 4.
5. $\frac{8}{17}$ by 4.
6. $16\frac{3}{4}$ by 5.
7. $172\frac{1}{2}$ by 3.
8. $875\frac{2}{3}$ by 6.
9. $935\frac{3}{4}$ by 8.
10. $729\frac{1}{2}$ by 9.
11. $637\frac{1}{2}$ by 9.
12. $875\frac{5}{11}$ by 12.
13. $1716\frac{2}{3}$ by 8.
14. $1729\frac{1}{2}$ by 3.
15. $2418\frac{3}{4}$ by 5.
16. $3516\frac{3}{7}$ by 5.
17. $2428\frac{3}{4}$ by 3.
18. $6375\frac{3}{8}$ by 4.
19. $4287\frac{7}{8}$ by 2.
20. $3281\frac{1}{7}$ by 8.
21. $5316\frac{2}{3}$ by 4.
22. $7144\frac{1}{2}$ by 5.
23. $1729\frac{3}{4}$ by 3.
24. $1749\frac{1}{4}$ by 9.
25. $8763\frac{1}{2}$ by 6.
26. $7385\frac{3}{4}$ by 8.
27. $4255\frac{7}{8}$ by 9.
28. $7134\frac{3}{4}$ by 7.
29. $9727\frac{1}{8}$ by 12.
30. $6345\frac{3}{8}$ by 16.

232. To divide by a fraction.

233. The **Reciprocal** of a number is 1 divided by that number. Thus, the reciprocal of 4 is 1 divided by 4, or $\frac{1}{4}$.

The **Reciprocal of a Fraction** is 1 divided by that fraction.

234. Principle.—*1 divided by a fraction is the fraction inverted.*

Thus, 1 divided by $\frac{3}{4}$ is $\frac{4}{3}$. This principle may be demonstrated as follows: In 1 there are 4 fourths. 1 fourth is contained in 4 fourths 4 times. Since $\frac{3}{4}$ is 3 times $\frac{1}{4}$, $\frac{3}{4}$ is contained in 1 $\frac{1}{3}$ as many times as $\frac{1}{4}$. Hence, $\frac{3}{4}$ is contained in 1 $\frac{1}{3}$ of 4 times, or $\frac{4}{3}$ times.

The reciprocal of a fraction is the fraction inverted.

Ex: At \$$\frac{3}{4}$ a yard, how many yards of cloth can be bought for \$5 ?

OPERATIONS.

$$5 \div \tfrac{3}{4} = \tfrac{20}{4} \div \tfrac{3}{4} = 6\tfrac{2}{3}$$

$$\text{Or, } 5 \div \tfrac{3}{4} = \tfrac{5}{1} \times \tfrac{4}{3} = \tfrac{20}{3} = 6\tfrac{2}{3}$$

Analysis.—Since 1 yard co \$$\frac{3}{4}$, as many yards can be bought for \$5 as \$$\frac{3}{4}$ is contained times in \$5. 5 is equal to $\frac{20}{4}$, and 3 fourths is contained in 20 fourths $6\frac{2}{3}$ times.

Or. \$$\frac{3}{4}$ is contained in \$1 $\frac{4}{3}$ times (*Prin.*), and in \$5, 5 times $\frac{4}{3}$ or $\frac{20}{3}$, equal to $6\frac{2}{3}$ times.

Ex. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be bought for $\$\frac{5}{6}$?

OPERATIONS.

$$\frac{5}{6} \div \frac{3}{4} = \frac{10}{12} \div \frac{9}{12} = 1\frac{1}{9}$$

Or, $$\frac{5}{6} \div \frac{3}{4} = \frac{5}{6} \times \frac{4}{3} = \frac{20}{18} = 1\frac{1}{9}$$

Or, $$\frac{5}{6} \div \frac{3}{4} = \frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{3} = \frac{10}{9} = 1\frac{1}{9}$$

ANALYSIS.—Since 1 yard cost $\$\frac{3}{4}$, as many yards can be bought for $\$\frac{5}{6}$ as $\$\frac{3}{4}$ is contained times in $\$\frac{5}{6}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$. $\frac{9}{12}$ is contained in $\frac{10}{12}$ $1\frac{1}{9}$ times.

Or, $\$\frac{3}{4}$ is contained in $\$1$ $\frac{4}{3}$ times (*Prin.*), and in $\$\frac{5}{6}$, $\frac{5}{6}$ times $\frac{4}{3}$ or $\frac{20}{18}$, equal to $1\frac{1}{9}$ times.

Ex. If $6\frac{2}{3}$ yards of cloth cost $5, what will 1 yard cost?

OPERATIONS.

$$5 \div \frac{20}{3} = (5 \div 20) \times 3 = \frac{3}{4}$$

Or, $$5 \div \frac{20}{3} = \frac{5}{1} \times \frac{3}{20} = \frac{15}{20} = \frac{3}{4}$$

Or, $$5 \div \frac{20}{3} = \frac{\cancel{5}}{1} \times \frac{3}{\underset{4}{\cancel{20}}} = \frac{3}{4}$$

ANALYSIS.—$6\frac{2}{3}$ yards are equal to $\frac{20}{3}$ yards. Since $\frac{20}{3}$ yards cost $5, $\frac{1}{3}$ of a yard will cost $\frac{1}{20}$ of $5 or $\$\frac{1}{4}$, and $\frac{3}{3}$ or 1 yard will cost 3 times $\$\frac{1}{4}$ or $\$\frac{3}{4}$.

Or, the price per yard equals the cost, divided by the quantity as an abstract number. 5 divided by $\frac{20}{3}$ equals 5 times 1 divided by $\frac{20}{3}$, or 5 times $\frac{3}{20}$ (*Prin.*), equal to $\frac{3}{4}$.

Ex. Divide 7552 by $78\frac{2}{3}$.

OPERATION.

```
78⅔ ) 7552
  3      3
-----------
236 ) 22656 ( 96
      2124
      ----
       1416
       1416
       ----
```

ANALYSIS.—Reduce both divisor and dividend to thirds as in the operation, omitting the common denominators. $\frac{22656}{3} \div \frac{236}{3}$ is the same as $22656 \div 236$.

Or, multiplying both divisor and dividend by the same number does not affect the quotient. Multiply both divisor and dividend by 3, and then divide as in simple numbers.

Ex. Divide $2195\frac{5}{6}$ by $175\frac{2}{3}$.

OPERATION.

```
175⅔ ) 2195⅚
  6        6
-------------
1054 ) 13175 ( 12½
       1054
       ----
        2635
        2108
        ----
         527
        ----  = ½
        1054
```

ANALYSIS.—Reduce both divisor and dividend to sixths, their least common denominator, reject the common denominator, and divide the numerators as in simple numbers.

Or, multiply both divisor and dividend by 6, the least common denominator, and divide as in simple numbers (see preceding analysis). $175\frac{2}{3} = 175\frac{4}{6}$.

235. Rule.—*Reduce the divisor and dividend to equivalent fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor.* Or,

Invert the terms of the divisor and proceed as in multiplication.

In dividing mixed numbers, multiply both divisor and dividend by the least common denominator, and divide as in simple numbers.

Note.—If both mixed numbers are small, reduce them to improper fractions, and apply the rule for division of fractions.

EXAMPLES.

236. Divide

1. 1 by $\frac{7}{8}$.
2. 16 by $\frac{4}{5}$.
3. 28 by $\frac{3}{4}$.
4. 49 by $\frac{7}{9}$.
5. 88 by $\frac{3}{4}$.
6. $\frac{3}{4}$ by $\frac{7}{8}$.
7. $\frac{5}{6}$ by $\frac{2}{3}$.
8. $\frac{9}{10}$ by $\frac{3}{5}$.
9. $\frac{7}{12}$ by $\frac{5}{8}$.
10. $\frac{3}{4}$ by $\frac{2}{3}$.
11. 28 by $4\frac{1}{2}$.
12. 33 by $3\frac{2}{3}$.
13. 64 by $5\frac{2}{3}$.
14. 73 by $8\frac{1}{9}$.
15. 45 by $7\frac{3}{7}$.
16. $8\frac{1}{2}$ by $3\frac{3}{4}$.
17. $6\frac{2}{3}$ by $3\frac{1}{9}$.
18. $4\frac{5}{7}$ by $3\frac{2}{3}$.
19. $7\frac{1}{2}$ by $8\frac{1}{3}$.
20. $9\frac{7}{9}$ by $18\frac{1}{3}$.
21. 875 by $33\frac{1}{3}$.
22. 625 by $83\frac{1}{3}$.
23. 516 by $34\frac{2}{5}$.
24. 917 by $43\frac{2}{3}$.
25. 864 by $86\frac{2}{5}$.
26. 702 by $30\frac{6}{7}$.
27. 920 by $73\frac{3}{5}$.
28. 720 by $43\frac{1}{5}$.
29. 700 by $37\frac{1}{3}$.
30. 560 by $26\frac{1}{4}$.
31. $682\frac{1}{2}$ by $45\frac{1}{2}$.
32. $847\frac{7}{8}$ by $89\frac{1}{4}$.
33. 984^3 by 75^3.
34. 862^2 by 18^3.
35. 731^1 by 56^1.
36. $431\frac{1}{4}$ by $18\frac{3}{4}$.
37. $983\frac{1}{3}$ by $29\frac{1}{2}$.
38. $504\frac{1}{6}$ by $36\frac{2}{3}$.
39. $583\frac{1}{3}$ by $43\frac{3}{4}$.

Find the value of the following complex fractions (**182**) and expressions of division :

40. $\dfrac{5\frac{1}{16}}{9};\ \dfrac{4\frac{2}{3}}{35};\ \dfrac{24\frac{3}{4}}{36}.$

41. $\dfrac{3\frac{1}{8}}{40};\ \dfrac{8\frac{2}{3}}{13};\ \dfrac{16\frac{2}{3}}{20}.$

42. $\dfrac{5\frac{1}{2}}{7\frac{1}{2}};\ \dfrac{\frac{3}{4}}{\frac{7}{8}};\ \dfrac{\frac{9}{10}}{\frac{3}{5}}.$

43. $\dfrac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{9}\text{ of }2\frac{1}{4}};\ \dfrac{\frac{1}{4}+3\frac{1}{2}}{5\frac{2}{3}-3\frac{1}{6}}.$

44. $\dfrac{18\frac{1}{2}\div 12\frac{1}{3}}{16\frac{1}{2}-15\frac{5}{6}};\ \dfrac{12\frac{1}{2}\times 11\frac{1}{5}}{68\frac{3}{4}+1\frac{1}{4}}.$

45. $\dfrac{175\frac{3}{4}-16\frac{7}{8}}{187\frac{1}{2}-186\frac{7}{8}};\ \dfrac{38\frac{2}{3}-30\frac{1}{3}}{16\frac{1}{3}+8\frac{2}{3}}.$

REVIEW EXAMPLES.

237. *1.* Reduce $\frac{288}{312}$ to its lowest terms.

2. Reduce $\frac{7}{8}$ to forty-eighths.

3. Reduce $727\frac{3}{8}$ to an improper fraction.

4. Reduce $\frac{13751}{8}$ to a mixed number.

5. Add $17\frac{1}{2}$, $37\frac{3}{4}$, $18\frac{2}{3}$, $49\frac{5}{6}$, $13\frac{3}{8}$, and $56\frac{5}{12}$.

6. From $1728\frac{1}{3}$ take $865\frac{3}{4}$.

7. Multiply $\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{14} \times \frac{3}{10} \times 16\frac{2}{3}$.

8. Multiply $1727\frac{3}{4}$ by 175.

9. Multiply 1727 by $175\frac{3}{4}$.

10. Divide $1\frac{7}{8}$ by $\frac{3}{16}$.

11. Divide 1736 by $144\frac{2}{3}$.

12. Divide $5779\frac{2}{3}$ by $275\frac{2}{3}$.

13. Divide $12346\frac{1}{4}$ by 7 ; by 35.

14. What is the cost of 1583 pounds sugar @ $11\frac{3}{4}$ cts. per pound ?

15. Add $\frac{3}{5}$ of $\frac{7}{8}$ of $4\frac{1}{6}$, $\frac{5}{8}$, $136\frac{2}{3}$, and $\frac{5\frac{3}{5}}{\frac{7}{8}}$.

16. A merchant sold a quantity of goods for $144, which was $\frac{3}{4}$ of the cost. What was the cost ?

ANALYSIS.—If $144 is $\frac{3}{4}$ of the cost, $\frac{1}{4}$ of the cost is $\frac{1}{3}$ ($\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$) of $144, or $48. $\frac{4}{4}$, or the total cost, is 4 times ($\frac{4}{4}$ is 4 times $\frac{1}{4}$) $48, or $192.

17. Required the value of 2993 pounds of sugar @ $9\frac{3}{8}$ cts. per pound ?

18. If $\frac{7}{8}$ of a ship is worth $$42430\frac{1}{2}$, what is the value of the whole ?

19. Bought $47\frac{3}{4}$ yards of cloth at $$4\frac{1}{2}$ per yard, and paid for it in wheat at $$2\frac{1}{4}$ per bushel ; how many bushels were required ?

20. Find the value of $31\frac{13}{16}$ pounds snuff @ 72 cts. per pound.

21. The less of two numbers is $777\frac{3}{4}$ and their difference $117\frac{3}{8}$; what is the greater number ?

22. A and B together have $1728 ; if A's money is equal to $\frac{3}{5}$ of B's, how much has each ?

23. A having $2146\frac{3}{4}$ yards of cloth, sold $\frac{5}{8}$ of it at $$1\frac{3}{4}$ a yard, and the remainder at $$2\frac{1}{2}$ a yard ; how much did he receive ?

24. A number being increased by $\frac{5}{8}$ of itself, the sum is 546 : what is the number ? (The number is $\frac{8}{8}$ of itself.)

25. A man had $5280 ; he bought goods with $\frac{3}{8}$ of it, and then lent $\frac{1}{4}$ of the balance to a friend ; how much had he left ?

26. Find the selling price of goods sold at a profit of $75, being $\frac{3}{4}$ of the cost.

27. Mr. A bought $117\frac{3}{4}$ acres of land at one time, and $87\frac{5}{8}$ at another ; after selling $110\frac{3}{8}$ acres, how much remained ?

28. If $8\frac{3}{4}$ tons of coal cost \$$30\frac{5}{8}$, what will $27\frac{1}{2}$ tons cost ? How many tons can be bought for \$$127\frac{3}{4}$?

29. A man paid \$$1145\frac{5}{8}$ for a horse and carriage. What was the value of each, the carriage being valued at $\frac{5}{8}$ as much as the horse ?

30. If $\frac{3}{4}$ of a farm is valued at \$$2253\frac{1}{2}$, what is the value of $\frac{2}{3}$ of it ?

31. What is the value of 2102^1 yards prints at 7^2 cents per yard ?

32. What number must be taken from $96\frac{3}{4}$, and the remainder multiplied by $16\frac{2}{3}$, that the product shall be $770\frac{5}{6}$?

33. What is the value of 164^2 yards muslin at $5\frac{3}{4}$ cents per yard ?

34. If 7 barrels of oil contain $313\frac{1}{4}$ gallons, how many gallons will $2\frac{3}{4}$ barrels contain ?

35. An executor collects $12724.84. He pays out $4096.48, and the residue he pays to the widow and her four children as follows : The widow receives a third part, and the remainder is divided equally among the children. Find the share of each.

36. What number increased by $\frac{3}{4}$ of itself will produce $2456\frac{1}{8}$?

37. Find the selling-price of goods, bought at $144, and sold at $\frac{1}{3}$ above cost.

38. A invests $\frac{5}{8}$ of his capital in real estate, and has $1725 remaining ; what is his capital ?

39. Bought a barrel of sugar containing 220 lbs., at $8\frac{1}{2}$ cents per pound. During the sale, it dried away $\frac{1}{10}$. Did I gain or lose, and how much, by selling it at $9\frac{1}{4}$ cents per pound ?

40. Multiply $2375\frac{1}{2}$ by $8\frac{1}{2}$; by $10\frac{1}{4}$.

41. Multiply $1727\frac{3}{4}$ by $18\frac{1}{2}$; by $107\frac{3}{8}$.

42. Multiply $377\frac{1}{4}$ by $16\frac{1}{2}$; by $37\frac{3}{4}$.

43. Multiply $875\frac{1}{2}$ by $22\frac{1}{4}$; by $9\frac{5}{8}$.

44. A merchant sold $12\frac{7}{8}$ yards of silk to one customer, $21\frac{3}{4}$ to another, $20\frac{3}{8}$ to another, and $28\frac{1}{2}$ to another ; at \$$2\frac{3}{8}$ per yard, how many dollars did he receive ?

45. An army loses $\frac{3}{16}$ of its number in battle and has 16042 remaining; how many did it originally contain?

46. What is the cost of 34 pieces prints, containing 1604^{3} yards, at 5^{1} cents per yard?

47. What is the value of 12 pieces prints containing 48, 48^{1}, 48^{2}, 48, 49^{2}, 48^{3}, 48, 49^{3}, 49^{2}, 48^{3}, 49^{2}, 48^{3} yards respectively at 4^{3} cents per yard?

48. A merchant purchased 29 pieces prints containing 48^{3}, 48^{2}, 41^{2}, 48^{2}, 48^{3}, 47, 49, 49^{2}, 52^{1}, 57^{3}, 48^{3}, 48^{2}, 38, 48^{2}, 48^{2}, 48^{2}, 47^{3}, 48^{2}, 48, 51, 48, 44^{1}, 51^{2}, 48, 42^{3}, 46^{2}, 48, 48^{2}, 48^{3} yards respectively; what was the cost at 5^{2} cents per yard?

49. There are 5280 feet in one mile, and $16\frac{1}{2}$ feet in one rod; how many rods in one mile?

50. A can do a certain piece of work in 10 days, and B can do it in 15 days; how long will it take them both to do it?

51. A market-woman bought 120 oranges at the rate of 5 for 2 cents, and sold $\frac{1}{2}$ of them at the rate of 3 for 1 cent, and the remainder at the rate of 2 for 1 cent. Did she gain or lose, and how much?

52. What is the duty on 22375 pounds sugar at $2\frac{13}{16}$ cts. per pound?

53. A farmer sold $1276\frac{18}{32}$ bushels oats at 44 cts. per bushel, $876\frac{22}{56}$ bushels corn at $52\frac{3}{4}$ cts., and $3381\frac{48}{60}$ bushels wheat at $1.32; how much did he receive?

54. How many bushels of corn at $54\frac{1}{4}$ cts. per bushel must a farmer exchange for 62 yards of sheeting at $8\frac{3}{4}$ cts. per yard, and 31 yards broadcloth at $1.75 per yard?

55. What is the value of 45^{3} yards damask at 77^{2} cts. per yard?

56. The salary of the President of the United States is $50000 per year; how much is that per day?

57. $1\frac{4}{16}$ pounds of beef and $1\frac{6}{16}$ pounds of flour are allowed to a ration; how much will 617 rations cost, if the price of beef is $11\frac{3}{8}$ cts. per pound, and of flour $3\frac{1}{4}$ cts. per pound?

58. What is the value of 36385 pounds of corn at $48\frac{3}{4}$ cents per bushel, each bushel containing 56 pounds?

59. What is the least common multiple of the nine digits?

60. The total production of gold and silver in the United States from 1792 to 1886 was $2,403,986,769. What was the average production per year?

DECIMALS.

238. A **Decimal** (from the Latin *decem*, ten) **Fraction** is a fraction whose denominator is 1 followed by one or more ciphers; as $\frac{3}{10}$, $\frac{16}{100}$, $\frac{7}{1000}$, $\frac{146}{10000}$.

239. Decimal fractions arise from dividing a unit into 10 equal parts, and then dividing these parts into 10 other equal parts, and so on.

Thus, if a unit be divided into 10 equal parts, each part is called a *tenth*. If a unit be divided into 100 equal parts, or 1 tenth into 10 equal parts, the parts are called *hundredths*. If a unit be divided into 1000 equal parts, or 1 hundredth into 10 equal parts, the parts are called *thousandths*.

240. All the rules, principles, operations, etc., of common fractions may be applied to decimal fractions. Since decimal fractions increase and decrease uniformly according to the scale of ten, a more simple notation, similar to that of integers, has been devised for them.

A hundred is written 100 ; a tenth part of a hundred (ten) is written 10, the 1 being written one place to the right ; a tenth part of one ten (one unit) is written 1, the 1 being written one place to the right ; in like manner, a tenth part of one unit (one-tenth) is written .1, the 1 being written one place to the right ; the tenth part of one-tenth (one hundredth) is writen .01, the 1 being written one place to the right, etc., etc.

Decimal fractions, like integers, decrease from left to right in a tenfold ratio, and increase from right to left in the same ratio.

241. In the decimal notation, the numerator only is written, the denominator being indicated by the position of a point (.) called the *decimal point*. The decimal point separates the integral from the fractional part.

242. The denominator of a decimal fraction is understood, and is 1 with as many ciphers annexed as there are figures in the decimal; thus,

Form of common fraction.		Form of decimal fraction.		
$\frac{7}{10}$	is written	.7	and is read	seven tenths.
$\frac{8}{100}$	" "	.08	" "	eight hundredths.
$\frac{16}{1000}$	" "	.016	" "	sixteen thousandths.

Hereafter, the first form, that of the common fraction, will be called a *fraction*, and the second, that of the decimal notation, a *decimal.*

243. The first place to the right of the point is called *tenths,* the second place *hundredths,* the third place *thousandths,* and so on.

244. The relation between integers and decimals is shown in the following

NUMERATION TABLE.

etc., etc.	*Billions.*	Hundred-Millions.	Ten-Millions.	*Millions.*	Hundred-Thousands.	Ten-Thousands.	*Thousands.*	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	*Thousandths.*	Ten-Thousandths.	Hundred-Thousandths.	*Millionths.*	Ten-Millionths.	Hundred-Millionths.	*Billionths.*	etc., etc.
	2	4	3	6	8	0	7	5	9	3	.	6	8	9	4	6	0	5	8	2	
	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	
	Orders of Integers.											Orders of Decimals.									

245. In the above table, observe that the first place to the left of units is called *tens,* and the first place to the right, *tenths;* the second place to the left of units is called *hundreds,* and the second place to the right, *hundredths,* etc. Hence the number of any order or place of the decimal, counting from the point, or from units' place, is the same as the number of ciphers in the denominator of the decimal.

246. A **Complex Decimal** has a fraction in its right-hand place.

Thus, .16⅔ $\left(\frac{16\frac{2}{3}}{100}\right)$ is a complex decimal, and is read 16⅔ hundredths, the fraction not being counted as a decimal place.

247. Principles.—1. *Annexing ciphers to a decimal does not alter its value.*

Annexing a cipher multiplies both the numerator and the denominator by 10, and hence does not alter the value of the decimal (**183**, 3). Thus, .7 ($\frac{7}{10}$) = .70 ($\frac{70}{100}$) = .700 ($\frac{700}{1000}$).

2. *Each removal of the decimal point one place to the right multiplies the value of the decimal by 10.*

Removing the point one place to the right does not change the numerator, but divides the denominator by 10, and hence multiplies the value of the decimal (**183**, 1). Thus, .072 ($\frac{72}{1000}$) becomes .72 ($\frac{72}{100}$) ; $\frac{72}{100} = \frac{72}{1000} \times 10$.

3. *Each removal of the decimal point one place to the left divides the value of the decimal by 10.*

Removing the point one place to the left does not change the numerator, but multiplies the denominator by 10, and hence divides the value of the fraction by 10 (**183**, 2). Thus, .72 ($\frac{72}{100}$) becomes .072 ($\frac{72}{1000}$); $\frac{72}{1000} = \frac{72}{100} \div 10$.

NUMERATION OF DECIMALS.

248. Rule.—*Read the decimal as if it were an integer, and give it the name of its right-hand order.*

EXERCISES.

249. Write in words, or read orally the following numbers :

1. .6.
2. .008.
3. .27.
4. .0375.
5. .0108.
6. .775.
7. .1007.
8. 17.6.
9. 8.029.
10. 24.000488.
11. 400.000088.
12. 76.7071.
13. 3000.0045.
14. .2045.
15. 375.18$\frac{3}{4}$.
16. 19.0033$\frac{1}{3}$.
17. 6.148$\frac{2}{3}$.
18. 648.6$\frac{2}{3}$.
19. 347.18005.
20. 808.008.
21. 600.06.

NOTATION OF DECIMALS.

250. Write sixty-four thousandths in the form of a decimal.

Analysis.—Since there are only two figures in the numerator 64, and the right-hand figure of the decimal must occupy the third decimal place to express thousandths, it is necessary to prefix a cipher to bring the right-hand figure into its proper place. Therefore write *point, naught, six, four* (.064) in the order named.

251. Rule.—*Prefix the decimal point, and decimal ciphers if necessary, to the numerator written as an integer, so that the right-hand figure will occupy the order named.*

Note.—Before writing, determine mentally the place of the right-hand figure and the number of ciphers required. Write in all cases from left to right.

EXERCISES.

252. *1.* What is the name of the third decimal order? The sixth? The first? The fourth? The second? The seventh?

2. How many decimal places are required to express hundredths? Millionths? Ten-thousandths? Tenths? Hundred-millionths? Hundred-thousandths?

3. How many ciphers must be written after the decimal point in writing 375 millionths? 27 hundredths? 875 thousandths? 446 ten-millionths? 37 ten-thousandths?

4. Write the following as decimals, so that the decimal-points stand in the same vertical line: 8 tenths; 16 hundredths; 175 thousandths; 1804 millionths; 56 ten-thousandths; 3004 ten-millionths; 1728 ten-thousandths.

5. Seventeen, and seventy-five hundredths.

6. Twenty-six, and twenty-six thousandths.

7. Two hundred forty-six ten-millionths.

8. Two hundred and forty-six ten-millionths.

9. Three hundred seventy-five, and eighteen hundred-thousandths.

10. Eight thousand, and sixty-five ten-thousandths.

11. Eight thousand sixty-five ten-thousandths.

12. $\frac{7}{10}$, $\frac{37}{100}$, $19\frac{3}{10}$, $218\frac{325}{1000}$, $\frac{1728}{10000}$.

13. $16\frac{75}{100}$, $19\frac{37}{1000}$, $345\frac{12}{1000}$, $\frac{75}{100000}$, $\frac{1234}{100000}$.

14. $28\frac{16}{1000}$, $37\frac{27}{100}$, $376\frac{2008}{1000000}$, $44\frac{1725}{100000}$, $\frac{5387}{1000000}$.

15. $170\frac{1256}{1000000}$, $16000\frac{225}{1000000}$, $38\frac{3}{100}$, $\frac{16225}{1000000}$.

16. $\frac{4}{10}$, $300\frac{75}{1000}$, $\frac{375}{1000}$, $1635\frac{216}{1000}$, $500\frac{63}{100}$.

17. $1027\frac{13}{100}$, $\frac{16}{100000}$, $\frac{214}{1000000}$, $387\frac{1012}{10000000}$.

18. $2016\frac{14}{1000}$, $\frac{306}{100000}$, $\frac{1707}{1000000}$, $104\frac{10002}{1000000}$.

19. $400\frac{14}{1000}$, $\frac{414}{1000}$, $8000\frac{6}{100000}$, $\frac{8006}{100000}$.

REDUCTION OF DECIMALS.

253. To reduce a fraction to a decimal.

Ex. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

$$4\ \underline{)\ 3.00}$$
$$.75$$

ANALYSIS.—$\frac{3}{4}$ equals $\frac{1}{4}$ of 3 units. 3 units equal 300 hundredths. $\frac{1}{4}$ of 300 hundredths equal 75 hundredths.

254. RULE.—*Annex decimal ciphers to the numerator, and divide by the denominator, pointing off as many decimal places in the quotient as there are ciphers annexed.*

255. A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator or divisor contain any prime factor other than 2 and 5, the divisor will not end. The decimals thus produced are called **Interminate** or **Repeating Decimals**, and the figures repeated, **Repetends**.

When a fraction is in its lowest terms, its numerator and denominator have no common factors (**187**). Annexing ciphers to the numerator introduces the factors 2 and 5 only; hence, if the denominator is an exact divisor of the numerator with the ciphers annexed, it must contain these prime factors and no others.

EXAMPLES.

256. Reduce to equivalent decimals:

1. $\frac{1}{2}$.
2. $\frac{7}{8}$.
3. $\frac{3}{4}$.
4. $\frac{3}{8}$.
5. $\frac{7}{16}$.
6. $\frac{25}{32}$.
7. $\frac{17}{40}$.
8. $\frac{2}{3}$.
9. $\frac{5}{6}$.
10. $\frac{7}{12}$.
11. $\frac{5}{7}$.
12. $\frac{4}{9}$.
13. $16\frac{5}{8}$.
14. $27\frac{12}{13}$.
15. $36\frac{23}{24}$.

257. To reduce a decimal to a fraction.

Ex. Reduce .075 to an equivalent fraction.

OPERATION.

$$.075 = \frac{75}{1000} \div \frac{3}{40}$$

ANALYSIS.—A decimal is changed to a fraction by writing its denominator, and omitting the decimal point and prefixed ciphers $\frac{75}{1000} = \frac{3}{40}$ (**189**).

Ex. Change $.83\frac{1}{3}$ to a simple fraction.

OPERATION.

$$.83\frac{1}{3} = \frac{83\frac{1}{3}}{100} = \frac{83\frac{1}{3} \times 3}{100 \times 3} = \frac{250}{300} = \frac{5}{6}$$

ANALYSIS.—Reduce the complex fraction $\frac{83\frac{1}{3}}{100}$ to a simple fraction by multiplying both terms by the denominator 3. (**183**, 3.)

258. RULE.—*Omit the decimal point, supply the proper denominator, and reduce the fraction to its lowest terms.*

EXAMPLES.

259. Reduce to equivalent fractions:

1. .25.
2. .75.
3. .375.
4. .625.
5. .875.
6. .125.
7. .016.
8. .128.
9. .00144.
10. .512.
11. .5625.
12. .1875.
13. $.12\frac{1}{2}$.
14. $.16\frac{2}{3}$.
15. $.33\frac{1}{3}$.
16. $.41\frac{2}{3}$.
17. $.066\frac{2}{3}$.
18. $.37\frac{1}{2}$.
19. $.104\frac{1}{6}$.
20. $.097\frac{2}{9}$.
21. $.0053\frac{4}{7}$.
22. $.44\frac{4}{9}$.
23. $.142857\frac{1}{7}$.
24. $.0833\frac{1}{3}$.
25. 28.0375.
26. $107.166\frac{2}{3}$.
27. 175.096.
28. 6.0175.

ADDITION OF DECIMALS.

260. Since decimals, like integers, increase and decrease uniformly according to a scale of ten, with the exception of placing the decimal point in the result (usually called *pointing off*), they may be *added, subtracted, multiplied,* and *divided* in the same manner as integers.

Ex. What is the sum of 28.7, 175.28, .037, 25.0045, and 4.08 ?

OPERATION.

$$\begin{array}{r@{}l} 28&.7 \\ 175&.28 \\ &.037 \\ 25&.0045 \\ 4&.08 \\ \hline 233&.1015 \end{array}$$

ANALYSIS.—Write the numbers so that units of the same order stand in the same column.

If the decimal points are in the same vertical line, tenths will necessarily be under tenths, hundredths under hundredths, etc. Add as in integers, and place the point in the result directly under the points of the numbers.

Ex. Add .6, .37¾, 16.048⅓, 8.1234$\frac{2}{7}$, and 24.125.

OPERATION.

.6	=	.6
.37¾	=	.3775
16.048⅓	=	16.0483⅓
8.1234$\frac{2}{7}$	=	8.1234$\frac{2}{7}$
24.125	=	24.125
		49.2742$\frac{13}{21}$

ANALYSIS.—Reduce the complex decimals as far as the decimal places extend in the other numbers. Since the fractions now express parts of the same fractional unit, they may be added.

In practice, the fractions may be rejected if the decimals are carried one place, at least, farther than accuracy is required.

261. RULE.—*Write the numbers so that their decimal points are in the same vertical line. Add as in integers, and place the decimal point in the result directly under the points in the numbers added.*

EXAMPLES.

262. *1.* Add ninety-seven hundredths; three hundred forty-seven thousandths; sixteen, and seventy-five hundred-thousandths; four hundred seventy-five, and two thousand thirty-seven millionths.

2. Add four, and eighty-one thousandths; thirty-seven, and two hundred one ten-thousandths; seven thousand eight hundred-thousandths; seven thousand, and eight hundred-thousandths; nineteen hundredths; three hundred sixty-four, and nine tenths; and fifty-six, and fifty-four thousandths.

3. Add three hundred seventy-five, and eight hundredths; eighteen thousandths; ninety-six, and eighty-four hundredths; four, and four tenths; and eight hundred seven ten-millionths.

4. What is the sum of 18 hundredths; 716 hundred-thousandths; 6342 millionths; 11567 ten-millionths; 625 ten-thousandths; 9 tenths; 99 hundredths; and 512 thousandths?

5. Add 81.86; 12.593; 4.004; 18.00129; .443; 400.043; .12875; 175.00175; 17.3008; 9000.0016; and .9016.

6. Find the sum of 99 ten-thousandths; 157½ thousandths; 789¾ millionths; 6 tenths; 18¾ hundredths; 1728 ten-millionths; and 88 hundredths.

7. Add $1728.64; $0.37½; $18.44½; $10.18¾; $6.25; and $0.16¼.

[illegible]

[illegible]

[illegible] and

[illegible]CTION OF DECIMALS.

[illegible]

[illegible] under the minuend [illegible] in the same column. Sub- [illegible] place the point in the result directly [illegible]

[illegible] has not as many dec- [illegible] to be annexed until [illegible]

[illegible]

[illegible] their *decimal* [illegible] *Subtract as in inte-* [illegible] *point in the remainder directly under* [illegible] *minuend and subtrahend.*

EXAMPLES.

[illegible] thousandths, subtract [illegible] thousandths.

[illegible] take twenty-nine hundred-

[illegible] and thirty-four ten-thousandths, sub- [illegible] thirty-four ten-thousandths.

[illegible] between

[illegible]	[illegible]	2.	[illegible].$864\frac{2}{3}$ and 16.[illegible]4.
[illegible]	[illegible]	3.	[illegible]44.$43\frac{1}{3}$ and 1[illegible]3.3875.
[illegible]	[illegible]	4.	[illegible]4.$37\frac{5}{8}$ and .$98\frac{3}{4}$.
[illegible]	[illegible]	5.	117.$48\frac{3}{4}$ and 49.$43\frac{1}{4}$.
[illegible]	[illegible]	6.	118.9871 and 389.[illegible]

MULTIPLICATION OF DECIMALS.

266. Ex. Multiply .144 by .12.

OPERATION.

.144
.12
.01728

Analysis.—.144 × .12 = $\frac{144}{1000} \times \frac{12}{100} = \frac{1728}{100000}$. Multiply the numerators of the two factors for the numerator of the product, as in multiplication of fractions. In the above multiplication of fractions, it will be observed that the number of ciphers in the denominator of the product equals the sum of the ciphers in the denominators of the two factors. Since each cipher represents a decimal place, the product should have as many decimal places as both factors.

If the number of figures in the product is less than the number of decimal places in the two factors, supply the deficiency by prefixing ciphers.

267. Rule.—*Multiply as in integers, and from the right point off as many decimal places in the product as there are decimal places in the two factors.*

Note.—To multiply a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand, if necessary.

EXAMPLES.

268. *1.* Multiply three hundred forty-four ten-thousandths by twelve thousandths.

2. Multiply one hundred ninety-two thousandths by four, and nineteen hundredths.

3. What is six hundredths of six hundred five millionths?

4. What is five hundredths of $864.32? Of $3645.75?

5. What is .058⅓ of 784.65? Of 943.25?

6. What is .02 × 1.106 × .25? 4.105 × .625 × .512?

Multiply

7. 8.716 by .39; by .047.

8. .00865 by .625; by 97.75.

9. .00128 by 8756.8; by 7.865.

10. 387.25 by .0147⅓; by .087½.

11. 58.625 by .488⅔; by .375.

Multiply

12. 17.28 by .016⅔; by 2.55⅙.

13. 64.325 by 1.44⅜; by .06¼.

14. 86.75 by 1.33⅓; by 5.76⅝.

15. 5.78 by .0885; by .66⅔.

16. 237.5 by .345⅓; by 4.468⅓.

17. Multiply 1728 by .33⅓; by .25; by .125; by .20.

18. Multiply .01837 by 1000; .00145 by 100000; .6874 by 100; 5.375 by 10; 17.056 by 10000. Find the sum of the products.

DIVISION OF DECIMALS.

269. Ex. Divide .01728 by 1.44.

FIRST OPERATION.

```
1.44 ) .01728 ( .012
        144
        ---
         288
         288
         ---
```

Analysis.—Dividing as in integers, without reference to the decimal points and prefixed ciphers, the quotient is 12. Since the dividend is the product of the divisor and quotient, it must contain as many decimal places as both of them. Hence the number of decimal places in the quotient must equal the number in the dividend less the number in the divisor.

SECOND OPERATION.

```
         .012
       -------
1.44 ) .01'728
        1 44
        ----
         288
         288
         ---
```

Analysis.—Since multiplying both divisor and dividend by the same number does not affect the quotient, make the divisor a whole number by placing the point two places to the right (or imagine the point to be omitted), and place the point of the dividend the same number of places to the right. (Compare 3rd analysis, page **72**). Indicate the new position of the point of the dividend by placing an index (′) or pointer between the figures as in the operation. 1.728 thousandths divided by 144 is 12 thousandths. Observe that the number of decimal places in the quotient is equal to the number in the dividend at the right of the pointer. In practice, place the point in the quotient when all the figures in the dividend at the left of the pointer have been divided. Notice in the operation that the point of the quotient is directly above the pointer of the dividend, and that each figure of the quotient is directly above the figure of the dividend which produced it.

270. Rule.—*Divide as in integers, and point off from the right of the quotient as many decimal places as the number in the dividend exceeds the number in the divisor.* Or,

Make the divisor a whole number, by placing the point to the right, and place the point of the dividend the same number of decimal places to the right. Divide as in integers, and place a decimal point in the quotient when the figures of the dividend have been used as far as the new position of the point in the dividend.

Notes.—1. If the number of figures in the quotient is less than the number of decimal places to be pointed off, supply the deficiency by prefixing ciphers.

2. If the divisor contains more decimal places than the dividend, before dividing make them equal by annexing ciphers to the dividend. If necessary to continue the division, more ciphers may be added.

3. If, after dividing all the figures of the dividend, there is a remainder the division may be continued by annexing ciphers (**247**, 1). The ciphers thus annexed must be regarded as decimal places of the dividend.

4. To divide a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers in the divisor, prefixing ciphers to the dividend, if necessary.

EXAMPLES.

271. *1.* Divide three thousand four hundred fifty-six hundred-thousandths by seventy-two hundredths.

2. Divide six, and twenty-five hundredths by twenty-five thousandths.

Divide	Divide
3. 35.88 by .345 ; by 4.16.	*8.* .0648 by .00425 ; by .0288.
4. .89958 by .47 ; by .319.	*9.* .31752 by .648 ; by .00384.
5. 12.6 by 14.4 ; by .125.	*10.* .1898 by .33⅓ ; by .0048⅜.
6. 96.3 by .20 ; by .25.	*11.* 85.2451 by 4.56⅝ ; by 8.27½.
7. 5.27 by 1.24 ; by .85.	*12.* 45.367 by .016⅔ ; by 1.080⅛.

13. Divide 17.28 by .20 ; by .25 ; by .33⅓ ; by .125 ; by .66⅔.

14. 321 is .178⅓ of what number ?

15. 186 is five hundredths of what number ?

16. What must 37.375 be multiplied by to produce 448.5 ?

17. What must 631.25 be divided by to produce 250 ?

18. Divide 176.824 by 100 ; 876.35 by 1000 ; 17380.5 by 10000 ; 2886.57 by 10 ; 375 by 1000000. Find the sum of the quotients.

NOTE.—To produce a result in hundredths or cents, the dividend must contain two decimal places more than the divisor ; to produce thousandths, three places, etc.

Find the results of the following examples in hundredths, and reduce the fractional remainders, if any, to their lowest terms.

Divide

19. $12.52 by $375.60 ; by $100.16.

20. $288 by $1728 ; by $720.

21. $232.50 by $3720 ; by $3875.

22. $60.40 by $2416 ; by $1812.

23. $72 by $3456 ; by $9000.

24. $212 by $1484 ; by $508.80

NOTE.—When the quotient is dollars and cents, it is customary in business operations to omit the fraction if less than ½ cent, and add 1 to the cents if the fraction is more than ½ cent.

In the following examples the results are carried to cents only and the fractions are omitted.

Divide	Divide
25. $18.08 by .05; by .04.	*28.* $720 by $.03\frac{1}{3}$; by .07.
26. $648 by .06; by 13.	*29.* $12.25 by $.06\frac{1}{8}$; by .08.
27. $17.28 by .48; by 21.	*30.* $960 by $.00\frac{3}{8}$; by .27.

Divide approximately to thousandths

31. 176.4 by 13; by .17.	*34.* 120.96 by 70; by 64.
32. 229.48 by 50.72; by 57.	*35.* 348.50 by 36; by .84.
33. 91.20 by 65; by 14.60.	*36.* 1728 by 12.16; by 17.5.

272. To find the value of goods sold by the hundred or thousand.

Ex. Find the cost of 864 pounds of meal at $1.15 per hundred pounds (*cwt.*).

OPERATION.

```
   8.64
   1.15
   ----
   4320
  9504
  -----
 $9.9360
```

ANALYSIS.—864 pounds equal 8 hundred weight and .64 of a hundred weight. Hence the cost of 864 pounds equals 8.64 times $1.15, or $9.94.

C. is the sign for hundred, and M. for thousand (**10**).

273. RULE.—*Reduce the quantity to hundreds by pointing two places at the right, or to thousands by pointing off three places. Multiply the price by this result and point off the product as in multiplication of decimals.*

NOTE.—If preferable, multiply the price by the quantity as given, and point off two additional places if the price is per hundred, and three additional places if the price is per thousand.

EXAMPLES.

274. *1.* Find the cost of 500 pounds of feed at $1.20 per *cwt.*

2. Find the cost of 4000 feet of boards at $8.50 per thousand feet.

3. Find the cost of 4375 feet of lumber at $11 per thousand.

4. What is the cost of 13280 bricks at $7 per thousand?

5. Find the cost of 6500 cigars at $48 per M.

6. What is the value of 640 pounds of hay at 85*c.* per *cwt.*?

7. Find the freight on 18480 pounds of merchandise at 62 cents per *cwt.*

8. What is the cost of 5967 *lbs.* meal at $1.10 per *cwt.*, and 4880 *lbs.* bran at 75*c.* per *cwt.*?

9. Find the cost of 26728 *lbs.* of feed at $1.05 per *cwt.*

10. Find the cost of 11760 feet joists at $14 per thousand.

11. Find the cost of 2 *cwt.* of oatmeal at $2.40 per *cwt.*, and 3 *cwt.* of cracked wheat at $3.84 per *cwt.*

12. What is the cost of 4 C. bolts at $2.70, and ½ C. bolts at $3.20?

13. Find the cost of insuring a house for $4500 at 35c. per $100.

14. Find the cost of 25 M. needles at $1.55 per M.

15. Find the value of 13450 feet of scantling at $18 per thousand.

16. What is the cost of 7½ M. envelopes at $2.20 per M.?

17. Find the cost of 12400 shingles at $16 per thousand.

REVIEW EXAMPLES.

275. *1.* Add 16 hundredths, 137 millionths, 48 ten-thousandths, and 2016 ten-millionths.

2. Add 16.07, 240.127⅔, 6.04$\frac{1}{7}$, 27.1234.

3. Reduce $\frac{13}{16}$ to a decimal.

4. From 175 take 16.083⅓.

5. What is ⅔ of $175.75?

6. What is .33 of 187.5?

7. Divide 43.75 by .0125.

8. Divide .06⅔ by 1.66⅔.

9. Change .8375 to a fraction.

10. Multiply 117.084 by 7.37⅔.

11. Reduce .083⅓ to a fraction.

12. From 375.1⅔ take 198.88⅔.

13. 1.75 is ⅞ of what number?

14. What is .33⅓ times 1728?

15. $3.75 is how many hundredths times $75?

16. $86.40 is how many hundredths of $2592?

17. 16.56 is .05 of what number?

18. What will 17280 bricks cost at $3.25 per M.?

19. If 278 barrels of pork cost $4378.50, what is the cost of 100 barrels?

20. Find the cost of 12456 feet of plank at $8.75 per M.

21. What is the value of 5 bbls. sugar, containing 312, 304, 301, 305, 304 pounds respectively, at 9⅜ cents per pound?

22. Find the cost of 13¾ pounds of crackers at 15½ cents per pound.

Analysis.—Instead of multiplying by the method given in Art. **226**, change one of the fractions to a decimal and then multiply. Thus, $.15½ = $.155. 13¾ × $.155 = $2.131¼, or $2.13.

Multiply according to the above method

23. $3.17½ by 13¼; by 19½. *26.* $4.12¼ by 26¼; by 11½.
24. $.68¾ by 24½; by 16¼. *27.* $1.79½ by 37¾; by 44⅛.
25. $.88½ by 32¾; by 36½. *28.* $1.37½ by 18¼; by 45½.

29. A merchant paid for merchandise during the year $137618.75, and sold merchandise to the amount of $146347.87. What was the gain, if the net market value of the merchandise remaining unsold was $24378?

30. A quartermaster has $8345 on hand, and receives $4379.62 from each of six sales of property; he turns over to quartermaster A $2875.28, and pays $120 for corn. Upon being relieved from duty, he turns over to quartermaster B one-third of the residue, and divides the remainder equally among three others, C, D, and E. What was paid over to each?

31. Merchandise on hand, Jan. 1, 1879, $46312.85; merchandise sold during the year, $317829.32; merchandise purchased in the same time, $301449.72; merchandise on hand, Dec. 31, 1879, $61378.12. What was the net gain or loss?

32. A farmer sold land for $22.50 an acre, as follows: to A, 98⅗ acres; to B, ⅜ of the number sold to A; and to C, ½ the number sold to A and B both. How much land was sold, how much did B and C each receive, and what was the amount realized?

33. At $28.75 per thousand, how many feet of lumber should be given for 2816 pounds of sugar at 7 3/16 cts. per pound?

34. A man bequeaths ⅓ of his property to his wife, ¼ to his son, ⅙ to his daughter, and the remainder, which is $36375, to charitable institutions. What is the amount bequeathed to each, and the total amount?

35. A gentleman after spending ⅓ of all his money, and ¾ of the remainder, had $177.50 remaining; how much had he at first?

36. A merchant bought 100 yards of cloth at $3.62½ per yard, and 87½ yards at $4.12½ per yard. At what average price per yard should he sell the whole, to realize a profit equal to ⅕ of the cost?

37. If 31¼ bushels of corn cost $17.50, how many bushels can be bought for $616?

DENOMINATE NUMBERS.

276. A Denominate Number is a concrete number (**145**), and may be either *simple* or *compound.*

Denominate numbers are used to express divisions of time, weights, measures, and moneys of different countries.

The scale of integers and decimals is uniform ; that of most denominate numbers is varying.

The moneys of nearly all countries excepting Great Britain, and the metric system of weights and measures have a uniform decimal scale.

277. A Simple Denominate Number refers to units of the same name and value ; as 7 inches, 4 pounds.

278. A Compound Denominate Number refers to units of different names, but of the same nature ; as 3 feet 6 inches, 4 pounds 8 ounces.

REDUCTION OF DENOMINATE NUMBERS.

279. Reduction of Denominate Numbers is the process of changing their denomination without changing their value.

280. To reduce denominate numbers from higher to lower denominations.

Ex. How many pence in £8 16*s.* 7*d.*?

OPERATION.

£	*s.*	*d.*
8	16	7
20		
160*s.*		
16*s.*		
176*s.*		
12		
2119*d.*		

ANALYSIS.—Since there are twenty shillings in 1 pound, in 8 pounds there are 8 times 20 shillings, or 160 shillings. (For convenience multiply by 20 as an abstract number.) 160 shillings plus 16 shillings equal 176 shillings. Since there are 12 pence in 1 shilling, in 176 shillings there are 176 times 12 pence, or 2112 pence. 2112 pence plus 7 pence equal 2119 pence. When possible, add mentally the number of the lower denomination to the product, as in the last step of this operation.

281. Rule.—*Multiply the number of the highest denomination given by the number of the next lower denomination required to make 1 of this higher, and to the product add the given number, if any, of such lower denomination.*

Treat this result, and the successive results obtained, in like manner until the number is reduced to the required denomination.

EXAMPLES.

282. Reduce

1. £9 13*s.* 10*d.* to pence.
2. 6 *gal.* 3 *qt.* 1 *pt.* to gills.
3. £112 18*s.* 5*d.* to farthings.
4. 6 *T.* 12 *cwt.* 65 *lb.* to pounds.
5. 8 *mo.* 16 *da.* to days.
6. £75 17*s.* 7*d.* to pence.
7. £245 15*s.* 3 *far.* to farthings.
8. 48 *bu.* 3 *pk.* 6 *qt.* to quarts.
9. 7 *T.* 9 *cwt.* 48 *lb.* to pounds.
10. 18 *lb.* 8 *oz.* to pennyweights.
11. 5 *mi.* 36 *rd.* 11 *ft.* to feet.
12. 456 miles to feet.

Reduce

13. 8 *bu.* [illegible] *pk.* [illegible] *qt.* to pints.
14. 13 *gal.* 3 *qt.* 1 *pt.* to pints.
15. 1 *mi.* 32 *rd.* 10 *ft.* to inches.
16. 29 *sq. rd.* to square feet.
17. 97 *sq. rd.* to square yards.
18. 5 *sq. mi.* to acres.
19. 11 *mo.* 24 *da.* to days.
20. 16 cords 112 *cu. ft.* to *cu. ft.*
21. £178 13*s.* 9*d.* to pence.
22. 2 *yr.* 8 *mo.* 22 *da.* to days.
23. 13 *T.* 17 *cwt.* 82 *lb.* to pounds.
24. £31 11*s.* 11*d.* to pence.

283. To reduce denominate numbers from lower to higher denominations.

Ex. Reduce 2119 pence to higher denominations.

OPERATION.

$$\begin{array}{r|l} 12 & \underline{2119d.} \\ 20 & \underline{176s.} + 7d. \\ & £8 \ + 16s. \end{array}$$

$2119d. = £8\ 16s.\ 7d.$

Analysis.—Since there are 12 pence in 1 shilling, in 2119 pence there are as many shillings as 12 pence are contained times in 2119 pence, or 176 shillings, and 7 pence remaining. Since there are 20 shillings in 1 pound, in 176 shillings there are as many pounds as 20 shillings are contained times in 176 shillings, or 8 pounds, and 16 shillings remaining. Therefore, 2119*d.* = £8 16*s.* 7*d.*

284. Rule.—*Divide the given number by the number of that denomination required to make 1 of the next higher reserving the remainder, if any, as part of the answer.*

Treat the quotient, and the successive quotients obtained, in like manner until the number is reduced to the required denomination. The last quotient and the several remainders will form the answer

EXAMPLES.

285. Reduce

1. 8475*d.* to pounds.
2. 9683 *cu. ft.* to cords.
3. 7534 *pts.* to bushels.
4. 9817 *pts.* to gallons.
5. 987 *in.* to yards.
6. 216 *da.* to months.
7. 875 rods to miles.
8. 6375 *hrs.* to weeks.
9. 9537 *sec.* to hours.
10. 6239 *in.* to yards

Reduce

11. 13387*d.* to pounds.
12. 987 *ft.* to yards.
13. 17416 *lbs.* to tons.
14. 1809 *in.* to yards.
15. 4711 pts. to bushels.
16. 8370*d.* to pounds.
17. 5316 *sq. rds.* to acres.
18. 7418 *oz.* to *cwt.*
19. 8716 *cu. ft.* to cords.
20. 4829*d.* to pounds.

REDUCTION OF DENOMINATE FRACTIONS.

286. **A Denominate Fraction** is a fraction whose integral unit is a denominate number.

The principles, analyses, and rules of denominate fractions are essentially the same as those of denominate integers; therefore, no special rules are necessary for their reduction. A sufficient number of illustrative examples are given to fully explain the different cases that may arise.

287. To reduce denominate fractions from higher to lower denominations.

Ex. Reduce $\frac{7}{16}$ of a £ to pence.

OPERATIONS.

$$\frac{7}{\underset{4}{\cancel{16}}} \times \frac{\overset{5}{\cancel{20}}}{1} = \frac{35}{4}s.$$

$$\frac{35}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{1} = 105d.$$

Or, $\frac{7}{\underset{4}{\cancel{16}}} \times \frac{\overset{5}{\cancel{20}}}{1} \times \frac{\overset{3}{\cancel{12}}}{1} = 105d.$

ANALYSIS.—Since there are 20 shillings in £1, in $\frac{7}{16}$ of a £ there are $\frac{7}{16}$ of 20 shillings, or $\frac{35}{4}$ shillings. Since there are 12 pence in 1 shilling, in $\frac{35}{4}$ shillings there are $\frac{35}{4}$ times 12 pence, or 105 pence. Or, multiply the given fraction by the numbers of the scale required to reduce its denomination to the required denomination.

Ex. Reduce $\frac{7}{16}$ of a £ to shillings and pence.

OPERATION.

$$\frac{7}{\underset{4}{\cancel{16}}} \times \frac{\overset{5}{\cancel{20}}}{1} = \frac{35}{4} = 8\tfrac{3}{4}s.$$

$$\frac{3}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{1} = 9d.$$

$$£\tfrac{7}{16} = 8s.\ 9d.$$

ANALYSIS.—Multiplying by 20, £$\frac{7}{16}$ = 8$\frac{3}{4}$ shillings. Reserve the integral part of the result, and reduce the fractional part to pence. Multiplying by 12, $\frac{3}{4}$ shilling = 9 pence. Hence, £$\frac{7}{16}$ = 8*s.* 9*d.*

293. To reduce denominate numbers to decimals of higher denominations.

Ex. Reduce .6 of a penny to the decimal of a £.

OPERATION.
12) .6 *d.*
20) .05 *s.*
£.0025

ANALYSIS.—Divide the given decimal by the numbers of the scale required to reduce pence to pounds.

If the answer is required in the form of a fraction, reduce the resulting decimal to a fraction by Art. **258.** £.0025 = £$\frac{1}{400}$.

Ex. Reduce 9 pence to the decimal of a £.

OPERATIONS.
12) 9. *d.*
20) .75 *s.*
.0375 £.
Or, £$\frac{9}{240}$ = £$\frac{3}{80}$ = £.0375.

ANALYSIS.—For first operation, as in previous example.

Or, since there are 240 pence in £1, 1 penny equals $\frac{1}{240}$ of a £, and 9 pence equal $\frac{9}{240}$, or $\frac{3}{80}$ of a £. £$\frac{3}{80}$ = £.0375 (**254**).

Ex. Reduce £18 12*s.* 9*d.* to the decimal of a £.

OPERATION.
12) 9. *d.*
20) 12.75 *s.*
£18.6375

ANALYSIS.—Write the denominations given in a vertical column, the lowest denomination at the top. Since there are 12 pence in 1 shilling, 9 pence are equal to .75 shilling; to which annexing the 12 shillings given we have 12.75 shillings. Since there are 20 shillings in £1, 12.75 shillings are equal to £.6375, to which annexing the £18, we have £18.6375. Hence £18 12*s.* 9*d.* = £18.6375.

EXAMPLES.

294. *1.* Reduce .875 of a shilling to pounds.

2. Change 12 *cwt.* to the decimal of a ton.

3. What decimal of a £ are 18*s.* 6*d.*?

4. Reduce £14 15*s.* 9*d.* to the decimal of a pound.

5. Reduce 116 *cu. ft.* to the decimal of a cord.

6. Reduce £247 14*s.* 9*d.* to pounds.

7. What decimal of an acre are 64 *sq. rds.*?

8. Reduce £27 10*s.* 6*d.* to pounds.

9. What is the cost of 16 *tons* 12 *cwt.* of "Nut" coal at $6.80 per ton, and 8 *tons* 16 *cwt.* of "Chestnut" coal at $6.10 per ton?

10. If 1 pound is equivalent to 4.87\frac{1}{2}$, what is the value of £123 16*s.* 9*d.* in U. S. money?

11. Reduce £25 12*s.* 6*d.* to the decimal of a £, multiply the result by .03, and reduce the resulting decimal to shillings and pence.

ADDITION OF DENOMINATE NUMBERS.

295. Denominate numbers are *added, subtracted, multiplied,* and *divided* by the same general methods as are employed for like operations in abstract numbers. The only difference arises from the use of a *varying* scale instead of the uniform scale of 10.

Ex. Add £5 11*s*. 4*d*., £7 14*s*. 9*d*., £6 16*s*. 8*d*., and £7 5*s*. 9*d*.

OPERATION.

£	*s.*	*d.*
5	11	4
7	14	9
6	16	8
7	5	9
27	8	6

ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to add at the right. The sum of the pence is 30*d*. = 2*s*. 6*d*. Write the 6*d*. under the column of pence, and add the 2*s*. to the column of shillings, obtaining for the sum 48*s*. = £2 8*s*. Write the 8*s*. under the column of shillings, and add the £2 to the column of pounds, obtaining for the sum £27; which write under the column of pounds, producing the entire sum, £27 8*s*. 6*d*.

EXAMPLES.

296. *1.* Add £16 5*s*. 4*d*., £12 8*s*. 9*d*., £13 14*s*. 8*d*., £42 0*s*. 7*d*., and 18*s*. 6*d*.

2. Add 3 *T.* 19 *cwt.* 46 *lb.*, 4 *T.* 13 *cwt.* 14 *lb.*, 18 *T.* 13 *cwt.* 24 *lb.*, and 42 *T.* 8 *cwt.* 82 *lb.*

3. Add £163 16*s*. 11*d*., £52 8*s*. 6*d*., £3 14*s*. 2*d*., £84 12*s*. 11*d*., £106 1*s*. 4*d*., and £49 13*s*. 8*d*.

4. Add [illegible] *yr.* 6 *mo.* 10 *da.*, 3 *yr.* 8 *mo.* 24 *da.*, 4 *yr.* 11 *mo.* 16 *da.*, 3 *mo.* 18 *da.*, and 1 *yr.* 8 *mo.* 8 *da.*

5. Add 8 *cd.* 106 *cu. ft.*, 3 *cd.* 85 *cu. ft.*, and 2 *cd.* 113 *cu. ft.*

6. Add 16 *hr.* 43 *min.* 48 *sec.*, 3 *hr.* 12 *min.* 40 *sec.*, 1 *hr.* 49 *min.* 13 *sec.*, and 5 *hr.* 19 *sec.*

7. Add 4 *bu.* 3 *pk.* 6 *qt.* 1 *pt.*, 10 *bu.* 2 *pk.* 7 *qt.* 1 *pt.*, 11 *bu.* 3 *pk.* 1 *qt.* 1 *pt.*, 9 *bu.* 2 *pk.* 5 *qt.* 1 *pt.*

8. 7 *yd.* 2 *ft.* 10 *in.*, 2 *yd.* 1 *ft.* 9 *in.*, 8 *yd.* 1 *ft.* 8 *in.*, 6 *yd.* 4 *in.*, 1 *yd.* 2 *ft.* 9 *in.*

9. Add 1 *lb.* 11 *oz.* 18 *pwt.* 14 *gr.*, 2 *lb.* 8 *oz.* 10 *pwt.*, 4 *lb.* 5 *oz.* 18 *gr.*, and 10 *oz.* 13 *pwt.* 12 *gr.*

10. Add 16 *gal.* 3 *qt.* 1 *pt.*, 45 *gal.* 2 *qt.*, 17 *gal.* 1 *qt.* 1 *pt.*, 4 *gal.* 3 *qt.*, 15 *gal.* 1 *pt.*, and 24 *gal.* 3 *qt.* 1 *pt.*

11. Add £17 16*s*. 8*d*., £37 13*s*. 5*d*., £46 7*d*., £11 5*s*. 10*d*., £8 4*s*., £38 19*s*. 3*d*., and £45 12*s*. 8*d*.

12. Add £175 14*s*. 9*d*., £37 9*s*. 3*d*., £5 10*s*. 9*d*., 17*s*. 3d., £55 17*s*., £3 6*s*. 9*d*., £44 18*s*. 5*d*., £218 15*s*. 6*d*., and £3 11*s* 11*d*.

SUBTRACTION OF DENOMINATE NUMBERS.

297. Ex. From £10 6s. 4d. take £8 15s. 3d.

OPERATION.

£	s.	d.
10	6	4
8	15	3
1	11	1

ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to subtract at the right. 3d. from 4d. leaves 1d., which write under the column of pence. Since 15s. cannot be subtracted from 6s., take £1 = 20s. from £10, leaving £9, and add it to the 6s., making 26s. 15s. from 26s. leaves 11s., which write under the column of shillings. £8 from £9 leaves £1, which write under the column of pounds. Hence the difference required is £1 11s. 1d.

EXAMPLES.

298. *1.* From £175 16s. 8d. take £87 12s. 6d.

2. From £84 10s. 2d. take £63 5s. 10d.

3. From £16 6s. 11d. take £12 12s. 8d.

4. From £48 10s. 8d. take £24 16s. 10d.

5. From 16 *yr.* 8 *mo.* 10 *da.* subtract 12 *yr.* 5 *mo.* 8 *da.*

6. From 80 *yr.* 10 *mo.* 16 *da.* take 76 *yr.* 5 *mo.* 24 *da.*

7. From 81 *yr.* 4 *mo.* 25 *da.* take 80 *yr.* 10 *mo.* 15 *da.*

8. From 82 *yr.* 3 *mo.* 20 *da.* take 79 *yr.* 8 *mo.* 26 *da.*

MULTIPLICATION OF DENOMINATE NUMBERS.

299. Ex. Multiply £7 16s. 8d. by 11.

OPERATION.

£	s.	d.
7	16	8
		11
86	3	4

ANALYSIS.—11 times 8d. are 88d. = 7s. 4. Write the 4d. under the pence, and add the 7s. to the product of shillings. 11 times 16s. are 176s., plus 7s. from the preceding product are 183s. = £9 3s. Write the 3s. under the shillings, and add the £9 to the product of pounds. 11 times £7 are £77, plus £9 from the preceding product are £86, which write under the pounds. Hence the entire product is £86 3s. 4d.

Ex. Multiply £8 12s. 6d. by .05.

ANALYSIS.—Reduce the multiplicand to the decimal of a pound by Art. **293**, perform the required multiplication, and reduce the result to shillings and pence by Art. **289**.

£8 12s. 6d. = £8.625. £8.625 × .05 = £.43125. £.43125 = 8s. 7.5d.

Or, reduce the multiplicand to pence by Art. **281**, perform the required multiplication, and reduce the result to shillings and pounds by Art. **284**.

£8 12s. 6d. = 2070d. 2070d. × .05 = 103.5d. 103.5d = 8s. 7.5d.

EXAMPLES.

300. *1.* Multiply £17 10*s.* 8*d.* by 7; by 9; by 11; by 15.

2. How many cords of wood in 12 loads, each load containing 2 *cd.* 108 *cu. ft.*?

3. What is the cost of 25 *yd.* of silk, at £1 2*s.* 6*d.* per *yd.*?

4. Find the weight of 24 spoons, each spoon weighing 1 *oz.* 13 *pwt.*

5. Multiply 1 *hr.* 38 *min.* 22 *sec.* by 10; by 12; by 15; by 18.

6. If 15 men perform a certain piece of work in 3 *da.* 16 *hr.* 52 *min.*, how long would it take one man to perform it?

7. What will 50 *gal.* of wine cost at 8*s.* 3*d.* per gallon?

8. What is .05 of £127 16*s.* 6*d.*? Of £145 15*s.* 9*d.*?

9. Multiply £138 8*s.* 9*d.* by .02½; by .04; by .06; by .07.

DIVISION OF DENOMINATE NUMBERS.

301. **Ex.** If 6 *yds.* of cloth are worth £8 18*s.* 6*d.*, what is 1 *yd.* worth?

OPERATION.

	£	*s.*	*d.*
6)	8	18	6
	1	9	9

ANALYSIS.—1 *yd.* is worth 1 *sixth* as much as 6 *yd.* ⅙ of £8 is £1 and £2 remaining. Write the £1 in the quotient, and reduce the £2 to shillings. £2=40*s.*, plus 18*s.* in the dividend = 58*s.* ⅙ of 58*s.* is 9*s.* and 4*s.* remaining. Write the 9*s.* in the quotient, and reduce the 4*s.* to pence. 4*s.* = 48*d.*, plus 6*d.* in the dividend = 54*d.* ⅙ of 54*d.* is 9*d.*, which write in the quotient. £1 9*s.* 9*d.* is the quotient required.

NOTE.—When the divisor is a denominate number, as in Ex. 2, reduce both divisor and dividend to the same denomination, and divide as in simple numbers.

EXAMPLES.

302. *1.* Divide £13 12*s.* 3*d.* by 11; by 9; by 33.

2. How many yards of muslin at 7*d.* per yard can be bought for £5 12*s.*? For £9 9*s.*? For £10 5*s.* 4*d.*? (See note.)

3. How many yards of silk at £1 19*s.* 2*d.* per yard can be purchased for £86 3*s.* 4*d.*? (See note.)

4. Divide 85° 18′ 30″ by 12; by 15; by 18; by 27.

5. Divide 322 *A.* 90 *sq. rd.* by 8; by 10; by 13; by 16.

6. If 42 *yd.* of cloth cost £20 16*s.* 6*d.*, what is the price of 1 *yd.*? Of 12 *yd.*? Of 20 *yd.*? Of 37 *yd.*?

7. Divide 17*s.* 3*d.* by .02½; by .04; by .05; by .09; by .15.

Reduce the dividend to the decimal of a pound or shilling, divide in the usual manner, and reduce the quotient to pounds, shillings, and pence.

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days.

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1582, and that all cente

The Julian and Gregorian [illegible] tyle and New Style. [illegible] ion years in the Gregorian calendar [illegible] ow 13 days. Thus when it is [illegible]

306. Rule for Leap Years. [illegible] *xcept centennial* [illegible]

All centennial [illegible]

307. The Calendar Months, [illegible] ontain, are as follows:

Season.		Days.	Season.		Days.
	1. January (Jan.)	[illegible]	[illegible]	[illegible]	[illegible]
WINTER.	2. February (Feb.)	28	[illegible]	[illegible]	[illegible]
	[illegible]	29		[illegible]	[illegible]
	3. March (Mar.)	[illegible]		[illegible]	[illegible]
SPRING.	4. April (Apr.)	30	[illegible]	[illegible]	[illegible]
	5. May	[illegible]		[illegible]	[illegible]
			[illegible]	[illegible]	[illegible]

The number of days [illegible] following lines:

[illegible]

EXAMPLES.

308. 1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible] per [illegible]

5. [illegible]

6. [illegible]

7. How many leap years from [illegible] From [illegible] From [illegible]

8. How long from [illegible] 30 sec. past [illegible]

9. Reduce [illegible]

10. Find [illegible] 8 hours.

309. To find the interval of time between two dates.

310. There are two methods in common use for finding the time between two dates: 1, by compound subtraction, in which the result is given in years, months, and days, and in which 12 months are considered a year, and 30 days a month; 2, the result is given in days, or in years and days, and the true number of days is taken for each month.

Ex. Find the time in months and days from Apr. 24 to Nov. 10.

OPERATION.

mo.	*da.*
11	10
4	24
6	16

ANALYSIS.—Represent the months and days by their numbers and find their difference by compound subtraction (**297**), writing the later date as the minuend and the earlier as the subtrahend.

In many examples the interval may be found mentally as follows: From Apr. 24 to Oct. 24 are 6 *mo.*; in Oct. there are 6 more days after the 24th (regarding each month as 30 days), and in November to Nov. 10th inclusive, there are 10 days. Hence the total time between the given dates is 6 *mo.* 16 *da.*

The above methods may be used for finding the exact interval in days by making the necessary corrections 6 *mo.* 16 *da.* = 196 *da.* From Apr. 24 to Nov. 10, there are 4 months containing 31 *da.* each; hence the true answer is 196 *da.* + 4 *da.*, or 200 *da.*

NOTE.—When the month of February is included, subtract 2 days in a common year, and 1 day in a leap year.

Ex. Find the time from May 18, 1884, to Mar. 2, 1890.

OPERATION.

yr.	*mo.*	*da.*
90	3	2
84	5	18
5	9	14

ANALYSIS.—As in preceding example.

Ex. What is the exact number of days from July 20, 1888, to Nov. 10, 1889?

OPERATION.

365 from July 20, 1888, to July 20, 1889.
11 remaining in July.
31 in August.
30 in September.
31 in October.
10 in November.
478 from July 20, 1888, to Nov. 10, 1889.

ANALYSIS.—In finding the interval between two dates the last day is counted, and not the first. Since the time is more than one year, write down 365 days as the number of days from the first date to the same date of the next year. Next write down the number of days in the month of July after the 20th, then the number of days in each of the full calendar months, and finally the number of days in November to Nov. 10 inclusive. The sum of these numbers will be the required time.

In solving examples by this method, the student should remember that the first quarter contains 90 days, the second quarter, 91 days, the third quarter, 92 days, and the last quarter, 92 days; the first six months, 181 days, and the last six months, 184 days.

EXAMPLES.

311. Find the time by compound subtraction from

1. Jan. 10 to Aug. 28.
2. Mar. 16 to Dec. 4.
3. Feb. 5, 1886, to Oct. 16, 1887.
4. Jan. 27, 1885, to July 4, 1887.
5. May 16, 1886, to Mar. 24, 1887.
6. June 28, 1885, to Apr. 10, 1886.
7. July 30, 1886, to May 12, 1887.
8. Aug. 16, 1887, to Jan. 1, 1888.

Find also the exact number of days between the above dates.

9. How many days from Jan. 1, 1888, to Jan. 1, 1906?

10. Ninety days after June 21 is what date?

OPERATIONS.

90		Or, 9	June.
9	June.	31	July.
81		31	Aug.
31	July.	71	
50		90	
31	Aug.	19	Sept.
19	Sept.		

Analysis.—Subtract from the given number of days, the number of days remaining in June, and from this remainder, subtract successively the number of days in the following months until the remainder is equal to or less than the number of days in the next following month. The last remainder represents the required date.

Or, write the remaining number of days in June, and the number of days in a sufficient number of months to produce about the given number of days. Take their sum and subtract it (if possible) from the given number of days. The remainder will be the day of the following month representing the required date. If the sum is greater than the given number, subtract the excess from the number of days in the last month written. The remainder will be the required date.

If the time be 30, 60, or 90 days, regard each 30 days as a calendar month, and correct by subtracting 1 day for each intervening month containing 31 days, and adding 2 days for February (in leap year 1 day). Thus 3 months after June 21 is Sept. 21, and by subtracting 2 days for July and August, the correct result is Sept. 19.

11. 63 days after Oct. 4 is what date?

12. 90 days after Mar. 24 is what date?

LINEAR MEASURES.

312. Linear or **Long Measure** is used in measuring distances; also the length, breadth, and height of bodies, or their linear dimensions.

In measuring length, the yard derived from the standard yard of England is the standard unit, the yards of the United States and England being identical. Theoretically, the yard is equal to $\frac{360000}{391393}$ of the length of a pendulum that vibrates seconds in a vacuum, at the level of the sea, in the latitude of London; that is, a pendulum that vibrates seconds under the above conditions is 39.1393 inches in length. The standard yard is, in fact, the distance between two points on a brass bar, preserved at Washington, the distance to be taken when the bar is at a temperature of 62° Fahrenheit.

TABLE.

12 Inches (*in.*, ″) = 1 Foot . . *ft.*, ′.
3 Feet = 1 Yard . . *yd.*
5½ Yards = 1 Rod . . *rd.*
320 Rods = 1 Mile . . *mi.*

mi.		*rd.*		*yd.*		*ft.*		*in.*
1	=	320	=	1760	=	5280	=	63360
		1	=	5½	=	16½	=	198
				1	=	3	=	36
						1	=	12

NOTES.—1. The inch is usually divided into halves, quarters, eighths, and sixteenths.

2. The foot and inch are divided by civil engineers and others into tenths, hundredths, thousandths, etc.

3. In measuring cloth, ribbon, and other goods sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

4. At the U. S. Custom Houses, the yard is divided into tenths and hundredths.

5. The mile (5280 *ft.*) of the above table is the legal mile of the United States and England, and hence it is sometimes called the statute mile.

6. 1 furlong = ⅛ mile = 40 rods. (Rarely used.)

313. The following denominations are also used:

1 Size = ⅓ Inch. Used by shoemakers.
1 Hand = 4 Inches. Used in measuring the height of horses.
1 Fathom = 6 Feet. Used in measuring depths at sea.
1 Cable-length = 120 Fathoms, or 240 yards.
1 Geographic Mile = 1.15+ Statute Miles. Used in measuring distances at sea.
1 Knot = 1 Geo. Mile. Used in determining the speed of vessels.
60 Geo. Miles, or 69.16 Stat. Miles = 1 Degree of latitude on a meridian, or of longitude on the equator.
360 Degrees = the Circumference of the Earth.

314. Surveyors' Linear Measure is used in measuring land, roads, etc.

The unit of this measure is a chain, 4 rods or 66 feet in length, called Gunter's Chain. It is divided into 100 parts called *links*, each link being 7.92 inches in length.

TABLE.

100 Links (*l.*) = 1 Chain . . . *ch.*
80 Chains = 1 Mile . . . *mi.*

mi.	*ch.*	*ft.*	*l.*	*in.*
1 =	80 =	5280 =	8000 =	63360
	1 =	66 =	100 =	792
		.66 =	1 =	7.92

NOTES.—1. Links are written decimally as hundredths of a chain.

2. For railroad and other purposes, engineers use a chain or tape 100 feet long, the feet being divided into tenths.

3. 1 rod = 25 links.

EXAMPLES.

315. *1.* Add 9 *ft.* 8 *in.*, 12 *ft.* 6 *in.*, 16 *ft.* 5 *in.*, 15 *ft.* 11 *in.*, 21 *ft.* 4 *in.*, 17 *ft.* 3 *in.*

2. In $\frac{5}{8}$ of a yard, how many inches?

3. How many feet in 17 miles? In 35 rods?

4. Find the difference between 5 *ft.* $4\frac{1}{2}$ *in.* and $16\frac{1}{2}$ hands.

5. Reduce 49175 *ft.* to higher denominations.

6. Reduce 32 *rd.* 4 *yd.* 2 *ft.* 10 *in.* to inches.

7. How many feet in $\frac{7}{8}$ of a mile?

8. In 4376 feet, how many chains? How many inches?

9. In 396 rods, how many chains? How many feet?

10. In 37.56 chains, how many feet? How many rods?

11. Children's size 1 of shoemakers' measure is $4\frac{1}{6}$ inches long: what is the length of boys' size 8, youths' size 1, and men's size 10? (Size 1 of the second series is one size longer than size 13 of the first series.)

12. How many fathoms in 1722 *ft.*? In 3136 *ft.*?

13. Reduce 48276 *ft.* to higher denominations.

14. Add 4.16 *ch.*, 3.75 *ch.*, 8.08 *ch.*, 17.28 *ch.*, 46.10 *ch.*, and 38.09 *ch.*

15. How much will it cost at $3.25 per rod to fence a field whose sides are 808 *ft.*, 975 *ft.*, 822 *ft.*, and 992 *ft.* respectively?

16. How many posts placed 8 *ft.* apart will be required to fence a railroad 14 miles in length? How many feet of wire will be required, the fence being 5 wires high?

SQUARE MEASURES.

316. Square Measure is used in measuring surfaces, as land, paving, painting, plastering, roofing, etc.

The unit of square measure is a square bounded by lines of some known length. Thus, a square inch is a square whose sides are one inch long; a square foot, a square whose sides are one foot long; etc.

TABLE.

144 Square Inches (*sq. in.*) = 1 Square Foot . . . *sq. ft.*
9 Square Feet = 1 Square Yard . . . *sq. yd.*
30¼ Square Yards = 1 Square Rod . . . *sq. rd.*
160 Square Rods = 1 Acre . . . . *A.*
640 Acres = 1 Square Mile . . . *sq. mi.*

NOTES.—1. 1 Rood = 40 *sq. rds.* = ¼ *A.* The rood has practically gone out of use.

2. All of the above, excepting the acre, are derived from the corresponding units of Linear Measure. Thus, 1 *sq. ft.* = 144 (12 × 12) *sq. in.*; 1 *sq. yd.* = 9 (3 × 3) *sq. ft.*; 1 *sq. yd.* = 30¼ (5½ × 5½) *sq. rd.*

3. The acre is the common unit of land measure, and is equivalent to a square whose side is 208.71 feet, or a rectangle 10 rods by 16 rods (165 *ft.* by 264 *ft.*).

4. Roofers, plasterers, and carpenters sometimes call 100 square feet a *square*.

317. Surveyors' Square Measure is used in measuring land.

TABLE.

10000 Square Links (*sq. l.*) = 1 Square Chain . . . *sq. ch.*
10 Square Chains = 1 Acre . . . . *A.*

NOTE.—In the vicinity of St. Louis, and in other parts of the Mississippi valley that were settled by the French, the old French *arpent* is still used as the unit of land measure. It contains about ⅚ of an English acre.

318. U. S. Public Lands are divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form *townships* of six miles square.

Townships are subdivided into *sections*, containing, as nearly as may be, 640 acres each, or 1 square mile.

Sections are subdivided into *half-sections, quarter-sections, half-quarter-sections,* and *quarter-quarter-sections.*

314. Surveyors' Linear Measure is used in measuring land, roads, etc.

The unit of this measure is a chain, 4 rods or 66 feet in length, called Gunter's Chain. It is divided into 100 parts called *links*, each link being 7.92 inches in length.

TABLE.

		mi.		ch.		ft.		l.		in.
100 Links (*l.*) = 1 Chain . . .	*ch.*	1	=	80	=	5280	=	8000	=	63360
80 Chains = 1 Mile . . .	*mi.*			1	=	66	=	100	=	792
						.66	=	1	=	7.92

NOTES.—1. Links are written decimally as hundredths of a chain.

2. For railroad and other purposes, engineers use a chain or tape 100 feet long, the feet being divided into tenths.

3. 1 rod = 25 links.

EXAMPLES.

315. *1.* Add 9 *ft.* 8 *in.*, 12 *ft.* 6 *in.*, 16 *ft.* 5 *in.*, 15 *ft.* 11 *in.*, 21 *ft.* 4 *in.*, 17 *ft.* 3 *in.*

2. In ⅝ of a yard, how many inches?

3. How many feet in 17 miles? In 35 rods?

4. Find the difference between 5 *ft.* 4½ *in.* and 16¼ hands.

5. Reduce 49175 *ft.* to higher denominations.

6. Reduce 32 *rd.* 4 *yd.* 2 *ft.* 10 *in.* to inches.

7. How many feet in ⅜ of a mile?

8. In 4376 feet, how many chains? How many inches?

9. In 396 rods, how many chains? How many feet?

10. In 37.56 chains, how many feet? How many rods?

11. Children's size 1 of shoemakers' measure is 4¼ inches long: what is the length of boys' size 8, youths' size 1, and men's size 10? (Size 1 of the second series is one size longer than size 13 of the first series.)

12. How many fathoms in 1722 *ft.*? In 3136 *ft.*?

13. Reduce 48276 *ft.* to higher denominations.

14. Add 4.16 *ch.*, 3.75 *ch.*, 8.08 *ch.*, 17.28 *ch.*, 46.10 *ch.*, and 38.09 *ch.*

15. How much will it cost at $3.25 per rod to fence a field whose sides are 808 *ft.*, 975 *ft.*, 822 *ft.*, and 992 *ft.* respectively?

16. How many posts placed 8 *ft.* apart will be required to fence a railroad 14 miles in length? How many feet of wire will be required, the fence being 5 wires high?

EXAMPLES.

321. *1.* Reduce 28140 square rods to acres.

2. How many square feet in 3 acres?

3. Reduce 4 *A.* 100 *sq. rd.* 20 *sq. yd.* to square yards.

4. Reduce 46.3125 *A.* to integers of lower denominations.

5. How many acres in the State of Wisconsin, whose area is 53924 square miles?

6. How many square feet in a lot 25 feet front and 100 feet deep? **(320.)**

7. How many square feet in a roof 20 *ft.* wide and 45 *ft.* long?

8. How many square feet in a floor 42 feet long and 33 feet wide? How many square yards?

9. How many square feet in a tight board fence 8 *ft.* high and 120 *ft.* long?

10. How many building lots, each 36 *ft.* by 110 *ft.*, can be made from a lot containing 5 acres?

11. How many acres in a farm 384 rods long and 245 rods wide?

12. How many acres in a rectangular field 28.50 chains by 46.38 chains?

13. How many acres in a rectangular piece of land 224 links by 448 links?

14. How many square yards in a floor 16 *ft.* 6 *in.* by 12 *ft.* 9 *in.*?

15. How many square feet of floor in a 3-story building 60 *ft.* by 98 *ft.*?

16. A ceiling, whose area is 720 *sq. ft.*, is 30 *ft.* long. What is its width?

17. What is the value of a field 320 *rd.* long and 160 *rd.* wide, at $22.50 an acre?

18. A rectangular lot contains 24 acres; what is its width, its length being 1056 feet?

19. What part of a square foot is a surface 3 *in.* by 8 *in.*? 4 *in.* by 9 *in.*? 8 *in.* by 12 *in.*?

20. How many square yards of oil-cloth will cover a floor 15 *ft.* long, 13½ *ft.* wide?

21. A railroad passes through 5808 feet of a farm. If the area occupied is 50 *ft.* wide, what is the cost of the right of way at $66 per acre?

22. How many square feet in 60 panes of glass each 24 *in.* by 30 *in.*?

23. How many square yards of paving in a street 1200 *ft.* long and 60 *ft.* wide?

24. How many square feet in a sidewalk 6 *ft.* wide and $\frac{5}{8}$ of a mile long?

25. What part of a square yard is a surface 6 *in.* × 8 *in.*? 10 *in.* × 14 *in.*? 14 *in.* × 20 *in.*?

26. How many shingles, 3 *in.* wide and 4 *in.* of the length exposed to the weather, would be required for a square yard of roofing?

27. How many shingles, 5 *in.* wide and 4 *in.* of the length exposed to the weather, would be required for a roof 60 *ft.* long and 24 *ft.* wide?

28. How many square feet in a piece of tin 20 *in.* × 14 *in.*?

29. How many pieces of tin, 14 *in.* × 20 *in.*, will be required for a roof 60 *ft.* long and 49 *ft.* wide, making no allowance for seams and waste?

30. How many brick, upper surface 4 *in.* × 8 *in.*, will be required for a walk, 6 *ft.* wide and 660 *ft.* long?

31. How many paving stones, 6 *in.* by 8 *in.*, will be required for a street 50 *ft.* wide and 1248 *ft.* long?

32. Find the value of a quarter-section (**318**) of land at $6.50 per acre.

33. $\frac{2}{3}$ of the land in a western township (**318**) is assessed at $6 per acre and the remainder at $8. What is the total assessment?

34. How much will it cost to build a road from the common corner of sections 6, 5, 7, and 8 (see diagram, Art. **318**) to the common corner of sections 2, 1, 12, and 11 at $1.25 per linear rod? How many acres of land will be occupied if the road is 4 rods wide?

35. How many square feet in the walls of a room, 16 *ft.* wide, 18 *ft.* long, and 9 *ft.* high?

NOTE.—There are 2 ends each 16 *ft.* × 9 *ft.*, and 2 sides each 18 *ft.* by 9 *ft.* The following method is frequently used by mechanics: Multiply the perimeter (the distance around the room) by the height. To find the perimeter, add twice the length to twice the width.

36. How many square inches of gold leaf would be required to cover a box 7 *in.* × 4 *in.* × 3 *in.*? (Use crayon box as an illustration.)

37. How many square yards of plastering surface in the sides and ends of a room, 9 *ft.* high, 16 *ft.* long, 15 *ft.* wide? How many square yards in the ceiling?

38. How many square yards of plastering surface in a room 12 *ft.* high, 20 *ft.* long, and 18 *ft.* wide, deducting 120 *sq. ft.* for doors and windows?

39. How many square feet of painting surface, excepting the bottom, on the outside of a car, 30 *ft.* long, 8 *ft.* wide, 7 *ft.* high?

40. How many square feet of tin plate would be required for making 1000 rectangular cans 8 *in.* × 6 *in.* × 15 *in.*, adding 7 *sq. in.* for seams and waste in making each can?

41. How many yards of paper border, 5 strips in a piece, would be required for a room 16 *ft.* 3 *in.* long and 12 *ft.* 6 *in.* wide, adding 2 *ft.* 6 *in.* for chimney, jambs, etc.?

42 How many squares (100 square feet) in a roof 40 *ft.* by 60 *ft.*? 45 *ft.* by 64 *ft.*?

NOTE.—Divide by 100 by pointing off 2 figures at the right.

43 How many shingles, exposed portion 5 *in.* by 5 *in.*, in a square of 100 square feet? If exposed portion is 4 *in.* by 4 *in.*, how many? 4 *in.* by 5 *in.*, how many?

44. How many sheets of tin 20 *in.* by 14 *in.* in a square of 100 square feet? 10 *in.* by 14 *in.*? 8 *in.* by 10 *in.*?

45. How many shingles would be required for a roof 60 *ft.* by 80 *ft.*, if 500 shingles will cover a square? How many, 700 shingles to a square? How many, 800 shingles to a square?

46. How many sheets of tin would be required for a roof 30 *ft.* by 50 *ft.*, if 50 sheets will cover a square? How many, 62 sheets to a square?

47. What part of a square yard in a piece of carpet 27 *in.* wide and 1 *yd.* long?

48. How much will it cost to carpet a floor 15 *ft.* by 18 *ft.* with carpeting ¾ *yd.* wide, at $1.60 per yard, making no allowance for waste in matching figures, etc.?

NOTE.—If no allowance is made for waste in cutting, divide the number of square yards in the floor by the number of square yards in one linear yard of carpet. Carpet dealers in estimating the number of yards of carpet required for a room, multiply the length of the room (plus a proper allowance for matching the design) by the number of full widths. Ingrain carpet is usually 1 *yd.* wide, Brussels, Moquette, Wilton, Velvet, and Axminster, ¾ *yd.* wide.

49. How many whole widths of carpet 1 *yd.* wide would be required for a room 14 *ft.* 8 *in.* wide, and how many inches would be folded under? If the room is 19 *ft.* 6 *in.* long, how many yards of carpet would be required, supposing the excess at the sides to be folded under, and 2½ *yds.* to be wasted in cutting and matching the figures?

50. How many yards of carpet border would be required for a room 21 *ft.* by 16½ *ft.*?

51. The height of a flight of stairs is 12 *ft.* How many steps, if they are each 8 *in.* high? How many yards of carpet would it be necessary to purchase if the tread of each step is 10 *in.*, and allowing one yard for moving? (Find what part of a yard is required for one step.)

52. How many square yards in a roll of paper 8 *yd.* long and 18 *in.* wide? How many *sq. ft.*?

53. How many rolls of paper, 8 *yd.* long, 18 *in.* wide, would be required for the sides and ends of room 22 *ft.* 6 *in.* long, 13 *ft.* 6 *in.* wide, and 9 *ft.* high, deducting 12 *sq. yd.* for doors and windows, and making no allowance for waste in cutting?

NOTE.—If no allowance is made for waste in cutting, divide the surface to be papered by the number of square feet (or square yards) in one roll of paper.

In practice, there is a great deal of waste in cutting and matching wall paper. If the room is 9 *ft.* high, but two whole strips could be cut from a roll 8 *yds.* long. If double rolls (16 *yds.* each) are used, 5 whole strips could be cut from each roll. It is therefore more economical to use double rolls.

Paper-hangers in estimating the number of rolls required for a room, calculate the number of full strips that will be necessary for the regular surface of the walls, and divide this number by the number of whole strips that can be cut from one roll. The ends of rolls are used for the surface above the doors, and above and below the windows, and other irregular places.

54. How many strips of paper, 18 *in.* wide, would be required for a surface 24 *ft.* wide?

55. How many whole strips of paper, 8 *ft.* 9 *in.* long, could be cut from a roll of paper, 8 *yd.* long? How many from a double roll, 16 *yd.* long?

56. How many rolls of paper (16 *yd.* long, 18 *in.* wide) would be required for the sides and ends of a room, 20 *ft.* long, 16 *ft.* wide, and 8 *ft.* 6 *in.* high, deducting 31 *ft.* for the width of doors, windows, mantels, etc.? (Paper-hangers' method.)

SOLID OR CUBIC MEASURE.

322. Solid or Cubic Measure is used in measuring solids, or bodies which have length, breadth, and thickness or depth.

The unit of cubic measure is a cube, each of whose edges is a unit of some known length. Thus, a cubic inch is a cube, each of whose edges is one inch; a cubic foot is a cube, each of whose edges is one foot; etc.

TABLE.

1728 Cubic Inches (*cu. in.*)	=	1 Cubic Foot	*cu. ft.*
27 Cubic Feet	=	1 Cubic Yard	*cu. yd.*
128 Cubic Feet	=	1 Cord	*cd.*

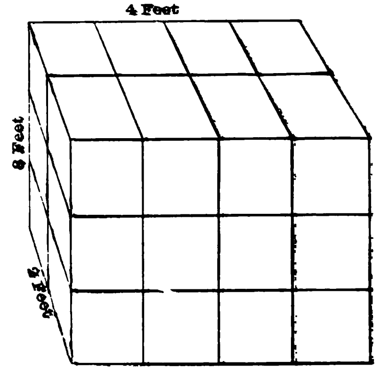
NOTES.—1. The above units, excepting the cord, are derived from the corresponding units of linear measure. Thus, 1 cubic foot contains 1728 (12 × 12 × 12) cubic inches; 1 cubic yard, 27 (3 × 3 × 3) cubic feet.

2. The U. S. measurement ton for freight contains 40 cubic feet.

3. The U. S. register tonnage (entire internal cubical capacity) of vessels is expressed in tons of 100 cubic feet each.

4. A perch of masonry is 1 rod long, 1½ feet thick, and 1 foot high, and is equal to 24¾ (16½ × 1½ × 1) or about 25 cubic feet.

323. The **Volume** or **Solid Contents** of a solid is an expression for that solid in terms of cubic or solid units.



The diagram represents a solid 4 feet long 3 feet broad, and 2 feet thick. Each small cube is a cubic foot. Since the end of the solid contains (3 × 2) 6 square feet of surface, it is evident, if a section 1 foot thick be cut off from this end, it can be divided into 6 cubes, with edges 1 foot in length, and therefore the section will contain 6 cubic feet; and since the whole solid is 4 feet long, and contains 4 like sections, it must contain 4 times 6 cubic feet, or twenty-four cubic feet. Hence the volume of a rectangular solid may be found by multiplying together the numbers expressing its length, breadth, and thickness, in the same denomination; or, more briefly,

To find the volume of a rectangular solid, multiply together its length, breadth, and thickness.

324. A **Rectangular Solid** is a solid having six rectangular sides or faces.

A *Cube* is a rectangular solid whose sides are six equal squares.

EXAMPLES.

325. *1.* How many cords in 15744 cubic feet?

2. How many cubic inches in 175 cubic feet?

3. Reduce 37368 cubic feet to cubic yards.

4. Add 7 *cd.* 49 *cu. ft.*, 13 *cd.* 92 *cu. ft.*, 12 *cd.* 28 *cu. ft.*, 16 *cd.* 110 *cu. ft.*, 3 *cd.* 16 *cu. ft.*, 14 *cd.* 80 *cu. ft.*

5. How many cubic yards in an excavation, 42 *ft.* long, 40 *ft.* wide, and 9 *ft.* deep?

6. How much will it cost to dig a cellar 36 *ft.* long, 30 *ft.* wide, and 6 *ft.* deep, at 30 cents per cubic yard?

7. What is a pile of wood, 20 *ft.* long, 4 *ft.* wide, and 7 *ft.* 6 *in.* high, worth at $5.75 per cord?

8. If a pile of bark is 40 *ft.* long and 4 *ft.* wide, how high must it be to contain 10 cords?

9. How many cords in a pile of bark, 22.5 *ft.* long, 4 *ft.* wide, and 4.8 *ft.* high?

10. How many cubic feet in a box, 4 *ft.* 6 *in.* high, 8 *ft.* long, and 3 *ft.* 9 *in.* wide?

11. How many cubic inches in a rectangular cistern, 6 *ft.* × 5 *ft.* × 4½ *ft.*?

12. How many tons in a shipment which occupies a space, 16 *ft.* by 15 *ft.* by 28 *ft.*? (**322**, 2.)

13. What is the freight of 350 *cu. ft.* of merchandise at $8 per ton? At 50 shillings per ton?

14. How many cubic feet in a vessel whose measurement is 2135 tons? (**322**, 3.)

15. How many perch of masonry in a wall, 40 *ft.* long, 9 *ft.* high, and 1½ *ft.* thick? (1 Perch = 25 *cu. ft.*) (**322**,4.)

16. How many bricks 2 *in.* × 4 *in.* × 8 *in.* in one cubic foot?

17. The space occupied by a bag containing 1000 standard silver dollars is 12 *in.* long, 9 *in.* wide, and 4 *in.* deep. How many cubic feet would be occupied by 1,000,000 such dollars?

18. How many cubic feet is a wall, 12 *in.* thick, 42 *ft.* long, and 30 *ft.* high? How many bricks would be required for the above wall allowing 21 to a cubic foot? What would be their value at $9 per thousand?

Common North River brick are 8 *in.* × 4 *in.* × $2\frac{1}{4}$ *in.* Brick manufactured in other localities are of various sizes.

Builders usually allow 7 common bricks for each square foot of the surface of the wall if the wall is one brick thick (4 *in.*), 14 to a square foot if 2 bricks thick (about 8 *in.*), 21 to a square foot if 3 bricks thick (about 12 *in.*), etc.

19. According to the above builder's rule, how many bricks would be required for a wall, 64 *ft.* long, 39 *ft.* high, and 2 bricks thick (about 8 *in.*)?

20. How many cubic feet of masonry in a cellar wall 2 *ft.* thick, 8 *ft.* high, outside measurement 25 *ft.* by 45 *ft* ?

Builders sometimes multiply the total outside measurement (perimeter or girth) by the height and thickness to find the number of cubic feet. By this method, the corners are counted twice. To find the exact length of the wall, from the total outside measurement subtract four times the thickness of the wall. (See note, Ex. 35, Art. **321**.)

21. How many bricks will be required to build a chimney 28 *ft.* high, if there are 5 courses of brick to each foot, and 16 bricks in each course (flue 4 *in.* by 12 *in.*, double wall) ? How many, 8 bricks in a course (flue 8 *in.* by 16 *in.*, single wall) ?

22. How much stone, lime, and sand will be required for a wall 144 *ft.* by 10 *ft.* by 2 *ft.*, if 128 *cu. ft.* of broken stone, $1\frac{1}{4}$ *bbls.* of lime, and a load (cubic yard) of sand will lay 100 *cu. ft.* of wall ?

23. How many bricks will be required for a house 24 *ft.* wide, 63 *ft.* long, and 30 *ft.* high, allowing 21 bricks for each square foot of surface (the walls being 3 bricks thick), if 225 square feet are deducted for doors and windows, and if the walls are considered 1 *ft.* thick in making deductions for the corners ?

BOARD MEASURE.

326. **Lumber** is measured by *board measure.* The board foot is 1 *ft.* long, 1 *ft.* wide, and 1 *in.* thick; hence it is $\frac{1}{12}$ of a cubic foot.

In measuring boards one inch or less in thickness, the number of square feet of surface which they would cover is measured.

Plank, joists, etc., more than one inch in thickness are reduced to inch boards and measured by board measure.

Boards, plank, scantling, joists, beams, and sawed timber generally are measured by *board* measure ; hewn and round timber are sometimes measured by *cubic* measure.

327. When lumber is not more than one inch thick, to find the number of feet board measure: *Multiply the length in feet by the width in inches, and divide the product by 12.*

16 pieces each containing a board foot could be cut from a board 16 *ft.* long, 12 *in.* wide. A board 11 *in.* wide is $\frac{11}{12}$ of a board, 12 *in.* wide and of the same length and thickness.

When more than 1 inch thick: *Multiply the length in feet by the width and thickness in inches, and divide the product by 12.*

Lumbermen use an automatic *rule* for measuring board, planks, etc. They also use table books for reducing logs or round timber to board measure.

EXAMPLES.

328. *1.* How many square feet would be covered by a board 16 *ft.* long and 9 *in.* wide?

2. If a stick of timber 24 *ft.* long, 8 *in.* wide, 3 *in.* thick, is reduced to boards one inch in thickness, how many square feet would they cover?

3. How many feet of boards would be required for a floor 20 *ft.* wide and 24 *ft.* long? What would be their value at $14 per thousand?

4. How many board feet in 475 cubic feet?

Find the number of feet, board measure, in each of the following boards, joists, beams, etc.:

	Length.	Width.	Thickness.		Length.	Width.	Thickness.
5.	12 *ft.*	12 *in.*	1 *in.*	*9.*	16 *ft.*	8 *in.*	$1\frac{1}{2}$ *in.*
6.	14 *ft.*	8 *in.*	$\frac{1}{2}$ *in.*	*10.*	20 *ft.*	10 *in.*	2 *in.*
7.	16 *ft.*	6 *in.*	$\frac{3}{4}$ *in.*	*11.*	12 *ft.*	6 *in.*	4 *in.*
8.	18 *ft.*	14 *in.*	$\frac{7}{8}$ *in.*	*12.*	24 *ft.*	9 *in.*	3 *in.*

13. How many square feet would be covered by 45 boards, 16 *ft.* long, 8 *in.* wide, if they are laid side by side? (First find the total width in feet.)

14. How many board feet in a pile of lumber 8 *ft.* high, 7 *ft.* wide, and 18 *ft.* long? Find its value at $15 per thousand feet.

15. How many feet, board measure, in 16 boards, each 18 *ft.* long, 10 *in.* wide, and 1 *in.* thick?

16. How many board feet in 24 joists, 20 *ft.* × 8 *in.* × 4 *in.*?

17. How many feet of boards, 4 *in.* wide and 1 *in.* thick, would be required for a fence 5 boards high and one mile long?

18. How many posts 8 *ft.* apart would be required for above fence?

NOTES.—1. *Cong.* is for the Latin *congius*, gallon ; *O.*, for the Latin *octavius*, one-eighth.

2. The symbols precede the numbers to which they refer: thus, *O.* 6 *f*℥ 10, is 6 pints 10 fluidounces.

EXAMPLES.

331. *1.* Reduce 8 *gal.* 3 *qt.* 1 *pt.* to pints.

2. Reduce 875 pints to gallons.

3. Add 4 *gal.* 2 *qt.* 1 *pt.*, 3 *gal.* 3 *qt.* 1 *pt.*, 9 *gal.* 1 *pt.*, 11 *gal.* 1 *qt.*

4. How many barrels in 100000 gallons?

5. 480 English gallons (**329**, 3) equal how many U. S. gallons? (Add $\frac{1}{5}$.)

6. How many cubic feet in 1000 gallons?

7. How many gallons in 300300 cubic inches?

8. How many gallons in a rectangular tank, 8 *ft.* × 4 *ft.* × 6 *ft.*?

OPERATION.—$8 \times 4 \times 6 \times 7\frac{1}{2} = 1440$, approximate result (see Art. **329**, Note 4). $8 \times 4 \times 6 \times 7.48 = 1436.16$, accurate result.

9. How many gallons in a rectangular cistern 6 *ft.* long, 4 *ft.* wide, 3 *ft.* high?

10. How many gallons in a rectangular cistern 16 *ft.* long, 4 *ft.* high, and 5 *ft.* wide?

11. How many gallons in a rectangular reservoir, 40 *ft.* long, 16 *ft.* wide, and 8 *ft.* deep?

12. How many gallons in a cylindrical vessel, 3 *ft.* in diameter and 9 *ft.* high?

OPERATION.—$3 \times 3 \times 9 \times 5\frac{7}{8} = 475\frac{7}{8}$ (see Art. **329**, Note 4).

13. How many gallons in a cylindrical tank 4 *ft.* in diameter and 16 *ft.* deep?

14. How many gallons in a circular reservoir, 40 *ft.* in diameter and 6 *ft.* deep?

15. How many gallons in a circular reservoir 60 *ft.* in diameter and 8 *ft.* deep?

16. Reduce *Cong.* 2 *O.* 6 *f*℥ 10 *f*ʒ 5 to fluidrachms.

17. From the sum of 51 *gal.* 2 *qt.* 1 *pt.*, and 45 *gal.* 1 *qt.* 1 *pt.*, subtract 27 *gal.* 1 *qt.*, and divide the result by 9.

18. How many bottles, each holding 1 *qt.* 1 *pt.* 2 *gi.*, can be filled from a barrel of cider?

DRY MEASURE.

332. Dry Measure is used in measuring dry articles; as salt, grain, fruits, etc.

The unit of this measure is the Winchester bushel, which contains 2150.42 cubic inches.

TABLE.

2 Pints (*pt.*) = 1 Quart . . . *qt.*
8 Quarts = 1 Peck . . . *pk.*
4 Pecks = 1 Bushel . . . *bu.*

bu.		*pk.*		*qt.*		*pt.*
1	=	4	=	32	=	64
		1	=	8	=	16

NOTES.—1. The half-peck or gallon of this measure contains 268.8 cubic inches, and is 37.8 cubic inches larger than the liquid gallon (268.8 − 231 = 37.8).

2. The imperial bushel of England contains 2218.19 cubic inches, and is equal to 1.03 Winchester bushels. In certain localities in Great Britain, 8 bushels are called a quarter.

3. Grain, seeds, etc., are usually sold by weight. For table of equivalents see Art. **338**.

4. 36 bushels = 1 chaldron of coke or charcoal.

5. 1 bushel is equivalent to about 9.3 (2150.42 ÷ 231) wine gallons.

EXAMPLES.

333. *1.* Reduce 2 *bu.* 3 *pk.* 5 *qt.* 1 *pt.* to pints.

2. Reduce 10000 pints to bushels.

3. Find the value of 5 bushels of nuts at 8*c.* per pint.

4. How many cubic inches in 75 bushels?

5. How many bushels in 322563 cubic inches?

6. How many bushels in 400 cubic feet?

NOTE.—Since a bushel is about 1¼ cubic feet, the following approximate rules may be used for all practical purposes:

To reduce cubic feet to bushels: *Deduct one-fifth, or multiply by .8*

The result will be too small by about 4½ bushels for every 1000 bushels of the result.

To reduce bushels to cubic feet: *Add one-fourth, or divide by .8.*

The result will be too great by about 4½ cubic feet for every 1000 cubic feet of the result.

Solve the above example both exactly and approximately, and compare the results.

7. How many bushels will a box 10 *ft.* long, 5 *ft.* wide, and 4 *ft.* high contain? (Approximate method.)

8. How many bushels of grain will a bin 14 *ft.* long, 5½ *ft.* wide, and 6 *ft.* high contain?

9. Find the capacity in bushels of a crib, 20 *ft.* long, 8 *ft.* high, 4 *ft.* wide at the bottom and 6 *ft.* wide at the top.

NOTE.—To find the average width, take one-half the sum of the top and bottom measurements.

10. A crib 24 *ft.* × 8 *ft.* × 6 *ft.* is filled with unshelled corn. How many bushels of shelled corn would this quantity produce, if two cubic feet of corn in the ear will make one bushel of shelled corn?

MEASURES OF WEIGHT.

334. Troy Weight is used in weighing gold, silver, coins, and jewels; also in philosophical experiments.

The unit of weight is the Troy pound, which contains 5760 grains. A cubic inch of distilled water weighs 252.458 of these grains, when the height of the barometer is 30 inches, and the temperature of the air and water 62° Fahrenheit.

TABLE.

24 Grains (*gr.*)	= 1 Pennyweight	*pwt.*
20 Pennyweights	= 1 Ounce . . .	*oz.*
12 Ounces	= 1 Pound . . .	*lb.*

lb.	*oz.*	*pwt.*	*gr.*
1 =	12 =	240 =	5760
	1 =	20 =	480

NOTE.—The carat, used in weighing diamonds, equals 3.2 Troy grains.

The term carat is also used to denote the fineness of gold, and means $\frac{1}{24}$ part. Thus, gold 18 carats fine contains 18 parts pure gold and 6 parts alloy.

335. Apothecaries' Weight is used in prescribing and compounding medicines not liquid.

The pound, ounce, and grain of this weight are the same as those of Troy weight, the division of the ounce being different.

TABLE.

20 Grains (*gr.*)	= 1 Scruple . .	*sc.* or ℈.
3 Scruples	= 1 Dram . .	*dr.* or ʒ.
8 Drams	= 1 Ounce . .	*oz.* or ℥.
12 Ounces	= 1 Pound . .	*lb.* or ℔.

℔	℥	ʒ	℈	*gr.*
1 =	12 =	96 =	288 =	5760
	1 =	8 =	24 =	480
		1 =	3 =	60

NOTES.—1. The symbols precede the numbers to which they refer; thus, ℥6 ʒ4, is 6 ounces 4 drams.

2. Drugs and medicines are sold in large quantities by Avoirdupois weight.

336. Avoirdupois Weight is used in weighing all articles, excepting gold, silver, precious stones, and medicines in small quantities.

The Avoirdupois pound contains 7000 Troy grains.

TABLE.

16 Ounces (*oz.*) = 1 Pound . . . . *lb.*
100 Pounds = { 1 Hundred-weight, or *cwt.*
{ 1 Cental . . . . *C.*
20 Hundred-weight = 1 Ton . . . . . *T.*

T.	*cwt.*	*lb.*	*oz.*
1 =	20 =	2000 =	32000
	1 =	100 =	1600
		1 =	16

NOTES.—1. The ounce is divided into halves and quarters.

2. The dram, $\frac{1}{16}$ of an ounce, is now little used, except by silk manufacturers.

3. The Long or Gross ton, formerly used, contained 2240 pounds ; the hundred-weight, 112 pounds ; and the quarter, 28 pounds.

These weights are still used in Great Britain, at the U. S. Custom Houses, in ocean freights, and by wholesale dealers in coal and iron.

337. Comparison of Troy and Avoirdupois weights.

5760 grains = 1 *lb.* Troy.
7000 grains = 1 *lb.* Avoirdupois.
480 grains = 1 *oz.* Troy.
437½ grains = 1 *oz.* Avoirdupois.

338. In buying and selling grain, seeds, and other produce, the bushel is regarded as a certain number of pounds. The Boards of Trade of the principal cities of the United States use the equivalents given in the following table :

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL.

Commodities.	Lbs.	Commodities.	Lbs.	Commodities.	Lbs.
Barley.........	48	Corn, shelled...	56	Peas..........	60
Beans..........	60	Corn in the ear.	70	Rye............	56
Buckwheat.....	48	Malt...........	34	Timothy Seed..	45
Clover Seed....	60	Oats...........	32	Wheat.........	60

In the Liverpool, San Francisco, and some other markets, produce is bought and sold by the *cental* of 100 pounds. Railway freight tariffs in the United States on grain, provisions, etc., are reckoned per *cwt.* or cental.

339. The following units are used in commerce :

1 Quintal of Fish = 100 *lbs.*
1 Barrel of Flour = 196 *lbs.*
1 Barrel of Pork or Beef = 200 *lbs.*
1 Gallon Petroleum = 6¼ *lbs.*
1 Keg of Nails = 100 *lbs.*

EXAMPLES.

340. *1.* Reduce 10000 grains to Troy pounds.

2. Reduce 2 *lb.* 8 *oz.* 16 *pwt.* to grains.

3. What is the weight in Troy ounces of 1000 silver dollars (**112**)? Of 1280 silver dollars?

4. Find the weight in Troy ounces of 1000 gold dollars (**112**).

5. Find the weight in Troy ounces of 2000 half-dollars (**112**).

6. What is the cost of a 14 *k.* watch chain weighing 37⅜ *pwt.* at $1.15 per pennyweight?

7. A watch case, 14 carats fine, and weighing 60 *pwt.*, contains how many ounces of pure gold?

8. Find the value of a diamond weighing $\frac{11}{16}$ of a carat, at $100 per carat.

9. How many Troy ounces of pure silver would be required for the coinage of 2,000,000 standard silver dollars (**113**)? How much copper?

10. Reduce ℔1 ℥9 ʒ6 ℈2 to grains.

11. How many powders, each containing 5 grains, can be made from 1 *lb.* Apothecaries (Troy) of quinine? How many from 1 *lb.* Avoirdupois?

12. Add 8 *lb.* 9 *oz.*, 10 *lb.* 7 *oz.*, 14 *lb.* 15 *oz.*, and 17 *lb.* 13 *oz.* Avoirdupois.

13. How many grains in 16 *lb.* Avoirdupois?

14. In 70 *lb.* Avoirdupois, how many pounds Troy? (**337**)

15. In 175 *oz.* Troy, how many ounces Avoirdupois?

16. Find the cost of 58 *lb.* 4 *oz.* of butter at 28*c.* per pound.

17. Find the cost of 875 pounds of feed at $1.15 per *cwt.* (**273**)

18. Find the cost of 17387 pounds of oats at $1.85 per cental.

19. Find the cost of 21370 pounds of hay at $8 per ton.

OPERATION.

$$\begin{array}{r} 2\,\underline{)\,21370} \\ 10.685 \\ \underline{8} \\ \$85.480 \end{array}$$

ANALYSIS.—21370 *lb.* = 10.685 (21370 ÷ 2000) tons. If 1 ton costs $8, 10.685 tons will cost 10.685 times $8, or $85.48. To divide by 2000, divide by 2 and place the point in the quotient three places to the left.

20. What is the value of 28140 pounds of straw at $6.50 per ton?

21. Find the cost of 16480 pounds of hay at $11.50 per ton.

NOTES.—1. 1 Crown = 5 shillings, or $\frac{1}{4}$ of a pound ($1.216+).

2. 1 Guinea = 21 shillings ($5.11). It is not now coined.

3. The gold coins of Great Britain are 22 carats ($\frac{11}{12}$), or .916$\frac{2}{3}$ fine. (The old carat system (**334**, note) is generally abandoned except for jewelry. 1 carat = .041$\frac{2}{3}$.) The silver coins of Great Britain are .925 ($\frac{37}{40}$) fine.

EXAMPLES.

342. *1.* Add £27 16*s.* 10*d.*, £6 10*s.* 8*d.*, £47 15*s.* 11*d.*, £25 7*s.* 6*d.*, £3 14*s.* 8*d.*, and £23 16*s.* 3*d.*

2. From £17 8*s.* 4*d.* subtract £10 12*s.* 8*d.*

3. Multiply £5 6*s.* 3*d.* by 8.

4. Reduce 8375*d.* to shillings and pounds.

5. Reduce £12 16*s.* 8*d.* to pence.

6. What is the cost of 466 yards of cloth at 9½*d.* per yard?

7. Find the value of 4120 *bu.* wheat at 4*s.* 4½*d.* per bushel.

8. In 47 guineas, how many pounds and shillings?

9. Divide £16 5*s.* 6*d.* by 9; by 7; by 31.

10. How many yards of cloth at 3*s.* 7*d.* per yard can be bought for £7? For £9 5*s.* 6*d.*?

11. What is the cost of 20 *yd.* silk at 10*s.* 6*d.* per yard?

12. Reduce £8 17*s.* 8*d.* to the decimal of a pound. (**293**)

NOTE.—The following method for reducing shillings and pence to the decimal of a pound is sufficiently accurate for most business purposes: *Write one-half of the greatest even number of shillings as tenths, and if there be an odd shilling write 5 hundredths; multiply the number of pence by 4, and write the product as thousandths. If the product is between 12 and 36, add 1 to the thousandths; if between 36 and 48, add 2 to the thousandths.* Thus, £8 17*s.* 8*d.* = £8 + £.85 + £.033 = £8.883.

Reduce mentally the following to the decimal of a pound:

13. 16*s.* 2*d.*

14. 18*s.* 5*d.*

15. 10*s.* 8*d.*

16. 17*s.* 4*d.*

17. 7*s.* 3*d.*

18. 13*s.* 11*d.*

19. Reduce £.821 of a pound to shillings and pence. (**289**)

NOTE.—This example can be performed mentally by reversing the operation explained in note to Ex. 12. *Multiply the number of tenths by 2, and write the product as shillings* (2 × 8 = 16). *Divide the number of thousandths expressed by the 2nd and 3rd figures by 4, and write the quotient as pence* (21 ÷ 4 = 5). £.821 = 16*s.* 5*d.*

If the second figure to the right of the point is 5 or more than 5, it is evident that there is an odd number of shillings, and the decimal must be separated into two parts before applying the above rule.

Thus, £.875 = £.85 + £.025. £.85 = 17*s.* (2 × 8½). £.025 = 6*d.* (25 ÷ 4).

Reduce mentally the following to shillings and pence:

20. £.425; £.637. *22.* £.255; £.183.

21. £.817; £.245. *23.* £.376; £.496.

24. Divide £15 16*s.* 8*d.* by .10; by .20; by .25; by .40.

25. Multiply £16 12*s.* 9*d.* by .05; by .06; .04.

26. If £1 sterling is worth $4.87, what is the value of £225 18*s.* 6*d.*? Of £140 8*s.* 8*d.*?

27. If £1 sterling is worth $4.88, how many pounds can be bought for $1000? How many for $1625?

MISCELLANEOUS TABLES.

343. The following table is used in counting certain articles:

12 Units = 1 Dozen . . . *doz.*
12 Dozen = 1 Gross . . . *gr.*
12 Gross = 1 Great Gross . *g.gr.*

g.gr.		*gr.*		*doz.*		*units.*
1	=	12	=	144	=	1728
		1	=	12	=	144

344. The following table is used in the paper trade:

24 Sheets = 1 Quire . . . . *qr.*
20 Quires = 1 Ream . . . . *rm.*

rm.		*qr.*		*sheets.*
1	=	20	=	480

Manufacturers and wholesale dealers usually sell paper by the pound.

EXAMPLES.

345. *1.* Find the value of 5 gross pencils at 4*c.* each.

2. A merchant buys 6 gross pens at 95*c.* per gross, and sells them at 1*c.* each. What is his profit?

3. How many sheets of paper in 12 quires? In 2 reams?

4. Find the difference between six dozen dozen and half a dozen dozen.

5. Combs are bought at $2.70 per dozen. How much is that piece?

6. Find the value of 306 eggs at 22*c.* per dozen.

7. At 1 cent each, what is the value of 20 great gross pens?

8. A grocer buys 81 dozen eggs at 22*c.* per dozen, and sells them at the rate of 9 for 25 cents. What is his profit?

9. If 32 pages of a book are printed on one sheet, how many reams of paper would be required for 2000 copies containing 384 pages each?

CIRCULAR MEASURE.

346. Circular or **Angular Measure** is used in measuring angles and arcs of circles. It is employed principally by surveyors in determining directions, by navigators in determining latitude and longitude of places, and by astronomers in making observations.

The unit of this measure is the *degree*, which is $\frac{1}{360}$ of the circumference of any circle.

TABLE.

60 Seconds (") = 1 Minute . . . . . . '.
60 Minutes = 1 Degree . . . . . . °.
360 Degrees = 1 Circle . . . . . . C.

NOTES.—1. A quadrant is one-fourth of a circle, or 90°.

2. A sextant is one-sixth of a circle, or 60°.

3. 1 minute of the circumference of the earth is called a nautical, or geographic mile, and is about 1.15 statute or common miles.

4. An arc of 1 degree of the circumference of the earth measured at the equator equals 69.16 statute miles.

EXAMPLES.

347. *1.* Add 74° 0′ 3″ and 77° 49′ 58″.

2. Add 12° 27′ 14″ and 122° 26′ 45″.

3. From 84° 29′ 31″ subtract 77° 0′ 45″.

4. Multiply 13° 11′ 16″ by 5 ; by 15.

5. Divide 76° 11′ 45″ by 15 ; by 12.

6. Divide 179° 42′ 15″ by 15 ; by 16.

7. Reduce 1,000,000″ to higher denominations.

8. Reduce 44° 16′ 40″ to seconds.

9. The angles of a triangle are 67° 18′ 40″, 72° 39′ 50″, and 40° 1′ 30″ respectively. What is their sum ?

LONGITUDE AND TIME.

348. The whole circle of the earth, or 360°, passes under the sun in 24 hours, and in 1 hour passes $\frac{1}{24}$ of 360°, or 15°; in 1 minute, $\frac{1}{60}$ of 15° (15 × 60′), or 15′; and in 1 second, $\frac{1}{60}$ of 15′ (15 × 60″), or 15″.

349. Comparison of **Longitude** and **Time.**

For a difference of	There is a difference of
15° in Longitude	1 *hr.* in Time.
15′ " "	1 *min.* " "
15″ " "	1 *sec.* " "
1° " "	4 *min.* " "
1′ " "	4 *sec.* "
1″ " "	$\frac{1}{15}$ *sec.* " "

350. Rule.—*1. The difference in longitude of two places, expressed in ° ′ ″, divided by 15 will produce their difference in time expressed in hours, minutes, and seconds.*

2. The difference in solar time of two places, expressed in hr. min. sec., multiplied by 15 will produce their difference in longitude expressed in ° ′ ″.

351. TABLE OF LONGITUDES.

Albany	73° 44′ 50″ W.	New York	74° 0′ 3″ W.
Ann Arbor	80° 43′ W.	New Orleans	90° 2′ 30″ W.
Boston	71° 3′ 30″ W.	Paris	2° 20′ 22″ E.
Berlin	13° 23′ 45″ E.	Philadelphia	75° 10′ W.
Calcutta	88° 19′ 2″ E.	Rome	12° 27′ 14″ E.
Cincinnati	84° 29′ 31″ W.	Richmond, Va	77° 25′ 45″ W.
Chicago	87° 37′ 45″ W.	San Francisco	122° 26′ 45″ W.
Jefferson City, Mo.	92° 8′ W.	St. Paul, Minn	95° 4′ 55″ W.
London	0° 5′ 38″ W.	St. Louis, Mo	90° 15′ 15″ W.
Mexico	99° 5′ W.	Washington, D. C	77° 0′ 15″ W.

352. Standard Time.—In 1883, the principal railroads and cities of the United States and Canada adopted the time of four different meridians as the standard time of four belts or sections comprising the whole of the above countries. The most eastern of these sections embraces the Eastern and Middle States, Maryland and Virginia, and extends about 7½° east and west of the meridian of 75° west of Greenwich (near Philadelphia). The time of this meridian, called Eastern time, is used in this section, and is 5 hours slower than Greenwich time. The time of the meridian of 90° west of Greenwich (near St. Louis) called Central time, is used in the next section, and is 1 hour slower than Eastern time. The time of the meridian of 105° west of Greenwich (near Denver), called Mountain time, is used in the Rocky Mountain region, and is 1 hour slower than Central time and 2 hours slower than Eastern time. The time of the meridian of 120° west of Greenwich, called Pacific time, is used in the Pacific slope, and is 3 hours slower than Eastern time.

EXAMPLES.

353. Find the difference in longitude between

1. New York and London.
2. Boston and Paris.
3. Chicago and San Francisco.
4. St. Louis and Calcutta.
5. Philadelphia and Berlin.
6. San Francisco and Calcutta.

Find the difference in solar time between

7. New York and Greenwich.
8. Chicago and New York.
9. Richmond and Calcutta.
10. Rome and London.
11. Paris and Albany.
12. Calcutta and Jefferson City.

Find mentally the difference in standard time between

13. Albany and Denver.
14. New York and Chicago.
15. Boston and San Francisco.
16. St. Louis and Richmond.
17. St. Paul and Sacramento.
18. Phila. and Portland, Me.

Find the difference between the standard time and the solar time of the following cities:

19. Boston.
20. Philadelphia.
21. San Francisco.
22. St. Louis.
23. Chicago.
24. St. Paul.

25. A navigator finds that when it is noon at his place of observation, it is 16 *min.* 34 *sec.* past 10 P.M. by his chronometer, Greenwich time; what is his longitude?

26. When it is 6 o'clock P.M., standard time, at Richmond, Va., what is the time at St. Louis, Mo.?

27. If the difference of solar time between two places is 1 *hr.* 18 *min.* 4 *sec.*, what is the difference of longitude?

28. When it is 20 *min.* past 2 P.M., standard time, at Boston, Mass., what o'clock is it at San Francisco?

29. When it is Monday, 10 P.M., standard time, in Chicago, what day and time is it in London (Greenwich time)?

30. When it is 9 o'clock P.M., solar time, in San Francisco, it is 3 *min.* $3\frac{2}{15}$ *sec.* past 11 A.M. in Calcutta; what is the longitude of San Francisco, if the longitude of Calcutta is 88° 19′ 2″ E.?

31. When it is noon, solar time, in Chicago, it is 5 *min.* $29\frac{1}{4}$ *sec.* of 1 P.M., solar time, in New York; what is the longitude of Chicago, the longitude of New York being 74° 3″ W.?

Notes.—1. The meter, like the yard, is used in measuring cloths, ribbons, laces, short distances, etc.

2. The kilometer is used in measuring long distances, and is about $\frac{5}{8}$ of a mile.

3. The centimeter and millimeter are use by artisans and others in measuring minute lengths. The other denominations are rarely used.

EXAMPLES.

359. *1.* Reduce 875275 meters to kilometers.

Analysis.—Since 1 kilometer equals 1000 meters, in 875275 meters there are as many kilometers as 1000 is contained times in 875275, or 875.275. To divide by 1000, place the point three places to the left (**270**, 4).

2. Reduce 675.318 kilometers to meters.

Analysis.—Since 1 kilometer equals 1000 meters, in 675.318 kilometers there are 675.318 times 1000, or 675318 meters. To multiply by 1000, place the point three places to the right (**267**, note).

3. Reduce 383.64 meters to centimeters; to kilometers.

4. Reduce 175.16 centimeters to kilometers; to meters.

5. Reduce to meters and find the sum of 876.2 decimeters, 30347 centimeters, 176.48 meters, 8.175 kilometers.

6. A ship sails 5712 kilometers in 48 days; how many kilometers does she sail per day?

7. What is the value of 56.4 meters of silk at $1.75 per meter?

8. 16 pieces of cloth contain 38.5 meters each; 18 pieces contain 39 meters each; and 24 pieces contain 41.2 meters each; how many meters in all?

9. How many meters of ribbon at 27 cents per meter can be purchased for $245.70?

10. If the forward wheels of a carriage are 3.5 meters in circumference, and the hind wheels 4.8 meters, how many more times will the forward wheels revolve than the hind wheels, in running a distance of 8.4 kilometers?

11. How much will it cost to sewer a street .64 *Km.* long, at $3.75 per meter?

12. How many meters of wire will be required to fence a rectangular field, .72 *Km.* long and .56 *Km.* wide, if the fence is 4 wires high?

13. How long will it take a railway train, running 60 *Km.* per hour, to go from New York to Chicago, the distance being 1440 *Km.*?

SQUARE MEASURE.

360. The unit of square measure is the *square meter.*

TABLE.

100 Square Centimeters, *sq. cm.*	= 1 Square Decimeter	=	15.5+ *sq. in.*
100 Square Decimeters, *sq. dm.*	= 1 SQUARE METER, *Sq. M.*	=	1.196+ *sq. yd.*

NOTES.—1. The square meter is used in measuring flooring, ceilings, etc.; the square decimeter and the square centimeter are used for minute surfaces.

2. Since units of square measure form a scale of hundreds, each denomination must have two places of figures.

361. The unit of **Land Measure** is the *are,* and is equal to a square dekameter (100 square meters), or 119.6 square yards.

TABLE.

	1 Centare....(1 square meter)	=	1550 *sq. in.*
100 Centares, *ca.* =	1 Are..........(100 square meters)	=	119.6 *sq. yd.*
100 Ares, A. =	1 Hectare....(10000 square meters)	=	2.471 acres.

NOTE.—The hectare is the ordinary unit for land.

EXAMPLES.

362. *1.* Write 16 *sq. m.,* 8 *sq. dm.,* 24 *sq. cm.,* having the square meter as the unit. (**360,** 2.) *Ans.* 16.0824.

2. Write 83 *sq. m.,* 9 *sq. dm.,* having the *sq. m.* as the unit.

3. In 47 ares how many square meters?

4. In 60.25 hectares how many centares?

5. How many square meters in a building lot 8 *m.* by 32 *m.*?

6. How many building lots, each containing 225 *sq. m.,* can be formed from a field containing 9 hectares?

7. How many hectares in a farm 1.024 *Km.* in width and 1.625 *Km.* in length?

8. What is the cost of a mirror 2.25 *m.* by 1.44 *m.,* at $3.84 per *sq. m.*?

9. How many lots 25 *m.* wide and 60 *m.* deep, or having an equivalent area, can be laid out from 6 hectares?

10. A man bought a piece of land for $6950.50, and sold it for $7603.30, by which transaction he made $6.80 a hectare; how many hectares were there?

CUBIC MEASURE.

363. The unit for measuring ordinary solids is the ***cubic meter.***

TABLE.

1000 Cu. Millimeters, *cu. mm.*	=	1 Cu. Centimeter	=	.061 *cu. in.*
1000 Cu. Centimeters, *cu. cm.*	=	1 Cu. Decimeter	=	61.027 *cu. in.*
1000 Cu. Decimeters, *cu. dm.*	=	1 Cu. METER	=	35.317 *cu. ft.* 1.308 *cu. yd.*

NOTES.—1. The cubic meter is used in measuring embankments, excavations, etc.; cubic centimeters and cubic millimeters for minute bodies.

2. Since units of cubic measure form a scale of thousands, each denomination must have three places of figures.

364. The unit of **Wood Measure** is the ***ster,*** and is equal to a cubic meter, or 35.317 cubic feet.

TABLE.

10 Decisters, *ds.*	=	1 Ster..........(1 Cubic Meter)	=	.2759 cord. 35.317 *cu. ft.*
10 Sters, *s.*	=	1 Dekaster, *Ds.*.(10 Cubic Meters)	=	2.759 cords.

EXAMPLES.

365. *1.* Write 29 *cu. m.*, 75 *cu. dm.*, having the cubic meter as the unit. (**363**, 2) *Ans.* 29.075 *cu. m.*

2. Write 17 *cu. m.*, 218 *cu. dm.*, 27 *cu. cm.*, having the cubic meter as the unit.

3. How many cubic meters in a box 3.5 *m.* by 3.2 *m.* by 2.5 *m.*?

4. Bought 12 sters of wood; having sold 8.7 cubic meters, how much remained?

5. There are 13 blocks of marble, each containing 370.16 *cu. dm.*; how many cubic meters in all?

6. How many cubic meters in an excavation 13.2 *m.* by 18.5 *m.* by 8.4 *m.*?

7. At $1.25 a cubic meter, what will it cost to dig a cellar 6.5 *m.* long, 5.4 *m.* wide, and 2.5 *m.* deep?

8. How many sters of wood in a pile of wood 2.5 *m.* high, 2 *m.* wide, and 16.5 *m.* long? What is the length of a pile of the same height and width containing 216 sters?

DRY AND LIQUID MEASURE.

366. The unit of **Dry** and **Liquid Measure** is the *liter*, which is equal to a cubic decimeter, 1.0567 wine quarts, or .908 dry quart.

TABLE.

		Dry Measure.	Liquid Measure.
	1 Milliliter......($\frac{1}{1000}$ of a liter)	= .06103 *cu. in.*,	or, .0338 *fl. oz.*
10 *ml.*	= 1 Centiliter.....($\frac{1}{100}$ of a liter)	= .6103 *cu. in.*,	or, .338 *fl. oz.*
10 *cl.*	= 1 Deciliter......($\frac{1}{10}$ of a liter)	= 6.1027 *cu. in.*,	or, .845 *gi.*
10 *dl.*	= 1 LITER.........(1 liter)	= .908 *qt.*,	or, 1.0567 *qt.*
10 *l.*	= 1 Dekaliter.....(10 liters)	= 9.08 *qt.*,	or, 2.6418 *gal.*
10 *Dl.*	= 1 Hectoliter ....(100 liters)	= 2.8375 *bu.*,	or, 26.418 *gal.*
10 *Hl.*	= 1 Kiloliter ......(1000 liters)	= 28.375 *bu.*,	or, 264.18 *gal.*

NOTES.—1. The liter is commonly used in measuring wine, milk, etc., in moderate quantities. For minute quantities the centiliter and milliliter are employed; and for large quantities the dekaliter.

2. For measuring grain, etc., the hectoliter (2.8375 bushels) is commonly used.

3. Instead of the kiloliter and milliliter, it is customary to use their equals, the cubic meter and cubic centimeter.

EXAMPLES.

367. *1.* How many liters in a vessel whose capacity is 1 cubic meter?

2. What is the cost of sixteen liters of milk at 8 cents a liter?

3. How many hectoliters of wheat can be bought for $396 at $5.50 per hectoliter?

4. How many hectoliters of grain can be put in a rectangular bin, 4 *m.* long, 3.5 *m.* wide, and 1.2 *m.* high?

5. How many liters in 63.5 dekaliters? In 83.75 hectoliters?

6. At $1.75 a liter, what is the cost of 85.6 dekaliters of wine?

7. How many hectoliters in 16 cubic meters?

8. How many bags, each holding 1 hectoliter, can be filled from a bin, 1.5 *m.* high, 2.4 *m.* wide, and 5 *m.* long?

9. A cistern 3.5 *m.* by 3.2 *m.*, and 9 *m.* deep, will hold how many dekaliters?

10. A merchant bought 4 hectoliters of nuts at $8.50 per hectoliter, and retailed them at 12 cents a liter; what was his profit?

WEIGHT.

368. The unit of weight is the *gram,* which is equal to the weight of a cubic centimeter of distilled water in a vacuum, at its greatest density (39.2° F.), or 15.432 grains.

TABLE.

		1 Decigram......($\frac{1}{10}$ of a gram)	=	1.543 *gr.* Tr.
10 *dg.*	=	1 GRAM..........(1 gram)	=	15.432 *gr.* Tr.
10 *g.*	=	1 Dekagram .....(10 grams)	=	.3527 *oz.* Av.
10 *Dg.*	=	1 Hectogram.....(100 grams)	=	3.5274 *oz.* Av.
10 *Hg.*	=	1 Kilogram......(1000 grams)	=	2.2046 *lb.* Av.
10 *Kg.*	=	1 Myriagram.....(10000 grams)	=	22.046 *lb.* Av.
100 Kilos	=	1 Quintal........(100000 grams)	=	220.46 *lb.* Av.
10 *Q.*, or 1000 Kilos	=	1 Tonneau, or TON ....(1000000 grams)	=	2204.6 *lb.* Av. 1.1023 *T.*

NOTES.—1. The above table is used in computing the weights of all objects from the smallest atom to the largest known body. The gram, kilogram (or kilo), and ton are principally used.

2. The gram is used in weighing letters, gold, silver, and medicines.

3. The kilogram, or kilo, like the pound, is used in weighing groceries and coarse articles. It is approximately 2½ pounds Av.

4. The ton is the weight of a cubic meter of water, and is used in weighing very heavy articles, as coal, iron, etc.

5. The pound of Germany, Austria, and Denmark is equal to ½ of a kilogram ; the centner, to 100 pounds, or ½ of a quintal.

EXAMPLES.

369. *1.* What is the weight in grams of a cubic meter of water ? Of a *cu. dm.* of water ?

2. A farmer sells to A 3.716 *T.* of hay, to B 4.325 *T.*, to C 8775 kilos ; how many tons does he sell ?

3. The U. S. 50-cent piece weighs 12.5 grams ; how many can be coined from a kilogram of standard silver ?

4. The U. S. 5-cent piece weighs 5 grams ; how many 5-cent pieces are equivalent in weight to 12 50-cent pieces ?

5. How much alloy must be used in making 1200 U. S. twenty-five-cent pieces ? (See Art. **113.**)

6. What is the cost of 75.6 kilos of sugar at 18 cents a kilo ?

7. How many powders, each containing 6 grams, can be made from .372 kilogram ?

8. What is the weight of 10 *cu. m.* of ice, it being .93 as heavy as water ?

[illegible]

[illegible] kilogram = 2.2046 pounds av.

[illegible] German pound = 1.1023 pounds av.

[illegible] = 2204.6 lb. av.

[illegible] grains Troy.

[illegible] ounce Troy.

[illegible] pounds Troy.

[illegible] How many

[illegible]

10. In 1000 *cu. m.* how many *cu. yd.*?
11. Reduce 1728 *gal.* wine to liters ; to dekaliters.
12. In 244 *sq. m.* how many *sq. yd.*? How many *sq. ft.*?
13. Reduce 220 *oz.* Av. to grams ; to kilograms.

372. APPROXIMATE VALUES.

When no great accuracy is required, we may consider—

1 decimeter = 4 inches.
1 meter = 39 inches.
5 meters = 1 rod.
1 kilometer = ⅝ mile.
1 square meter = 10¾ square feet.
1 hectare = 2½ acres.

1 cu. met. or ster = 1⅓ cu. yd. or ¼ cord.
1 liter = 1 quart.
1 hectoliter = 2⅓ bushels.
1 gram = 15⅓ grains.
1 kilogram = 2⅕ pounds.
1 ton = 2200 pounds.

APPROXIMATE RULES.

373. To reduce avoirdupois pounds to kilograms:

Divide by 2, and then deduct one-tenth.

NOTE.—Answer too small by about 8 kilos for every 1000 kilos of the result. If $\frac{1}{11}$, instead of $\frac{1}{10}$, be deducted, the answer will be too great by 2 kilos for every 1000 kilos of the result.

374. To reduce avoirdupois pounds to half-kilograms, or German pounds:

Deduct one-tenth.

NOTE.—Answer too small by about 8 German pounds for every 1000 German pounds of the result. If $\frac{1}{11}$ be deducted, the answer will be too great by 2 German pounds for every 1000 German pounds of the result

375. To reduce yards to meters:

Deduct one-twelfth.

NOTE.—Answer too great by 2½ *m.* for every 1000 *m.* of the result.

376. To reduce square yards to square meters:

Deduct one-sixth.

NOTE.—Answer too small by about 3 *sq. m.* for every 1000 *sq. m.* of the result.

377. To reduce cubic yards to cubic meters:

Divide by 1.3.

NOTE.—Answer too great by about 6 *cu. m.* for every 1000 *cu. m.* of the result.

378. To reduce U. S. gallons to liters:

Multiply by 4, and then subtract one-twentieth (5 per cent.).

Note.—Answer too great by about 4 *l.* for every 1000 *l.* of the result.

379. To reduce U. S. bushels to hectoliters:

Divide by 3, and then add one-twentieth (5 per cent.).

Note.—Answer too small by about 7 *hl.* for every 1000 *hl.* of the result.

380. To reduce kilograms to avoirdupois pounds:

Multiply by 2, and then add one-tenth.

Note.—Answer too small by about 2 *lb.* for every 1000 *lb.* of the result.

381. To reduce German pounds, or half-kilograms, to avoirdupois pounds:

Add one-tenth.

Note.—Same error as in Art. **380**.

382. To reduce meters to yards:

Add one-twelfth and 1% of the original number.

Note.—Answer too small by only ¼ *yd.* for every 1000 *yd.* of the result.

Dealers in dry goods add only $\frac{1}{12}$ in reducing meters to yards, and thus make the result too small by about 9½ *yd.* for every 1000 *yd.* of the result.

If $\frac{1}{11}$ be added, the answer will be too small by about 2½ *yd.* for every 1000 *yd.* of the result. If $\frac{1}{10}$ be added, the answer will be too great by about 6 *yd.* for every 1000 *yd.* of the result.

383. To reduce square meters to square yards:

Add one-fifth.

Note.—Answer too great by about 8 *sq. yd.* for every 1000 *sq. yd.* of the result.

384. To reduce cubic meters to cubic yards:

Multiply by 1.3.

Note.—Answer too small by about 6 *cu. yd.* for every 1000 *cu. yd.* of the result.

385. To reduce liters to U. S. gallons:

Multiply by 2.11, and then divide by 8.

Note.—Answer too small by about 1.7 *gal.* for every 1000 *gal.* of the result

386. To reduce hectoliters to U. S. bushels:

Multiply by 3, and then subtract one-twentieth (5 per cent.).

Note.—Answer too great by about 4 *bu.* for every 1000 *bu.* of the result.

REVIEW EXAMPLES

387. *1.* Add 17⅘, 26¼, 35⅔, 48¾, and 8$\frac{7}{18}$; multiply the sum by 59; subtract 2309$\frac{7}{12}$ from the product; and divide the remainder by 162⅔.

2. Divide fourteen, and twenty-five hundredths by one hundred twenty-five thousandths; add nineteen, and sixty-four hundredths to the quotient; and multiply the sum by eight, and five tenths.

3. Find the time by Compound Subtraction and in exact days from March 24 to Sept. 18.

4. How many lengths of pipe, each 16 *ft.* (′) 3 *in.* (″) long, will be required for a well 130 *ft.* deep?

5. A horse trots a mile in 2 *min.* 45 *sec.* How many feet is that per second?

6. A grass plot 13 *ft.* by 54 *ft.* is surrounded by a stone walk 1½ *ft.* wide. The stone walk is surrounded by a gravel road 7½ *ft.* wide. How many square feet are covered by the grass, the stone, and the gravel respectively? (Make diagram.)

7. What is the cost of 15669 pounds meal at $1.10 per *cwt.*?

8. What is the cost of 16450 pounds of hay at $15.50 per ton?

9. How many square rods in a triangular piece of land, 360 *rd.* long and whose perpendicular width is 240 *rd.*?

NOTE.—To find the area of a triangle, take one-half the product of the length (base) and the height or width (altitude).

10. How many square feet in the gable of a house, 40 *ft.* long and 24 *ft.* high?

11. How many feet of siding would be required for a house 40 *ft.* long, 24 *ft.* wide, 18 *ft.* high, with two gables each 24 *ft.* wide and 12 *ft.* high, adding one-fifth for the lap and waste in cutting?

12. How much will it cost to make an excavation, 40 *ft.* long, 30 *ft.* wide, and 9 *ft.* deep, at 32*c.* per cubic yard.

13. How many feet of 2-inch plank, making no deduction for the corners, would be needed to build a rectangular tank, without a cover, 10 *ft.* long, 6½ *ft.* deep, 8 *ft.* wide?

14. The circumference of any circle is equal to the diameter multiplied by 3.1416 (about 3$\frac{1}{7}$). Find the circumference of a circle, whose diameter is 5 feet.

15. The area of any circle equals the square of the radius multiplied by 3.1416 (3⅐), or the square of the diameter multiplied by .7854. What is the area of a circle whose diameter is 6 *ft.*? Whose radius is 5 *ft.*?

16. How many feet of 2-inch plank would be required to make a cylindrical cistern without a cover, 7 *ft.* in diameter and 8 *ft.* high ?

17. How many pounds in 16 *T.* 3 *qr.* 18 *lb.* (Long Ton Table)?

18. How many quarts in 3 *bbl.* 24 *gal.* cider ?

19. In 27318 pounds of corn, how many bushels ? What is the value of the same at 48¾ cents per bushel ?

20. What is the value of 27318 pounds of corn, at 87.1 cents per cental ?

NOTE.—Examples 19 and 20 illustrate the present and the cental systems of buying and selling produce, and show the calculations saved by using the latter.

21. Paid $222.75 for boards at $13.50 per M.; how many feet were purchased ?

22. What is the value of 27315 *ft.* of lumber at $12 per M.?

23. A quartermaster purchased 75000 pounds of corn, at 31¼ cents per bushel ; 32113 pounds of oats, at 32½ cents per bushel ; and 79500 pounds of hay, at $22.37½ per ton (2000 pounds). What was the total cost of the purchase ?

24. A farmer sold 18360 pounds of corn, at 64 cents per cental ; 22450 pounds of oats, at 94 cents per cental ; and 36650 pounds of hay, at $1.31 per cental. How much was realized from the sale ?

25. Reduce £19 16*s.* 9*d.* to the decimal of a pound.

26. If £1 sterling is worth $4.87, what is the value of £225 18*s.* 6*d.*?

27. From £16 12*s.* 9*d.* deduct .05 of itself.

28. What is the value of 20 *yd.* silk at 10*s.* 6*d.* per yard ?

29. The difference in the local time of two places is 3 *hr.* 43 *min.* 12 *sec.*; what is the difference in longitude ?

30. What is the capacity in liters of a cistern 25 meters long, 2.2 meters wide, and 3 meters deep ?

31. The specific duty on Brussels carpet is 44 cents per square yard ; what is the duty per square meter ?

32. The duty on tallow candles is 2½ cents per pound ; what is the duty per kilogram ?

ALIQUOT PARTS.

388. An **Aliquot Part** of a number or quantity is a number that will divide it without a remainder; as 20 of 60, $12\frac{1}{2}$ of 100, 4 of 12, etc.

Any fraction having 1 as its numerator is an aliquot part of a unit.

Many of the ordinary business computations can be shortened by the use of aliquot parts.

EXAMPLES.

389. *1.* Find the cost of 217 pounds of sugar at $8\frac{3}{4}$*c.* per pound? At $9\frac{3}{8}$*c.*? At $7\frac{5}{8}$*c.*?

OPERATION.

$$\begin{array}{rl} 217 & \\ \underline{.08\frac{3}{4}} & \\ 1736 & \\ 109 & (\frac{1}{2}) \\ \underline{54} & (\frac{1}{4} = \frac{1}{2} \text{ of } \frac{1}{2}) \\ 18.99 & \end{array}$$

NOTE.—$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ ($\frac{1}{2}$ of $\frac{1}{2}$). $\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$ ($\frac{1}{2}$ of $\frac{1}{4}$). $\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$ ($\frac{1}{4}$ of $\frac{1}{2}$). $\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

Find the cost of

2. $\frac{3}{4}$ *doz.* elbows at $2.75 per dozen.
3. 141 *lbs.* raisins at $8\frac{5}{8}$*c.* per pound.
4. 200 *lbs.* lead at $6\frac{3}{4}$*c.* per pound.
5. 228 *lbs.* putty at $2\frac{3}{8}$*c.* per pound.
6. 207 *lbs.* currants at $5\frac{7}{8}$*c.* per pound.
7. $102\frac{5}{12}$ ($\frac{1}{4} + \frac{1}{6}$) *ft.* tubing at 16*c.* per foot.
8. 877 *lbs.* paper at $3\frac{3}{4}$*c.* per pound.
9. 102 *lbs.* oatmeal at $3\frac{3}{8}$*c.* per pound.
10. 700 *lbs.* soap at $5\frac{3}{4}$*c.* per pound.
11. 503 *lbs.* ham at $10\frac{3}{4}$*c.* per pound.
12. 644 *lbs.* lard at $11\frac{7}{8}$*c.* per pound.
13. 2957 *lbs.* sugar at $8\frac{5}{16}$*c.* ($\frac{1}{4} + \frac{1}{4}$ of $\frac{1}{4}$) per pound.

390. Aliquot parts of 100.

$2\frac{1}{2} = \frac{1}{40}$.
$3\frac{1}{3} = \frac{1}{30}$.
$4 = \frac{1}{25}$.
$5 = \frac{1}{20}$.
$6\frac{1}{4} = \frac{1}{16}$.
$10 = \frac{1}{10}$.
$12\frac{1}{2} = \frac{1}{8}$.
$16\frac{2}{3} = \frac{1}{6}$.
$20 = \frac{1}{5}$.
$25 = \frac{1}{4}$.
$33\frac{1}{3} = \frac{1}{3}$.
$50 = \frac{1}{2}$.
$37\frac{1}{2} = \frac{1}{4} + \frac{1}{8}$ ($\frac{1}{2}$ of $\frac{1}{4}$).
$62\frac{1}{2} = \frac{1}{2} + \frac{1}{8}$ ($\frac{1}{4}$ of $\frac{1}{2}$).
$75 = \frac{1}{2} + \frac{1}{4}$ ($\frac{1}{2}$ of $\frac{1}{2}$).
$87\frac{1}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
$18\frac{3}{4} = \frac{1}{8} + \frac{1}{16}$ ($\frac{1}{2}$ of $\frac{1}{8}$).
$31\frac{1}{4} = \frac{1}{4} + \frac{1}{16}$ ($\frac{1}{4}$ of $\frac{1}{4}$).

NOTE.—In the following commercial problems, use as few figures as possible.

EXAMPLES.

391. *1.* Find the cost of 13756 pounds of meal at $1.05 per *cwt.*

OPERATION.

$$\begin{array}{r} 137.56 \\ \underline{6.88} \\ 144.44 \end{array}$$

ANALYSIS.—At $1 per *cwt.* the cost would be $137.56. 5*c.* = $\frac{1}{20}$ of $1. To divide by 20, divide by 2 and place the quotient figures one place to the right.

2. Find the value of 16345 *lbs.* of feed at $1.10 per *cwt.* (10*c* = $\frac{1}{10}$ of $1).

3. What is the cost of 12 *doz.* hats at $4.12½ ($\frac{1}{8}$) per *doz.*?

4. Find the cost of 471$\frac{50}{60}$ ($\frac{1}{2}+\frac{1}{3}$) bushels of corn at 41*c.*

5. What is the cost of 96 *doz.* buttons at $1.75 ($\frac{3}{4} = \frac{1}{2}+\frac{1}{4}$) per dozen?

6. Find the cost of 711$\frac{20}{32}$ ($\frac{1}{2}+\frac{1}{8}$) bushels oats at 39*c.* per bushel.

7. Find the cost of 24 boxes note paper at 16⅔*c.* per box.

8. Find the cost of 24116 *lbs.* bran at $1.20 per *cwt.*

9. Find the cost of 1750 *lbs.* soap at 5½*c.* per pound.

10. Find the cost of 131 *lbs.* coffee at 16½*c.* per pound. (16½ = 12½ + 4.)

11. Find the cost of 60½ *lbs.* crackers at 12½*c.* per pound.

12. Find the cost of 4880 *lbs.* feed @ 75*c.* per *cwt.*

13. Find the cost of 20 half-barrels fish at $4.25 per half-barrel. At $5.35 ($\frac{1}{4}+\frac{1}{10}$) per half-barrel.

14. Find the cost of 75 books at 25*c.* each.

15. Find the cost of 36 pairs shoes at $2.25 per pair. At $2.50 per pair.

16. Find the cost of 3019 *lbs.* bran at 62½*c.* per *cwt.*, and 24375 *lbs.* feed at $1.05 per *cwt.*

PERCENTAGE.

392. **Percentage** is a term applied to all operations in which 100 is used as the basis of computation.

It is also the name given to any number of hundredths of a number.

393. **Per Cent.** (%) is an abbreviation of the Latin *per centum,* meaning *on* or *by the hundred.*

Thus, 5% means 5 of every hundred, or 5 hundredths ($\frac{5}{100}$, or .05).

394. Any *per cent.* may be expressed in the form of a *decimal* or *fraction.*

Thus 5 *per cent.* = 5% = 5 hundredths = .05 = $\frac{5}{100}$ = $\frac{1}{20}$. The first two forms are used in the statements of questions; the others in the operations.

395. In percentage, three elements are considered, viz: the *Base,* the *Rate,* and the *Percentage.* Any two being given, the other can be found.

396. The **Percentage** is the result obtained by taking a certain number of hundredths of a number.

397. The **Base** is the number of which a certain number of hundredths are taken.

398. The **Rate** is the number of hundredths, or the number per cent.

Thus, in the statement, 6% of 300 is 18, the Percentage is 18, the Base 300, and 6 per cent. (.06) is the Rate.

399. **Applications of Percentage.**—The principles of percentage are applied to many of the most common business transactions. Among the most important of these are Trade Discounts, Commission, Insurance, Profit and Loss, Duties, Interest, and Exchange.

400. To find the percentage, the base and rate being given.

Ex. What is 6 per cent. of 300 ?

OPERATION.
300 Base.
.06 Rate.
18.00 Percentage.

ANALYSIS.—6% means 6 hundredths. 6% of 300 is equivalent to .06 of (or times) 300. .06 × 300 = 18. The percentage is the product of two factors, the base and the rate.

Or, 1% of 300 is 3, and 6% is 6 times 3, or 18.

To find 1% of any number, place the point two places to the left.

401. RULE.—*1. To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLES.

402. What is
1. .03 × 1728 ?
2. .16 times 375 ?
3. 4% of 448 ?
4. 6 per cent. of 387 ?
5. .06 of 945 ?

What is
6. 8% of $414 ?
7. .08 of $716 ?
8. 1.12 × $575 ?
9. 107% of $385 ?
10. 9% of $456 ?

What is
11. 16% of $375.60 ?
12. 8% of $414.60 ?
13. 6% of $875.75 ?
14. 113% of $913.25 ?
15. 32% of $485.50 ?

16. What is the difference between 2½% of $16000 and 5% of $8475 ?

17. A merchant bought goods amounting to $375.60, and sold them so as to gain 30% of the cost; how much did he gain ?

18. A lawyer collected $2875, and charged 5% for his services; how much did he retain for his services, and how much did he pay over ? The amount paid over is what per cent. of the amount collected ?*

19. An agent sells a house and lot for $16450, and receives 2% for his services; what does he pay to the owner of the property ?

20. What is the duty, at 25% of the value, on twelve watches worth $75 each ?

21. Jan. 10, a merchant buys a bill of goods amounting to $876.40 on the following terms: 4 months, or less 6% if paid in 10 days. How much would settle the bill Jan. 18 ?

22. A merchant, failing in business, pays 43% of his indebtedness. He owes A $3750, and B $6280. How much does he pay each ?

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* In order to prepare the student for examples in which the conditions are the reverse of those in this example, the teacher should ask oral questions similar to the above in all examples in which an amount or difference is involved.

23. A commission merchant sold 450 barrels of flour at \$5.30 per barrel. How much should he send to the miller, if he charges $2\frac{1}{4}$ per cent. for making the sale?

24. A manufacturer's list price of cans is \$2.50 per dozen. He sells a dealer 48 dozen at a discount of 30%. How much does he receive for them?

25. A broker sells merchandise amounting to \$916.64, at a commission of $1\frac{1}{8}$ per cent. What is his commission?

OPERATION.

9.166	1%.
1.146	$\frac{1}{8}\%$.
10.312	$1\frac{1}{8}\%$.

ANALYSIS.—1% of \$916.64 is \$9.166, to which add $\frac{1}{8}$ of itself as in the operation. (See Art. **389**.)

According to the above method, find

26. $\frac{1}{4}\%$ of \$375.60.

27. $\frac{3}{4}$ $(\frac{1}{2}+\frac{1}{4})\%$ of \$875.

28. $1\frac{1}{4}\%$ of \$1176.40.

29. $1\frac{3}{8}$ $(\frac{1}{4}+\frac{1}{8})\%$ of \$287.96.

30. $1\frac{1}{2}\%$ of \$5275.

31. $1\frac{3}{4}\%$ of \$3075.75.

32. A contracts to make an excavation containing 3456 cubic yards, at 28¢ per cubic yard for earth, and \$1.20 per cubic yard for rock. When completed, it is found that 16% is rock, and the remainder earth. How much does he receive for the work?

403. When the rate is an aliquot part of 100, it is generally more convenient to use the equivalent fraction. Thus,

$50\% = .50 = \frac{1}{2}$.
$33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$.
$25\% = .25 = \frac{1}{4}$.
$20\% = .20 = \frac{1}{5}$.

$16\frac{2}{3}\% = .16\frac{2}{3} = \frac{1}{6}$.
$12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$.
$10\% = .10 = \frac{1}{10}$.
$8\frac{1}{3}\% = .08\frac{1}{3} = \frac{1}{12}$.

$6\frac{1}{4}\% = .06\frac{1}{4} = \frac{1}{16}$.
$5\% = .05 = \frac{1}{20}$.
$3\frac{1}{3}\% = .03\frac{1}{3} = \frac{1}{30}$.
$2\frac{1}{2}\% = .0?\frac{1}{2} = \frac{1}{40}$.

EXAMPLES.

404. What is

1. $\frac{1}{4}$ of 1728?
2. 25% of 3472?
3. .25 of 6418?
4. $\frac{25}{100}$ of 7264?

What is

5. $33\frac{1}{3}\%$ of \$375?
6. $12\frac{1}{2}\%$ of \$848?
7. $2\frac{1}{2}\%$ of \$6480?
8. 20% of \$9875?

What is

9. 50% of \$487.20?
10. 5% of \$9742?
11. 10% of \$1764.30?
12. $37\frac{1}{2}\%$ $(\frac{3}{8})$ of \$875.60?

13. From a bill of goods amounting to \$475.60, 5% is deducted for cash. What is the net amount of the bill?

OPERATION.

475	60
23	78
451	82

ANALYSIS.—5% is $\frac{1}{20}$. To divide by 20, divide by 2 and place the quotient figures one place to the right.

14. A commission merchant pays $2375.40 for a quantity of grain, and charges 2½% for his services. What is the total cost ?

15. Mr. B's tax is $175.60. If the collector is allowed 5% additional, what is the total amount paid ?

16. The gross amount of a bill of tinware is $97.40. What is the net amount, if the trade discount is 33⅓% ?

17. A house is sold for $16400, and 25% of the purchase money is allowed to remain on bond and mortgage. What is the amount of the mortgage ?

18. A house worth $7200 is insured for 62½% (⅝) of its value. What is the amount of the insurance ? What is its cost at ⅜% ?

405. To find the base, the percentage and rate being given.

Ex. 18 is 6% of what number.

OPERATION.
Rate. Percentage.
.06) 18.00
Base. 300

ANALYSIS.—The question "18 is 6% of what number?" is equivalent to "18 = .06 × what number?" If 18 is the product of two factors and one of the factors is .06, the other may be found by dividing 18 by .06.

Or, since the Percentage = the Base × the Rate, the Base = the Percentage ÷ the Rate.

Or, if 18 is 6% of a certain number, 1% is ⅙ of 18, or 3 ; and the number, or 100%, is 100 times 3, or 300.

NOTE.—The student should remember that the Percentage = the Base × the Rate, and that when the Percentage and one of its factors are given, in the operation of finding the other, the Percentage becomes the dividend and the given factor, the divisor. If the relation between the given terms is indicated in the form of an equation, the student will have no difficulty in determining which number should be the divisor in solving the great variety of examples which occur in percentage and its applications.

406. RULE.—*To find the base, divide the percentage by the rate expressed decimally.*

EXAMPLES.

407. Find the unknown term in the following equations:

1. 48 = 6 times ——.
2. 48 = .06 × ——.
3. 324 is 6% of ——.
4. 448 = .08 of ——.
5. .04 of —— = 375.
6. 6% of 380 = ——.
7. 72 is 4% of ——.
8. 184 = .23 × ——.
9. 175 is .07 times ——.
10. $576 × .06 = ——.

11. $324 = 8% of ——

12. $144 = .16 × ——.

13. 52% of $440 = ——.

14. $875 = 25% of ——.

15. $144 = ⅛ of ——.

16. 12½% of $475 = ——.

17. $325 is 13% of ——.

18. $48.60 is 2½% of ——.

19. The product of two factors is 75; if one of the factors is .03, what is the other factor?

20. The percentage is 60, and the rate 2½%; what is the base?

NOTE.—Use the fractional method when convenient. See Art. **402.**

21. $18.08 are 5% of what?

22. $324 are 10% of what?

23. $37.56 are 2½% of what?

24. $17.28 are 25% of what?

25. 165 *ft.* are 33⅓% of what?

26. £240 are 3⅓% of what?

27. $12.25 are 6¼% of what?

28. $96 are ⅝% of what?

29. Mr. A invests 42% of his capital in real estate, and has $53070 left; what is his capital?

ANALYSIS.—100% of any amount is the amount itself. If 42% of his capital is invested in real estate, the remainder, $53070, must be 58% (100% − 42%) of his capital.

30. A has 35% of his property invested in stocks, 10% in horses and cattle, 18% in grain, and the remainder, which is $24235, in real estate. What is the total value of his property?

31. A horse was sold for $658, which was 16⅔% more than it cost; what was the cost?

NOTE.—The cost of the horse was $\frac{100}{100}$, or 100% of itself; since the gain was 16⅔% of the cost, the selling price (the cost plus the gain) was 116⅔% of the cost. $658 is 116⅔% of what number?

What number increased by

32. 25% of itself is 500?

33. 8% of itself is $1004.40?

34. 125% of itself is 999?

What number decreased by

35. 5% of itself is $307.80?

36. 40% of itself is 3726?

37. 25% of itself is $342.60?

38. A merchant sells goods for $555.50, which is 10% more than they cost him. What did they cost?

39. In a cargo of oranges, consisting of 4275 boxes, 33⅓% are damaged. How many are damaged?

40. A bankrupt whose assets are $23625, pays 40% of his liabilities. What are his liabilities?

41. A farm was sold at a commission of 1¼%. If the agent's commission was $80.80, what was the price of the farm?

16. A house, insured for $16500 in several companies, is damaged by fire to the extent of $7260. What % of its insurance will each company pay ?

17. A tax of $18480 is levied upon a township whose valuation is $3,696,000. What is the rate of the tax, and what tax should a man pay whose assessment is $8500 ?

18. The annual interest on a mortgage of $7500 is $337.50. What is the rate per cent. ?

19. A merchant with a capital of $24000 gains $3840 in one year. His gain is what per cent. of his capital ?

20. A bankrupt has liabilities to the amount of $12600 and his assets are only $7087.50. What % dividend will he pay ?

21. The dividend of a manufacturing company, whose capital stock is $125000, is $6000. What % does it pay ?

22. A man's salary is $1600 per year and his living expenses $1300. What % of his salary does he save ?

23. A house cost $8000, and rents for $750 per year. If the taxes and other expenses are $230 per year, what % does it pay on the investment ?

24. The cost of insuring a cargo for $8500 is $63.75. What is the rate of the insurance ?

25. 1 meter = 1.0936 yards. The meter is what % greater than the yard ?

26. A man subscribes for 36 shares ($100 each) of a gas company. He pays in $1980. What % is still due ?

27. The Avoirdupois pound (**334**) is what % greater than the Troy pound (**336**) ?

28. A merchant bought a quantity of goods for $425. Being damaged he sold them for $340. What % of the cost did he lose ?

29. A horse and wagon are worth $600. What is the value of each, if the wagon is worth 87½% as much as the horse ?

30. A man bought a watch for $160 and sold it for $180. His gain was what % of the cost ?

31. A tea merchant mixes 40 *lbs.* tea at 45*c.* per pound with 50 *lbs.* at 27*c.* per pound. He sells the mixture at 42*c.* per pound. What % profit on the cost does he make ?

32. A consignment of flour was sold for $3148, of which $3124.39 were the net proceeds. What was the rate % of the commission ?

33. Mr. A's house is worth $12500. He pays $30 for insuring it for ¾ of its value. What per cent. does he pay ?

days. How much would settle the account May 16? The amount paid May 16, is what per cent. of the full amount of the bill? The above discount is equivalent to what rate per cent. per annum?

20. Mar. 16, a merchant buys a bill of goods amounting to $2475 on the following terms: 4 months, or less 5% if paid in 30 days. Apr. 15, he makes a payment of $1000, with the understanding that he is to have the benefit of the discount of 5%. With what amount should he be credited on the books of the seller? How much would be due July 16, the expiration of the 4 months?

Note.—As in Ex. 19, the amount paid within 30 days is 95% of that part of the bill which it settles or cancels.

21. A bought a bill of merchandise July 24, 1879, amounting to $6287.45 on the following terms: 6 months, or less 4% 30 days. He paid on account Aug. 23, 1879, $5000, with the understanding that the payment would cancel an equitable amount of the bill. How much was due Jan. 24, 1880?

22. Paid for transportation $664.95 on an invoice of goods amounting to $8866. What per cent. was the value of the goods thereby increased? What per cent. must be added to the invoice cost to make a profit of 20% on the full cost?

23. What is 3% of £247 13*s.* 6*d.*?

OPERATION.

$$\begin{array}{lrrr} & \pounds & s. & d. \\ & 247 & 13 & 6 \\ & & & .03 \\ \hline \pounds & 7|.41 & .39 & .18 \\ & 20 & & \\ \hline s. & 8|.59 & & \\ & 12 & & \\ \hline d. & 7.26 & & \end{array}$$

Analysis.—Multiply the number of each denomination by .03, as in the margin, and then reduce the decimal parts to integers of lower denominations (**289**).

Or, reduce shillings and pence to the decimal of a pound (see note, Ex. 12, Art. 342), take the required per cent., and reduce the decimal result to lower denominations. Thus,

£247 13*s.* 6*d.* = £247.675
£247.675 × .03 = £7.43025 = £7 8*s.* 7*d.*

When the rate per cent. is an aliquot part of 100, use the equivalent fraction (**403**). Thus, 5% of £247 13*s.* 6*d.* = $\frac{1}{20}$ of £247 13*s.* 6*d.* = £12 7*s.* 8*d.*

24. Find 3% of £384.
25. Find 6% of £440 16*s.*
26. Find 5% of £375.
27. Find $2\frac{1}{2}$% of £64 16*s.*
28. Find 4% of £75 12*s.* 6*d.*
29. Find 10% of £37 8*s.* 9*d.*
30. 16*s.* is $2\frac{1}{2}$% of what?
31. £1 8*s.* 4*d.* is 4% of what?

12. A lot of dry goods was sold at an advance of 18%. If the gain was $436.50, what was the cost? (Gain = .18 × cost; hence, gain ÷ .18 = cost.)

13. A farm was bought for $7200, and sold at a gain of $900; what was the gain per cent.? (Gain = gain % × cost; hence, gain % = gain ÷ cost.)

14. A man paid for merchandise $875, and sold it for $1015; what per cent. did he gain?

15. A man paid for merchandise $1015, and sold it for $875; what per cent. did he lose?

16. Find the rate % of profit on goods bought for $324 and sold for $364.50.

17. A painting was sold for $2343, at a gain of 32%; what was the cost? [Selling price = 1.32 (100% + 32%) × cost; hence, cost = selling price ÷ 1.32.]

18. Find the cost of goods sold at an advance of 12½%, being a profit of $76.

19. How much was paid for a farm sold for $9878, at 12% below cost?

20. What is the profit on iron sold for $4520, at an advance of 13% on cost?

21. What is the selling price of tea which cost 32 cents per pound and is sold at a profit of 37½%?

22. Sold drugs for $168, at an advance of 75%; what was the profit?

23. A merchant sold for $2576 a lot of dry goods for which he paid $3360. What was the per cent. of loss?

24. A mixture is made of 1 gallon of wine at 50 cents a gallon, 3 at 90 cents, 4 at $1.20, and 12 at 40 cents. What per cent. would be gained by selling the mixture at $1.60 a gallon?

25. If, by selling tea at 47½ cents per pound, I lose 5%, at what price must I sell it to gain 15%?

26. If, by selling goods for $126, I lose 16%, what per cent. would I have lost or gained if I had sold them for $168?

27. A merchant's price is 25% above cost price. If he allows a customer a discount of 12% on his bill, what per cent. profit does he make?

28. If cloth, when sold at a loss of 25%, brings $5 per yard, what would be the gain or loss per cent. if sold at $6.40 per yard?

29. Goods that cost $168 are sold at an advance of 25%; what is the selling price?

30. At what price must ribbon be sold per yard so as to gain 20%, if 22½ yards cost $6.75?

31. A merchant gave $25000 for seven houses. What per cent. does he gain by selling them at $7000 each?

32. A woman buys a certain number of apples at the rate of 3 for 1 cent, and as many more at the rate of 2 for 1 cent. What per cent. does she gain or lose, if she sells them at the rate of 5 for 2 cents?

33. Eggs are bought at 27 cents per dozen, and sold at the rate of 8 for 25 cents. What is the per cent. of profit?

34. A merchant by selling goods for $364, loses 9%. For what ought they to be sold to gain 8%?

35. A drover bought 160 sheep for $400, and sold ¾ of them at $2.25 each. At what price must he sell the remainder so as to gain 10% on the whole?

36. A merchant sells goods to a customer at a profit of 60%, but the buyer becoming bankrupt pays only 70¢ on the dollar. What % does the merchant gain or lose by the sale?

37. If a merchant adds to the cost price of his goods a profit of 12½%, what is the cost of an article which he sells for $7.20?

38. Sold a horse at a gain of 33⅓%, and with the proceeds purchased another horse, which I sold for $120, at a loss of 20%. What was the gain or loss?

39. A merchant's retail price for boots is $4.75 per pair, by which he makes a profit of 33⅓%. He sells to a wholesale customer at a discount of 20% from the retail price. What per cent. does he gain or lose, and what does he receive per pair?

40. 40 head of cattle weighing 52770 pounds are purchased in Chicago at $4.80 per *cwt.*, and are sold in New York at 10½ cents per pound, to dress 56 pounds to the hundred-weight. What was the total cost? The total selling price? What is the gain per cent., making no allowance for transportation?

NOTE.—The quantity bought or sold does not affect the gain or loss per cent.

41. A speculator sold two building lots for $4800 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose, and how much?

DISCOUNTS.

415. It is customary in many branches of business for manufacturers and dealers to have fixed price-lists of certain kinds of merchandise; and when the value changes, instead of changing a long price-list, the rate of discount is changed. The fixed price is called the *List Price,* and the discount allowed the *Trade Discount.*

Books are usually sold by publishers and jobbers at certain discounts from the retail prices.

416. Many kinds of merchandise are sold at "time" prices, subject to certain rates of discount if paid at an earlier period.

1. Thus, the following or similar announcements are usually found upon the bill-heads of wholesale dealers: "Terms, 4 months, or 30 days less 5%"; or, "Terms 60 days, or 1% discount in 30 days, or 2% discount in 10 days."

2. In the same business house, certain goods are sold on long credit, and others on short credit.

3. When no rate of discount has been offered, merchants are generally willing, when bills are paid before maturity, to deduct the interest on the amount of the bill for the remainder of the time at the legal rate per annum.

Ex. The list-price of a scale is $80; what is the net price if a discount of 25% and 10% is allowed?

OPERATION.

$80 List-price.
20 25%, or $\frac{1}{4}$.
60
6 10%, or $\frac{1}{10}$.
54 Net-price.

ANALYSIS.—The first rate of discount is reckoned upon, and deducted from the list price, and the others are deducted from the successive remainders.

The result is not affected by the order in which the discounts are taken. A discount of 25% and 10% is the same as a discount of 10% and 25%.

EXAMPLES.

417. *1.* The gross amount of a bill of shoes is $82.68. What is the net amount, the rate of discount being 5%? (See Ex. 1, Art. **391.**)

2. A stove is sold for $45 less 30%; required the net price?

NOTE.—If the discount is not required, multiply by .70 (100% — 30%); the product will be the net price. To multiply by .70, multiply by 7 and place the figures of the product one place to the right.

3. What is the value of 466 *lb.* O.W. casing @ 45 *cts.* per pound, less 1¼ per cent.?

27. If a merchant buys goods at a certain price 10 and 5 off, and sells them at the same price, 5 off, what per cent. profit does he make?

28. What per cent. profit does a merchant make who buys at a discount of 20, 10, and 12½%, and sells at the list price?

29. What must be the marked price of goods costing $32, that I may deduct 20% from it, and still gain 25% on the cost?

ANALYSIS.—First find the selling price, and then the marked or list price. If the cost is $32, the gain will be 25% (¼) of $32, or $8; and the selling price will be $32 + $8, or $40. If the goods are sold at a discount of 20%, the selling or net price is 80% of the marked price. $40 = .80 of $50.

30. What must be the asking price for books that cost $1.60, in order to abate 20%, and still make a profit of 25%?

31. What must be the list price of goods that cost $18, in order to make a profit of 33⅓%, if they are sold at a discount of 40%?

32. Find the list price of goods that cost $75 and are sold at a discount of 60 and 10%, at a profit of 20%.

33. A manufacturer sells his goods at a discount of 30 and 10%, and thereby gains 12½%. What is the list price, if the cost is $28?

34. A hardware dealer sells certain goods at a discount of 75 and 12½%, and gains 20%. What is the list price, if the cost is $2.80?

35. What per cent. must be added to cost price in order to give a discount of 25%, and make a profit of 20%?

ANALYSIS.—Assuming 100% as the cost price, the selling price is 120% of the cost. 120% is 75% (100% − 25%), or ¾, of 160%. 160% − 100% = 60%, the per cent. to be added to the cost price.

36. What advance on cost would be necessary in order to give a discount of 20%, and still make a profit of 20%?

37. At what per cent. above cost must goods be marked, so that when sold at a discount of 5%, there would be a profit of 25%?

38. If goods are bought at a discount of 2 10's and a 5 from a manufacturer's list price, and sold at a discount of 12½% (⅛), what is the gain per cent.?

39. I purchase books at $2 each less 33⅓%, and 5% for cash. What is the net cost, and what per cent. discount may be given on the list price to produce a net profit of 10%?

BILLS.*

418. A **Bill** is a detailed statement of merchandise sold, or of services rendered. Bills of merchandise state the place and date of the sale, the names of the buyer and seller, the terms of the sale, the quantity, price, and distinguishing marks and numbers of the merchandise and other details.

The terms *Bill* and *Invoice* are used by many interchangeably. The term Invoice is applied more particularly to statements rendered by consignees to commission merchants, showing marks, numbers, values, and accrued charges of goods shipped; to bills rendered to jobbers; and to bills received from foreign countries.

EXAMPLES.

419. Copy and extend the following bills:

(1. Canned Goods.)

WILMINGTON, DEL., *Nov. 16, 1889.*

Messrs. WM. DOLTON & Co.,

Bought of JAMES MORROW & SON.

Cases.	*Doz.*						
2	4	3 lb. Peaches - - - - - -	2^{25}[a]	9	00		
1	2	2 " Saco Corn - - - - -	1^{85}	*	**		
1	2	2½ " Salmon - - - - - -	3^{85}	*	**		
2	4	3 " Tomatoes, B. & L. - - -	1^{80}	*	**		
1	2	2½ " Col. Pears - - - - -	4^{00}	*	**		
1	2	2½ " Apricots - - - - - -	4^{00}	*	**		
		Ctg. -			50	$**	**

[a] Price per dozen.

(2. Flour.)

BUFFALO, N. Y., *Dec. 6, 1888.*

Messrs. DANIEL CROUSE & SONS,

Bought of SCHOELLKOPF & MATTHEWS.

Interest charged on all accounts after 30 days. We allow no Expressage or Exchange.

	20	Bbls. Flour "Sunlight" Sacks	- $7.05	***			
	25	" " " Bbls.	- 7.25	***	**		
	25	" " "Victor" Sacks	- 6.05	***	**		
	25	" " " Bbls.	- 6.25	***	**		
	15	" " "Dakota" Sacks	- 5.30	**	**		
	5	" " "Superior" Sacks	- 8.55	**	**		
20 bags		2177 lb. S. Meal - - - - - -	1.20[a]	**	**		
70 "		$264\frac{9}{32}$ bu. Oats - - - - - -	.56[b]	***		***	**

[a] $1.20 per hundredweight. [b] 56c. per bushel.

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* It is suggested that a part of these bills be reserved for review. One or two of them may be given each week as a general exercise.

(3. Window Glass.)

PITTSBURGH, *May 14, 1888.*

EUREKA GLASS CO.,

Bought of CUNNINGHAMS & CO.

Terms 30 days.—If not promptly paid, interest will be charged from date of bill.

2	Bx's 7 × 9 "A" - - - - - - - - -	$7^{\underline{30}}$	**			
15	" 8 × 10 " - - - - - - - - -	$7^{\underline{50}}$	***	**		
2	" 9 × 12 " - - - - - - - - -	$7^{\underline{50}}$	**			
1	" 10 × 14 " - - - - - - - - -	$7^{\underline{50}}$	*	**		
2	" 12 × 26 " - - - - - - - - -	$8^{\underline{50}}$	**			
2	" 13 × 36 " - - - - - - - - -	$10^{\underline{75}}$	**	**		
2	" 14 × 20 " - - - - - - - - -	$8^{\underline{50}}$	**			
2	" 15 × 30 " - - - - - - - - -	$9^{\underline{75}}$	**	**		
			***			
	Less 60 and 20% - - -		***			**

(4. Provisions.)

CLEVELAND, O., *Oct. 9, 1886.*

Messrs. L. C. MAGAW & SON,

Bought of J. P. ROBISON & CO.

Terms Net Cash.—*No goods sold on 30 days.*

10	Bbls. S. M. Pork - - - - - - - -					$17^{\underline{00}}$	***			
5	" Mess Beef - - - - - - - -					$10^{\underline{75}}$	**	**		
5	" Hams	90[a]	1376[b]	—98[c]	****[d]	14¢	***	**		
3	" Shoulders	58	744	—57	***	9¢	**	**		
1	" Dr. Beef	33	241	—22	***	14¢	**	**		
1	Tc. Lard		406	—60	***	11¢	**	**	***	**

[a] Number of pieces. [b] Gross weight. [c] Tare. or weight of barrel or tierce. [d] Net weight.

(5. Fish.)

GLOUCESTER, MASS., *Sept. 28, 1886.*

Messrs. DANIEL WEIDMAN & CO.,

Bought of CLARK & SOMES.

Subject to sight draft without notice after thirty days.

2	Qtl. New Geo. Cod - - - - - -	5.75	**	**		
1	Bbl. Ex. $1 Mackerel - - - - - -	20.00	**	**		
10	Kits 15 lbs. Ex. $1 Mackerel - - -	1.80	**	**		
10	" 20 lbs. Bay $1 " - - -	1.80	**	**		
2	Bbls. $2 Shore " lg. - -	12.00	**	**		
10	Kits 20 lbs. $2 Shore " " - -	1.50	**	**		
5	Halfs New Labrador Herring - - -	3.82	**	**		
3	" Round Shore " - - -	2.95	*	**		
	Box $^{.38}$, ctg. in Boston $^{.90}$		*	**	$****	**

(6. Groceries.)

Day Book, 115-797.

NEW YORK, *Feb. 1, 1889.*

Messrs. EDWARDS & Co.,

Bought of H. K. & F. B. THURBER & Co.

M $4385	1	Cask Old Prunes 1544 − 134 = **** lbs. -	4¾	**	**
	3	Boxes Old Muscatel Raisins - - - - -	$1^{\underline{65}}$	*	**
	3	" New " " - - - - - -	$2^{\underline{10}}$	*	**
	4	" " Layer " - - - - - -	$1^{\underline{25}}$	*	**
	1	" Cream Tartar, ¼ foil - - 20 lbs. -	.39	*	**
	2	" Yeast-Cakes, 3 doz. ea., - 6 doz. -	.65	*	**
	25	lbs. Whole Pepper - - - - - - - - -	.16	*	
	10	" Nutmegs $1 - - - - - - - - -	$1^{\underline{00}}$	**	
	1	Box O. K. Mustard, ¼'s - - - 12 lbs. -	.25	*	
	1	" " " ½'s - - - 12 " -	.25	*	
		Cartage on all - -		1	
				***	**

1st item—"M $4385" is mark and number upon the cask, 1544, gross *wt.*; 134, tare or weight of cask. 5th item—¼ foil, put up in ¼ *lb.* packages and wrapped in tin foil.

(7. Groceries.)

NEW YORK, *Aug. 13, 1886.*

Messrs. HORTON, CRARY & Co.,

Bought of AUSTIN, NICHOLS & Co.

W. B.	1	Bag $^{.20}$ Rio Coffee - - - - - - -		132	23	30	56
A $99	1	" $^{.20}$ " - - - - - - -		131	21½	**	**
	1	Bbl. $^{.25}$ Roa. Java Coffee	121 21 -	100	25½	**	**
	2	" $^{.50}$ " Rio "	112—22 221 109—20 42 -	***	24	**	**
H. R. P.	1	Case Conc. Lye - - - - - - - - - - -				5	50
Union.	2	Boxes Yeast Cakes, ea. 3 - - - - -		*	65	*	**
	25	lbs. Spice, Bag 20¢ - - - - - - - -			15½	*	**
	5	Mats Cassia - - - - - - - - - -		21½	26	*	**
A. N. & Co.	1	Keg Gr. Mustard - - - - - - -		50	35	**	**
	10	lbs. White Glue - - - - - - - - - - -			40	*	**
A. N. & Co.	5	Bbls. X. C. Sugar - -	257—20 **** 269—20 ** 256—21 —— 253—18 253—20 -	****	11¾	***	**
$134	1	" W. D. Syrup - - - -	47 ½	***	60	**	**
$114	1	" C. D. " - - - - -	45 ½	***	50	**	**
		Ctg. - - -				1	50
						$***	**

The small figures at the right of the words "bag" and "*bbl.*" are the prices of the same. 3rd item—121 *lbs.*, gross *wt.*, 21 *lbs.* tare, 100 *lbs.* net *wt.* 4th item—112 and 109, gross weights; 22 and 20, tare; 221, total gross weight; 42, total tare. 12th item—½, ½ gallon allowance for leakage.

(8. Dry Goods.)

NEW YORK, *March 20, 1889.*

Messrs. MARSHAL FIELD & Co.,

Bought of H. B. CLAFLIN & Co.

Terms Cash in 30 days less 5%, or 4 months' note delivered within 30 days, and payable at Bank in New York exchange.

			yd.	c.		
2875	1	Bale Boott M. Brown - - - - -	800	6^3	54	
8039	1	" Continental C. do. - - - - -	800	7^1	**	
3369	1	" Pequot A. 36 in. - - - - -	967	7^2	**	**
1290	1	" Great Falls E. - - - - - -	1111	7^1	**	**
1590	1	" Atlantic H. - - - 1038 - .07^3	$ **	**		
		Less 4% -	*	**	**	**
6888	1	" Boott F. F. - - - - - - - -	800	7^1	**	
2179	1	" Pepperell 600 Drill - - - -	622	7^3	**	**
2507	1	Case Blackstone A. A. - - - - - -	1649	7^1	***	**
6515	1	" Dwight Anchor - - - - - - -	1139	9	***	**
2985	1	" Great Falls Q. - - - - - - -	1492	8	***	**
1650	1	" Pearl River Ticking - - - -	708	15^2	***	**
		Cooperage -				75
					***	**

How much would settle the above bill April 19, 1889 ?

(9. Dry Goods.)

NEW YORK, *March 23, 1888.*

Messrs. DAVIDGE, LANDFIELD & Co.,

Bought of TEFFT, WELLER & Co.

			yd.	c.		
2	Naumkeag Bl. Jean - -	48 47 - - -	95	9	8	55
4	Roll Cambric - - - - -	46 46 40^1 46 - -	****	5^2	*	**
3	Pepperell Drill - - - - -	47^3 30^3 - - - 44^3	****	8	*	**
1	Lowell $^{10}/_4$ Brown - -	- - - -	38	14^2	*	**
3	Continental C. - - - - -	40 40 - - - 40	***	7^2	*	
5	New Market N. - - - -	45^3 45 45^1 58^1 - 46^2	****	6^1	**	**
2	Champion Cheviot - -	48^1 50^1 - - -	***	9	*	**
2	Otis B. B. Dk Stripe - -	57^3 57^1 - - -	***	10	**	**
1	Hamilton 30 in. Tick -	- - - -	48^3	11^2	*	**
2	Thorndyke C. - - - - -	58^3 61 - - -	****	8^2	**	**
2	Wamsutta C. Blea. - -	58^1 68^1 - - -	****	12	**	**
8	Andros L. - - - -	52 52 49 51^1 51^2 51^3 51 52^2 -	***	7^3	**	**
1	Pepperell $^{10}/_4$ - - - - - - - - - -		36^3	22	*	**
					***	**

1st item—2 pieces Naumkeag Bleached Jean containing 48 and 47 yards respectively; total, 95 yards at 9 cents per yard.

(10. Dry Goods.)

Book 174, Page 148. New York, *March 30, 1888.*

Mr. James Morgan, *Milwaukee, Wis.*

Bought of H. B. Claflin & Co.

Terms: Net 60 Days, or 1% discount in 30 days, or 2% discount in 10 days, N. Y. Funds. No Exchange allowed.

$4641	53	Pc's Gordon Prints (Job)				
		21^2 48^2 38 49^1 48^2 48^3 37^3 48 48				
		44 49^2 44^3 48^2 49^2 49^3 49^2 42 56				
		48^2 49^1 28^2 49^1 45 48^3 49^1 28 48^3				
		37 33^2 49^2 52 38^3 40 48 49^1 49^1				
		24 48^2 48^2 52 48^1 49 47^2 48^1 48^2				
		49^1 49^2 48^3 48^2 48^2 43^2 49^1 49^2 -	*****			
$2601	54	Pc's Do.				
		48^3 48 49 42 22^1 49^1 49 48^2 53^2				
		48^2 47^3 48^3 48^2 49 44 49 49^2 48^2				
		49^2 49 49 48^2 47^3 47 48^2 49^1 56				
		50^2 49^1 41^1 48^1 50 27^1 49 48^2 48^3				
		21^3 29^1 51^3 46^3 48^2 48^2 28^2 48^2 49^1				
		49^2 45^2 47 48^2 40^2 50^1 39^2 48^2 46^1	*****			
$4765	61	Pc's Do.				
		30^2 49^2 42 49^2 32 48 46 48^2 46^2				
		42^3 47^2 22^1 33 46 48 49^2 48^2 48				
		42 42 48 28 48^1 49^2 48^2 49 49				
		49^2 48^2 28^2 49^2 43 49^1 48^2 49^2 48				
		38^2 29 25 26^3 49^1 49^3 49^1 49 48^2				
		34^3 48^3 45 49 49^1 49^2 48^1 36 48				
		29^2 49^3 48^2 31^1 48^2 49 48^1 - -	*****			
			*****	$.04^2$	***	**

How much would settle the above bill April 8, 1888? How much April 28, 1888?

(11. Dry Goods.)

New York, *March 20, 1888.*

Messrs. Jordan, Marsh & Co.

Bought of A. T. Stewart & Co.

Job.	8	Cases Gordon Fancy				
J. U.		$4561 2810				
S. B. R.		4157 2902^1				
H. Z.		3473 2787^2				
S. J. L.		4224 2880^2				
G. Q.		2777 2821^1				
J. B.		3504 2842^2				
J. Z.		3970 2883^1				
J. H.		4198 2863^1 - - ****** .05	****	**		
		Less 5% -	**	**	****	**

1st column, distinguishing number of each case. 2d column, number of yards in the several cases.

[illegible]

(B. Dry Goods.)

[illegible]

[illegible] & Co.

[illegible] & Co.

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]				25

[illegible]

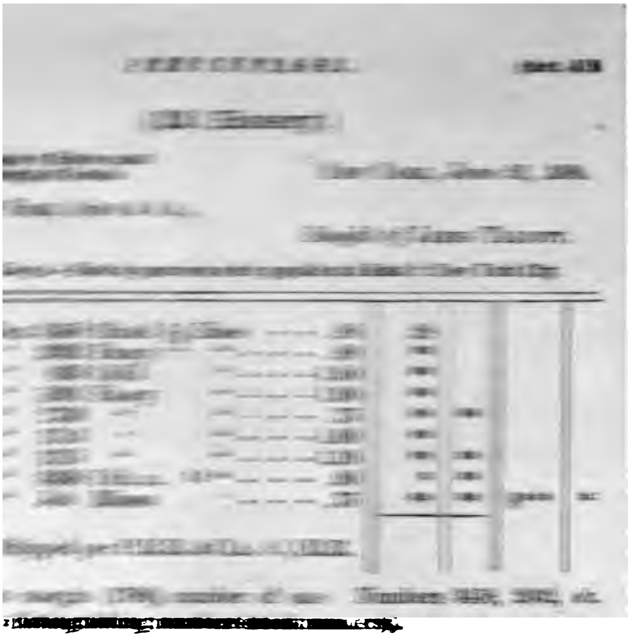
(B. Dry Goods.)

[illegible]

[illegible] & Co.

[illegible] & Co.

[illegible]



Chicago, Ill., Aug. 7, 187

Bought of [illegible] Bros.

[illegible]	.50		2		
[illegible]	.50	*			
Primary [illegible]	.06	*	**		
Spellers	.22	*	**		
3d Readers	.45	*	**		
4th "	.70	*	**		
5th "	1.15	*	**		
6th "	1.35	*	**		
Brown's Copy Books	1.80	**	**		
		**	**		
Less 33⅓ %		**	**	**	**
Geographies No. 1	.35	2	10		
" " 2	.63	*	**		
" " 3	1.10	*	**		
" " 4	2.00	**			
		**	**		
Less 25 %		*	**	**	**

(14. Hardware.)

PHILADELPHIA, PA., *Aug. 18, 1889.*

MESSRS. N. RUTTER, SON & CO.,

Bought of BIDDLE HARDWARE CO.

Terms 60 days.

24	Sets W'd Wh'l Bed Casters $1 2 in. -	.18	*	**		
	50% - -				*	**
1	Doz. Russell's S.B. Knives 14 in. $1540 -				11	
	2.40 2.55 3.15 3.20					
200	Carriage Bolts ¼ × 1 2¼ 5¼ 5½		**	**		
	5.95 6.25 6.50 6.85					
100	" " ⅜ × 5¼ 5 6¼ 6¾		**	**		
	7.15 7.45					
100	" " ⅜ × 7¼ 7¾ - - - -		**	**		
	7.90 8.05					
100	" " ⅜ × 8½ 8¾ - -		**	**		
	7.25 7.75 9.25					
100	" " ½ × 2 2½ 4 - - -		**	**		
	11.25 11.75 13.25					
100	" " ½ × 6 6½ 8 - - -		**	**		
			***	**		
	75 & 12½% -				**	**
½	C. Machine Bolts ½ × 8	8.70	*	**		
	15.10 16.60					
½	" " " ¾ × 6 7 - - -		**	**		
99	lbs. " " ¾ × 11	.10¾	**	**		
			**	**		
	60 & 10% -				**	**
					**	**

3rd item—200 bolts of each of the following sizes: ¼ *in.* thick × 1 *in.* long, ¼ *in.* thick × 2¼ long, ¼ *in.* thick × 5¼ long, ¼ *in.* thick × 5½ *in.* long. The numbers 2.40, 2.55, 3.15, and 3.20 represent the prices per hundred of the several sizes.

(15. Watches and Jewelry.)

NEW YORK, *Mar. 7, 1887.*

MR. CHARLES BABCOCK,

Bought of A. S. GARDNER & CO.

Terms: Net Cash 4 months, or less 5% 30 days. with Exchange on New York.

H 658	1	18 k. Ancre full Engrd. & Enld. S. W.	90			
20422	1	14 k. Russell flat C. B. "	46	50		
	1	18 k. Plain Ring 3¾ dwts. - - - 1^{25}	*	**		
	2	14 k. Guards with slides $\frac{37¾}{222}$, $\frac{56}{208}$ - 1^{15}	***	**		
	1	Pr. Solid Roman Sl. Buttons 908 - -	10	50	***	**

How much would settle the above bill, Apr. 2, 1887?

The letters and numbers on the margin refer to the numbers of the watches. 4th item—numbers 222 and 208 refer to the style numbers (stock numbers) of the guards (chains), and the numbers above (87¾ and 56) express the weights in pennyweights; $1.15 per pennyweight.

(16. Tinware.)

ROCHESTER, N. Y., *Oct. 16, 1889.*

Messrs. McCARTHY & REDFIELD,

Bought of JOHN H. HILL.

Terms 60 days. If paid in 10 days 2 per cent. discount.

2	Doz. $21 Pieced Dish Pans - -	8.25	**	**		
½	" 9 in. Wash Boilers - - -	36.00	**			
3	" Pieced Bread Pans 3 x 9 x 3 -	2.00	*			
3	" " " " 5 x 9 x 2 -	2.00	*			
3	" $13 Pieced Cups - - - -	.90	*	**		
2	" $25 Dippers - - - -	1.75	*	**		
6	Nests $021 Flaring P'ls & Dippers	1 14	*	**		
			**	**		
	20 & 12½% -		**	**	**	**
1	Doz. Champion Nutmeg Graters				1	75
1	" Nests $4 Fancy Cov'd Pails		6	00		
1	" $4 Burnished Tea Pots - -		6	75		
			**	**		
	25 & 12½% -		*	**	*	**
2	Doz. $10 Pudding Pans - - -	4.25	*	**		
½	" $200 Pressed Kettles - -	5.50	*	**		
			**	**		
	37½% -		*	**	*	**
6	Enameled Kettles Ea. $\overset{.75}{4}$—$\overset{.90}{5}$ qt. -		*	**		
12	" " " $\overset{1.10}{6}$—$\overset{1.30}{8}$ qt. -		**	**		
			**	**		
	60% -		**	**	**	**
	N.Y.C. & H.R.R.R. 975 lbs. @ 12¢				**	**

What amount would be due on the above bill Oct. 26, 1889?

(17. Wooden Ware.)

CHICAGO, *July 9, 1887.*

Messrs. OLIVER & BACON,

Bought of JAMES S. BARRON & Co.

Terms Cash, with exchange on Chicago or New York.

6	Oak Churns $1 - - - - - -	1.60	*	**		
	Less 20% -		*	**	*	**
2	Doz. 1 bu. Corn Baskets - - -	4.00			*	
½	" Potato Mashers - - - -	1.00				**
1	" 6 ft. Ladders - - - - -		4			
1	" 8 ft. " - - - - -		5			
1	" 10 ft. " - - - - -		6			
			**			
	Less 50 & 10% -		*	**	*	**
2	" $10 Shoe Brushes - - -	2.25			*	**
					**	**

NOTE.—In the preparation of bills in the usual form, from the following items, use your own name as the seller, the name of your teacher as the buyer, and the present date and place.

18. ½ *gro.* Table Knives and Forks @ \$8.40. $\frac{1}{12}$ *doz.* Cheese Knives @ \$9.60. ½ *doz.* Razors each #100 \$9, #101 \$10, #102 \$10.50. ½ *doz.* Pocket Knives each #337 \$6, #427 \$7.50, #204 \$3.75. 6 sets Champion Irons @ \$1.50. 1 *doz.* Tacks each #1 22*c.*, #2 22*c.*, #3 25*c.*, #4 27*c.* ⅔ *doz.* Panel Saws @ \$20 less 20%. Box and drayage, 75*c.* Terms: Cash. (See Ex. 14.)

19. 2293 *lbs.* S. Meal @ $\$1^{45}$ per *cwt.* 200 *bbl.* Dakota Flour @ $\$7^{25}$. 11890 *lb.* Feed @ $\$1^{40}$. 170 Bags (To be returned) @ 25*c.* Expressage on Empty Bags, 25*c.* Car #30808. (See Ex. 2.)

20. 2 *doz.* Wrought Butts each 3½ × 3 \$2.40, 3½ × 4 \$3, less 55 and 10%. $\frac{10}{12}$ *doz.* Locks #184 @ \$48 less 30%. 1 *doz.* Locks #476 \$15 ; 1 *doz.* Knobs #700 \$6.50 ; 1 *doz.* Escutcheons #16 \$2, less 45 and 5%. (On last three items.) 3 *doz.* Sash Fasteners #15 @ \$2.25. 4 *doz.* Solid Eye Mattocks @ \$15.50, less 33⅓(⅓)%. Box and cartage, 63*c.* Terms: 60 days. (See Ex. 14.)

21. 2 *doz.* Smith's Bitters @ \$7.25. 110 *lb.* Epsom Salts @ 3¼*c.* 2 *doz.* Sweet Oil #3 @ \$1.75. 2 *doz.* Sweet Oil #4 @ \$1.25. 2 *doz.* Paregoric 2 *oz.* @ \$1.25. 2 *doz.* Laudanum 2 *oz.* @ \$2.25. 2 *doz.* S. M. Oil @ \$1.25. 2 *doz.* Extract Lemon 2 *oz.* @ \$2. 2 *doz.* Brown's Syrup @ \$1.75. 2 *doz.* Fancy Soap @ 67*c.* 2 *doz.* Castor Oil @ \$1.75. 2 *doz.* Golden Liniment @ \$1.85. Cartage 50*c.* Terms: Net Cash, without discount.

22. 2 *bbl.* Prunes 248—20, 285—20, @ 6*c.* 2 *bbl.* Rice 240—19, 229—20, @ 6⅛*c.* 5 *bbl.* "A" Sugar 319, 306, 288, 319, 306 *lb.* @ 9⅝*c.* 5 *bbl.* Yellow C Sugar 314—19, 319—20, 329—20, 311—21, 328—19, @ 8¾*c.* 1 *bbl.* Cut Loaf Sugar 236—20 @ 10⅝*c.* Cartage, \$1.25. (See Ex. 7, 11th item.)

23. 8 lengths 8″ Drive Pipe 129′ 10″ @ \$2.25 (per foot); 44 lengths 5⅝″ Casing 801′ 8″ @ 70*c.*; 233 lengths 2″ O. W. Tubing 4507′ @ 21*c.*, less 7½ and 2%. Less freight 29400 *lb.* @ 23*c.* per *cwt.*

24. 12 *Pr.* Women's Grain *B't.* #443 Shoes @ \$1.25. 12 *Pr.* *Wos.* Kid *B't.* #407 @ \$1.50. 12 *Pr.* *Wos.* Kid *B't.* #406 @ \$1.75. 12 *Pr.* Misses Kid *B't.* #301 @ \$1.50. 12 *Pr.* Misses Goat *B't.* #302 @ \$1.60. 12 *Pr.* *Wos.* Goat *B't.* #428 @ \$1.75. 12 *Pr.* Children's Goat *B't.* #200 @ \$1.30. 12 *Pr.* *Ch.* Grain *B't.* #202 @ \$1.20. 12 *Pr.* *Ch.* Glove Kid *B't.* #222 @ \$1.10.

COMMISSION AND BROKERAGE.

420. Commission or **Brokerage** is an allowance made to an agent for transacting business for another; as, the sale or purchase of property, the collection or investment of money, etc.

An additional percentage is usually charged by commission merchants for guaranteeing the payment of sales made on credit.

421. The party who transacts the business is called a **Commission Merchant**, or **Broker**; and the one for whom he acts is called the **Principal**.

NOTES.—1. Commission Merchants usually have possession of the subject-matter of the negotiation, and make sales and purchases in their own name.

2. Brokers do not have possession of the merchandise bought or sold, and generally make contracts in the name of those who employ them and not in their own. They simply effect bargains and contracts.

The name *broker* is often erroneously applied to dealers in stocks, bonds, etc., who buy and sell on their own account only.

422. A **Consignment** is a quantity of merchandise sent by one party to another. The party who sends it is called the **Consignor**; and the party to whom it is sent, the **Consignee**.

423. The **Net Proceeds** of a consignment is the balance due the consignor after all charges or expenses have been deducted.

The whole amount realized from a sale is called the *gross proceeds*. The commission is usually a certain per cent. of this amount.

424. An **Account Sales** is a detailed statement rendered by the Commission Merchant to the Consignor, showing the sales of certain goods, the charges or expenses attending the same, and the difference or net proceeds.

The charges embrace freight, cartage, inspection, advertising, storage, insurance, commission and guarantee, etc.

425. An **Account Purchase** is a detailed statement rendered by the Commission Merchant to his Principal, showing the cost of certain goods bought, and the charges or expenses attending the purchase.

426. Commission or brokerage is usually computed at a certain per cent. of the amount realized or invested, or of the amount involved in the transaction. In such cases the general principles of percentage are applied.

NOTES.—1. In buying and selling stocks, bonds, etc., the par value, and not the actual value, is taken as the base.

2. The commission for buying and selling some kinds of merchandise is usually computed at a certain price per unit of weight or measurement; as, grain per bushel, cotton per bale, etc.

EXAMPLES.

427. *1.* A commission merchant sold goods to the amount of $864; what was his commission at 2½ (¼ of 10) %?

2. A salesman sells goods at a commission of 2½%; what must be his sales, that he may have a yearly income of $5000?

3. What is the brokerage for selling 850 bales of cotton at the rate of $25 per 100 bales?

4. A lawyer collected a note of $2375; how much did he pay to the owner of the note, his commission being 5%?

5. My agent in Chicago purchases for me 600 barrels of flour at $3.75 per barrel; how much do I owe him, his commission for purchasing being 2%?

6. An officer collected $17850, and deposited $17493 in the Treasury, retaining the remainder as his commission. What was the rate per cent. of the commission?

7. Sent to a commission merchant in Toledo $2080.80 to invest in flour, his commission being 2% on the amount expended; how many barrels of flour would be purchased at $4.25 per barrel?

8. A commission merchant sells merchandise amounting to $3325; how much is paid to the consignor of the merchandise, the charges being, for transportation $117.50, for advertising $10, for storage $15, for commission 2½%?

9. My agent in Chicago buys for me 1187.76 centals wheat at $2.123 per cental. What is his commission at ½ per cent.?

10. A commission merchant purchased for me $9^{\underline{28}}$ bushels of clover seed at $8.55 per bushel. How much should I send to him in settlement, if his commission for purchasing is 1 per cent.?

11. A broker buys 8375 pounds of leather at 26 cents per pound. What is his brokerage at ¾%, and what is the net amount received by the seller, the brokerage being paid by him?

12. A freight broker procures transportation for 375 tons of merchandise at $3.50 per ton; what is his brokerage at 5%?

13. A collector deposits $28117, retaining 3% on the whole amount collected. What amount did he collect and what was his commission ?

14. A lawyer, collecting a note at a commission of 5% thereon, received $6.25 ; what was the face of the note ?

15. An agent sold 6 mowing-machines at $120 each, and 12 at $140 each. He paid for transportation $72, and, after deducting his commission, remitted $2208 to the manufacturer. What was the % of his commission ?

16. A merchant instructs his agent in Cincinnati to buy pork to the amount of $5000. The charges on the pork being $16, and the agent's commission 1½%, how much must be remitted to settle the bill ?

17. What are the net proceeds of the sale of 12372 pounds of leather at 22 cents per pound, the charges being $31, and a commission of 2½% being paid for selling and 2½% for guaranteeing payment ?

18. A real estate agent, who charged 2½% for making the sale, paid to the owner of a house and lot $42412.50 ; what was the value of the property ?

19. A commission merchant sells 240 *bbl.* of potatoes at $3.75 per *bbl.*, and 260 *bbl.* at $3.60 per *bbl.* How much is due the consignor, the commission being 12½ cents per barrel ?

20. John Smith is a disbursing agent of the United States. Jan. 1, 1880, there is in his hands $11870.63. Feb. 1, he pays out $3220.34, on which he is entitled to a commission of 1⅜%. Mar. 1, he receives $3750.87. May 1, he pays out $3795.01, on which he is entitled to a commission of 2½%. Make a statement of his account, showing balance due the United States.

21. A lawyer collected 75% of an account of $3416, charging 5% commission. What amount should he pay over ?

22. A, having a claim against the government of $10970, agreed to pay an agent 8 per cent. of the amount collected. The amount collected was 22 per cent. less than the amount of the claim. How much was received by A ?

23. B sends $2240.70 to his agent in Cleveland, requesting him to invest in provisions after deducting his commission of 3% for purchasing ; what was the sum invested ?

24. A broker received $62.50 for selling some bonds, charging ¼% brokerage. What was the par value of the bonds ?

Copy the following account, and make the necessary extensions, etc.

(25. Account Sales.)

NEW YORK, *Oct. 19, 1889.*

Sold for account of A. W. RANDOLPH & CO.,

By DAVID DOWS & CO.

1880.							
Sept.	12	100 Bbls. "Sunshine" - - - -	5.75	***			
"	18	125 " "Pride of the West" -	6.25	***	**		
"	30	150 " "Sunshine" - - - -	6.	***			
Oct.	14	75 " "Pride of the West" -	6.50	***	**		
"	18	50 " " " -	6.60	***		****	**
		Charges.					
Sept.	10	Transportation 500 Bbls. @ 27¢ - - -		***			
"	10	Cartage 400 " @ 5¢ - - -		**			
Oct.	19	Storage 400 " @ 3¢ - - -		**			
"	19	Insurance $\frac{1}{16}$% - - - - - - - - -		*	**		
"	19	Commission and Guarantee 5% - - -		***	**	***	**
		Net proceeds - - - - - -				****	**

26. According to the above form, prepare an Account Sales of 10 *bbl.* Yellow C Sugar, 3031 *lb.* @ 8*c.*; 10 *bbl.* Standard A Sugar, 2957 *lb.* @ $9\frac{1}{16}$*c.*; 10 *bbl.* Soft A Sugar, 2839 *lb.* @ $8\frac{3}{4}$*c.*; 2 *Tc.* Lard, 713 *lb.* @ $9\frac{1}{2}$*c.*; 1 *Tc.* Rice, 608 *lb.* @ 7*c.* Charges as follows: Cooperage, \$1.80; Cartage, \$3.60; Commission, $1\frac{1}{4}$%. Present date and place; Student & Co., commission merchants; and sold for account of Teacher & Co.

27. If an agent's commission is \$145.20, when he sells \$5808 worth of goods, how much would it be when he sells \$7416 worth?

28. A creditor receives on a debt of \$1725, a dividend of 60%, on which he allows his attorney 5%. He receives a further dividend of 25%, on which he allows his attorney 6%. What is the net amount that he receives?

29. A gentleman left a sum of money to be divided equally among 7 persons, subject to an inheritance tax of 5%, which caused a deduction of \$364 from the whole amount. What did each receive?

30. An agent's commission for the month was \$128.40. If his sales had been \$864 more, his commission would have been \$150. Find the amount of his sales.

31. A man allows his agent 5% on his gross rentals, and receives a net rental of \$3488.40. If the gross rental is 6% of the value of the property, what is the value of the property?

(32. Account Purchase.)

TOLEDO, O., *Mar. 6, 1887.*

Purchased by A. L. BACKUS & SONS,
For account and risk of L. A. & W. B. SHAW.

9	Bags "Montauk" - - - - - -	.21	*	**		
2^{27}	Bu. Mammoth Clover Seed - -	9^{00}	**	**		
9^{28}	" Clover Seed - - - - - - -	8^{50}	**	**		
9^{81}	" Timothy Seed - - - - - -	1^{75}	**	**	***	**
	Charges.					
	Cartage - - - - - - - - - -			25		
	Commission 1% - - - - - - -		*	**	*	**
	Charge your % - - - - - - -				***	*

NOTE.—The small figures at the left represent pounds. See Art. **338**.

33. According to the above form, prepare an Account Purchase of 3 Half-Chests Gunpowder Tea, 165 *lb.* @ 35*c.*; 2 *Hfc.* Oolong Tea, 86 *lb.* @ 20*c.*; 20 bags Rio Coffee, 2388 *lb.* @ 13*c.*; 2 mats Java Coffee, 133 *lb.* @ 19½*c.*; 1 *Hhd.* P. R. Molasses, 143 *gal.* @ 54*c.* Charges as follows: Drayage, $1.75; Commission, 1%. Commission Merchants, Student & Co.; bought for Teacher & Co.; present place and date.

34. After paying an auctioneer 5%, a man received $1172.30 for his furniture. What were the gross sales?

35. A bankrupt's assets are $17415, and his liabilities, $48375. I place my claim of $2560 in the hands of my attorney for collection. How much do I receive if the attorney retains 5% commission?

36. A landlord received $822 as the net rental of a house, after his agent had paid $60 for repairs and charged 2% commission on the gross rental. What was the gross rental?

37. C of New York sells for D of Atlanta, a quantity of cotton, amounting to $7317.83, and charges a commission of 2½%. By instructions, he invests the proceeds in dry goods, after deducting a commission of 1½% on the amount expended. What was the total commission?

38. A commission merchant sold 300 bales of cotton, averaging 462 *lb.* to the bale, at 15.7¢, his commission being 25¢ per bale, and the charges $161. He purchased for the consignor dry goods amounting to $2576.37, charging a commission of 1½%. How much was still due the consignor?

INTEREST.

428. Interest is a sum charged for the use of money, or its equivalent; or more strictly speaking, it is *the use of money,* or the service rendered in its use.

429. The **Principal** is the sum for the use of which interest is charged.

430. The **Rate** is the per cent., or number of hundredths, of the principal, charged for its use for a certain time, usually for one year (per annum). When no time is mentioned with the rate in the contract, a year is understood.

431. The **Amount** is the sum of the principal and interest.

If $1000 is loaned for one year at 6% per annum, $60 would be the interest, $1000 the principal, and $1060 the amount.

432. Simple Interest is interest on the principal only for the full time.

433. Compound Interest is interest not only on the principal, but on the interest also after it becomes due.

If $1000 is loaned Jan. 1, 1881, for 2 years, the amount due Jan. 1, 1883, at 6% simple interest, would be $1000 (Principal) plus $120 (Simple Interest), or $1120. At compound interest the amount due Jan. 1, 1882, would be $1060 ($1000 + $60); the amount due Jan. 1, 1883, would be $1060 plus $63.60 (6% of $1060), or $1123.60. The simple interest for 2 years would be $120; the compound interest for the same time, $123.60. When the word interest is used alone, simple interest is understood.

434. Legal Interest is the interest according to the rate per cent. fixed by law for cases in which the rate per cent. is not specified. By special agreement between parties in certain States, interest may be received at a rate higher than the legal rate. In most of the States, this rate is limited. See Art. **436**.

435. Usury is the taking of a higher rate of interest than that allowed by law. A person taking usury is liable to certain penalties differing in the several States.

(82. Account Purchase.)

TOLEDO, O., *Mar. 6, 1887.*

Purchased by A. L. BACKUS & SONS,
For account and risk of L. A. & W. B. SHAW.

9	Bags "Montauk" - - - - - -	.21	*	**		
$2^{\underline{21}}$	Bu. Mammoth Clover Seed - - -	$9^{\underline{00}}$	**	**		
$9^{\underline{28}}$	" Clover Seed - - - - - -	$8^{\underline{50}}$	**	**	***	**
$9^{\underline{21}}$	" Timothy Seed - - - - -	$1^{\underline{75}}$	**	**		
	Charges.					
	Cartage - - - - - - - - - -			25		**
	Commission 1% - - - - - - -		*	**	*	
	Charge your % - - - - - - -				***	*

NOTE.—The small figures at the left represent pounds. See Art. **338**.

33. According to the above form, prepare an Account Purchase of 3 Half-Chests Gunpowder Tea, 165 *lb.* @ 35*c.*; 2 *Hfc.* Oolong Tea, 86 *lb.* @ 20*c.*; 20 bags Rio Coffee, 2388 *lb.* @ 13*c.*; 2 mats Java Coffee, 133 *lb.* @ 19½*c.*; 1 *Hhd.* P. R. Molasses, 143 *gal.* @ 54*c.* Charges as follows: Drayage, $1.75; Commission, 1%. Commission Merchants, Student & Co.; bought for Teacher & Co.; present place and date.

34. After paying an auctioneer 5%, a man received $1172.30 for his furniture. What were the gross sales?

35. A bankrupt's assets are $17415, and his liabilities, $48375. I place my claim of $2560 in the hands of my attorney for collection. How much do I receive if the attorney retains 5% commission?

36. A landlord received $822 as the net rental of a house, after his agent had paid $60 for repairs and charged 2% commission on the gross rental. What was the gross rental?

37. C of New York sells for D of Atlanta, a quantity of cotton, amounting to $7317.83, and charges a commission of 2½%. By instructions, he invests the proceeds in dry goods, after deducting a commission of 1½% on the amount expended. What was the total commission?

38. A commission merchant sold 300 bales of cotton, averaging 462 *lb.* to the bale, at 15.7¢, his commission being 25¢ per bale, and the charges $161. He purchased for the consignor dry goods amounting to $2576.37, charging a commission of 1½%. How much was still due the consignor?

1. **Finding the time in months and days (Compound Subtraction, Art. 310), and regarding the months as twelfths of a year, and the days as thirtieths of a month or 360ths of a year.** This method, although implied by the general interest laws* of the State of New York, is not uniform, since it allows the same interest for February with its 28 days as for March with its 31 days. Its results are sometimes greater and sometimes less than those of accurate interest.

2. Finding the exact time in days (**310**) and regarding the days as 360ths of a year. Since a day is $\frac{1}{365}$ of a year, this method produces too great a result. It is used by merchants and bankers generally, and by many banks† in discounting notes. 6% by this method is equivalent to $6\frac{1}{12}$% accurate interest.

3. **Accurate Interest.**—Finding the exact time in days (**310**) and regarding the days as 365ths of a year. This method is used by the United States government, and by some merchants and banks; but, on account of its inconvenience when interest tables are not used, it is not generally adopted.

NOTES.—1. By the first method, the time from July 10 to Sept. 10, would be 2 months, and the interest would be $\frac{2}{12}$ or $\frac{1}{6}$ of the interest for one year. On $10000 at 6% for 2 months, the interest would be $100 ($\frac{1}{6}$ of .06 of $10000).

2. By the second method, the interval between the same dates would be 62 days, and the interest would be $\frac{62}{360}$ of the interest for one year. On $10000 at 6% for $\frac{62}{360}$ of a year, the interest would be $103.33 ($\frac{62}{360}$ of .06 of $10000).

3. By the third method, the interval between the same dates would be 62 days as in the second method, and the interest would be $\frac{62}{365}$ of the interest for one year. On $10000 at 6% for $\frac{62}{365}$ of a year, the interest would be $101.92 ($\frac{62}{365}$ of .06 of $10000).

4. The difference between ordinary interest and accurate interest for the same number of days is $\frac{1}{73}$ of the former, or $\frac{1}{72}$ of the latter. Thus in the above example, the difference between the results, $1.41 ($103.33-101.92), is $\frac{1}{73}$ of $103.33, or $\frac{1}{72}$ of $101.92.

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* "For the purpose of calculating interest, a month shall be considered the twelfth part of a year, and as consisting of thirty days; and interest for any number of days less than a month shall be estimated by the proportion which such number of days shall bear to thirty"

† According to the banking laws of the State of New York, banks are authorized in discounting notes to charge interest in advance for the exact number of days which the note has to run (Ch. XVIII, Title 2, § 300).

This law appears to conflict with the law quoted above which implies that the time shall be found in months and days. It does not state whether the days shall be regarded as 360ths or 365ths of a year.

EXAMPLES.

441. Find the interest of

1. $875 for 2 *yr.* at 7%.
2. $642.50 for 3 *yr.* at 6%.
3. $1010.10 for 6 *yr.* 6 *mo.*, at 8%.
4. $3010.75 for 3 *yr.* 4 *mo.*, at 7%.
5. $3745.80 for 4 *yr.* 1 *mo.*, at 6%.
6. $816.40 for 5 *yr.* 3 *mo.*, at 5%.
7. $1275 for 7 *yr.* at 6%.
8. $2789.40 for 3 *yr.* 2 *mo.*, at 4½%.
9. $456.75 for 4 *yr.* 8 *mo.*, at 5%.
10. $10180 for 3 *yr.* 4 *mo.*, at 10%.

In the following examples, find the time by Compound Subtraction (**310**).

11. What is the interest of $6488 from May 3, 1889, to Sept. 3, 1891, at 7%?

12. What is the amount of $396.60 from Aug. 16, 1890, to Dec. 16, 1892, at 8%?

13. Compute the interest of $250.75 from Nov. 20, 1892, to July 20, 1894, at 4½%.

14. Loaned on interest, New York, Dec. 16, 1880, $1739.75 (no rate specified); what amount should I receive, June 16, 1881?

442. To find the ordinary interest (360 days to the year) for any rate and time.*

443. First or Day Method.

The interest of $1 for 1 year at 36% is $.36.
" " $1 " 1 day at 36% is $.001.
" " $1 " 1 " 6% is $.000⅙.
" " $1 " 1 " 9% is $.000¼.
" " $1 " 1 " 4% is $.000⅑.
" " $1 " 1 " 12% is $.000⅓.
" " $1 " 1 " 3% is $.000$\frac{1}{12}$.
" " $1 " 1 " 4½% is $.000⅛.

Ex. What is the interest of $1735 for 73 days at 6%?

OPERATION.

```
  $1735
     73
   5205
  12145
6)126655
 $21.109
```

ANALYSIS.—The interest of $1735 for 73 days is equivalent to the interest of 73 times $1735, or $126655 for 1 day. Since the interest of $1 for 1 day is ⅙ of a mill, the interest of $126655 for 1 day is as many mills as 6 is contained times in 126655, or 21109 mills, or $21.11.

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* **The student should be taught at least two** (to be selected by the teacher) **of the following methods of reckoning interest.**

[illegible]

[illegible] 29 days at 3%.

[illegible] days at 12%.

[illegible] at 5%.

[illegible] at 7%.

[illegible] at 8%.

[illegible] 7%.

[illegible] at 10%.

[illegible] Compound Subtraction

[illegible] at [illegible] At 4½%.

[illegible] at 7½%.

[illegible] at 6%. At 12%.

17. [illegible] Feb. 1, 1888, at 6%. At 10%.

Second or 6% Method.

446. At 6%, the interest of one dollar for 1 year is \$0.06. For 1 month, $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of \$0.06, or \$0.00½ (.005). For 1 day, $\frac{1}{30}$ of a month, it is $\frac{1}{30}$ of \$0.005, or \$0.000⅙.

Ex. What is the interest of \$864, at 6%, for 2 *yr.* 7 *mo.* 20 *da.*?

OPERATION.

$$\begin{array}{rcl@{\qquad}r}
2 \times .06 & = & .12 & 864 \\
7 \times .00\frac{1}{2} & = & .035 & \underline{.158\frac{1}{3}} \\
20 \times .000\frac{1}{6} & = & \underline{.003\frac{1}{3}} & 288 \\
 & & .158\frac{1}{3} & 6912 \\
 & & & 4320 \\
 & & & \underline{864} \\
 & & & \$136.800
\end{array}$$

ANALYSIS.—If the interest of \$1 for 1 *yr.* is \$.06, for 2 *yr.*, it is twice \$.06, or \$.12. If the interest of \$1 for 1 *mo.* is \$.00½, for 7 *mo.*, it is 7 times \$.00½, or \$.035. If the interest of \$1 for 1 day is \$.000⅙, for 20 days, it is 20 times \$.000⅙, or \$.003⅓. Hence the interest of \$1 for 2 *yr.* 7 *mo.* 20 *da.* is \$.12 + \$.035 + \$.003⅓, or \$.158⅓. The interest of \$864 is 864 times \$.158⅓, or \$136.80.

447. RULE.—*Take as the interest of one dollar, six cents for each year, one-half cent (or five mills) for each month, and one-sixth of a mill for each day. Multiply the principal by the sum of these amounts (as an abstract number). The product will be the interest at 6%.*

To find the amount, add the principal to the interest.

NOTES.—1. In using this method, to multiply by ⅔, write ⅓ twice; to multiply by ⅚, take ½ and ⅓.

2. If the time is less than 60 days, and the rate is 6% or less, reckon the interest on the nearest number of dollars. The result will be sufficiently accurate.

448. The interest for any other rate may be found from the interest at 6% as follows: At 1%, divide by 6; at 1½%, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract ⅓; at 4½%, subtract ¼; at 5%, subtract ⅙; at 5¼%, subtract ⅛; at 5½%, subtract $\frac{1}{12}$; at 6½%, add $\frac{1}{12}$; at 6¾%, add ⅛; at 7%, add ⅙; at 7½%, add ¼; at 8%, add ⅓; at 9%, add ½; at 10%, divide by 6, and multiply by 10 by placing the point one place to the right; at 12%, multiply by 2. At any per cent., divide by 6 and multiply by the rate.

449. For examples to be worked by the above method, see Art. **445** and Art. **459**

Third or Month Method.

450. **Ex.** What is the interest of $875 for 2 *yr.* 8 *mo.* 14 *da.*, at 4% ?

OPERATION.

```
     $875
      .04
12 ) 35.00    Int. for 1 yr.
     2.917    Int. for 1 mo.
     32.4⅔    Number of mos.
       972
       972
     11668
     5834
    8751
  $94.7052
```

Analysis.—The interest of $875 for 1 *yr.* at 4% is $35 (.04 × $875), and for 1 *mo.* is $\frac{1}{12}$ of $35, or $2.917. 2 *yr.* 8 *mo.* = 32 *mo.* 14 *da.* = .3⅔ *mo.* (To reduce days to tenths of a month, divide the number of days by 3 and place the point one place to the left.) If the interest for 1 *mo.* is $2.917, for 32.3⅔ *mo.*, it is 32.3⅔ times $2.917, or $94.71.

451. Rule.—*Multiply the principal by the rate per cent., and divide the product by 12. The result will be the interest for one month at the given rate per cent. Multiply this result by the total number of months, the years being reduced to months and the days being expressed as tenths of a month.*

452. For examples to be worked by this method, see Art. **445** and Art. **459.**

Fourth or 60-day Method at 6%.

453. 6% for 12 months or 1 year, is equivalent to 1% for 2 months (60 days), or $\frac{1}{6}$ of one year. 1% of any amount is readily ascertained by placing the point two places to the left. Hence the interest of any sum at 6% per annum for 2 months, or 60 days, may be found by placing the point two places to the left. The interest for 6 days may be found by placing the point three places to the left.

Note.—It will be found advantageous to use a perpendicular line as a separatrix in solving examples by this method. All necessity for pointing off will then be dispensed with, and confusion prevented.

Ex. 1. What is the interest of $1236 for 80 *da.*, at 6%?

OPERATION.

$12	36 = int. for 60 *da.*
4	12 = " " 20 *da.*
$16	48 = " " 80 *da.*

ANALYSIS.—The interest of $1236 at 6% for 60 *da.* is found to be $12.36, by the process already explained. If the interest for 60 *da.* is $12.36, for 20 *da.* ($\frac{1}{3}$ of 60), it will be $\frac{1}{3}$ of $12.36, or $4.12. Hence for 80 *da.*, it will be $12.36 plus $4.12, or $16.48.

Ex. 2. Find the interest of $864 for 1 *yr.* 10 *mo.* 15 *da.*, at 6%.

OPERATION.

$8	64 = int. for 2 *mo.*, or 60 *da.*
	11
95	04 = int. for 22 *mo.*
2	16 = " " 15 *da.*
$97	20 = required interest.

ANALYSIS.—The interest of $864 at 6% for 2 *mo.* is $8.64. For 1 *yr.* 10 *mo.* (22 *mo.*), it will be 11 times $8.64, or $95.04. If the interest for 60 *da.* is $8.64, for 15 *da.* ($\frac{1}{4}$ of 60), it will be $\frac{1}{4}$ of $8.64, or $2.16. Hence the interest for the given time will be $95.04 plus $2.16, or $97.20.

Ex. 3. What is the interest of $375.60 for 55 days, at 6%?

OPERATION.

$3	75.60 = int. for 60 *da.*
	313 = " " 5 "
3	443 = " " 55 "

ANALYSIS.—55 days = 60 days less 5 days. The interest for 60 days is $3.756, for 5 days ($\frac{1}{12}$ of 60), $.313, and for 55 days, $3.443 ($3.756 − $.313).

454. Aliquot Parts of 60.—1 = $\frac{1}{60}$; 2 = $\frac{1}{30}$; 3 = $\frac{1}{20}$; 4 = $\frac{1}{15}$; 5 = $\frac{1}{12}$; 6 = $\frac{1}{10}$; 10 = $\frac{1}{6}$; 12 = $\frac{1}{5}$; 15 = $\frac{1}{4}$; 20 = $\frac{1}{3}$; 30 = $\frac{1}{2}$.

NOTES.—1. To divide by 10, place the figures of the basis one place to the right.

2. To divide by 20, 30, or 60, divide by the first figure and write the quotient figures one place to the right.

455. Ex. 1. What is the interest of $976 for 26 days, at 6%?

OPERATION

$9	76 = int. for 60 *da.*
3	253 = " " 20 *da.*
	976 = " " 6 *da.*
4	229 = " " 26 *da.*

ANALYSIS.—As 26 is not an aliquot part (**388**) of 60, take 20, which is $\frac{1}{3}$ of 60, and 6, which is $\frac{1}{10}$ of 60. Divide the basis which is the interest for 60 *da.* by 3 to find the interest for 20 *da.* ($3.253); and the same sum by 10, to find the interest for 6 *da.* ($.976). (See Art. **454**, 1.) The sum of these two results will be the interest for 26 days.

Ex. 2. Find the interest of $732.80 for 2 *yr.* 8 *mo.* 27 *da.*, at 6%?

OPERATION.

$7	32.8	= int. for 2 *mo.*, or 60 *da.*
	16	
43	968	
73	28	
1	832	= int. for 15 *da.*
1	466	= " " 12 *da.*
$120	546	= required int.

ANALYSIS.—The interest for 2 *mo.*, forming the basis, is $7.328. Multiply this by 16 to find the interest for 32 *mo.* (2 *yr.* 8 *mo.*). 27 = 15 + 12. To find the interest for 15 *da.*, divide the basis by 4 (15 = $\frac{1}{4}$ of 60); and the same sum by 5 to find the interest for 12 *da.* By adding these results, we have the interest for the given time at 6%.

456. If the number of days given is not an aliquot part of 60, it will need to be so separated that the component parts will be aliquot parts of 60.

Numbers not aliquot parts of 60, with best divisions: 7 = 6+1; 8 = 6+2; 9 = 6+3; 11 = 6+5, or 10+1; 13 = 10+3; 14 = 12+2; 16 = 10+6; 17 = 12+5, or 15+2; 18 = 12+6. (The interest for 18 days may be found by multiplying the basis by 3, and placing the figures of the product one place to the right); 19 = 15+4, or 10+6+3; 21 = 15+6; 22 = 20+2 (2 = $\frac{1}{10}$ of 20); 23 = 20+3; 24 = 12+12 (or multiply by 4 and place the figures of the product one place to the right); 25 = 20+5 (5 = $\frac{1}{4}$ of 20); 26 = 20+6; 27 = 15+12; 28 = 12+12+4 (4 = $\frac{1}{3}$ of 12), or 20+6+2; 29 = 12+12+5, or 20+6+3.

457. RULE.—*Draw a perpendicular line two places to the left of the decimal point; the result will be the interest at 6% for 2 months, or 60 days, the dollars being on the left, and the cents on the right of this line. Multiply this result by one-half the total number of months. To this product, add that proportion of the interest for 60 days, which the given number of days is of 60.*

NOTE.—The interest at any other rate per cent. may be found as in Art. **448.**

458. The interest at 6% may be found for 6 days by placing the point three places to the left (**453**). In many examples, when the time is less than 100 days, the process is shortened by taking as the basis the interest for 6 days instead of 60 days.

Ex. Find the interest of $375 for 8 days at 6%. $425 for 79 days. $500 for 47 days.

1ST OPERATION.

$	375	6 *da.*
	125	2 *da.*
	500	8 *da.*

2ND OPERATION.

$	425	6 *da.*
5	525	78 *da.* (13 × 6)
	071	1 *da.*
5	596	79 *da.*

3RD OPERATION.

$	500	6 *da.*
4	000	48 *da.*
	083	1 *da.*
3	917	47 *da.*

EXAMPLES.

459. Find the interest of the following at 6%. (See Arts. **453** and **454**.)

1. $864 for 60 days.
2. $396 for 20 days.
3. $290 for 72 days.
4. $785 for 66 days.
5. $636 for 62 days.
6. $400 for 90 days.
7. $525 for 61 days.
8. $600 for 10 days.
9. $728 for 65 days.
10. $340 for 15 days.
11. $1275 for 50 days.
12. $2345 for 30 days.
13. $1728 for 63 days.
14. $375.60 for 5 days.
15. $414.80 for 54 days.
16. $1024 for 59 days.
17. $2375 for 90 days.
18. $1000 for 57 days.
19. $2480 for 63 days.
20. $5000 for 56 days.

What is the interest of

21. $375.60 for 8 *mo.* 20 *da.*, at 6%? At 5%? (See Ex. 2, Art. **453**.)
22. $1727 for 7 *mo.* 15 *da.*, at 6%? At 3%? (**448**)
23. $449.38 for 1 *yr.* 4 *mo.* 12 *da.*, at 6%? At 7%?
24. $285 for 1 *yr.* 5 *mo.* 10 *da.*, at 6%? At 5%?
25. $432.65 for 2 *yr.* 2 *mo.* 6 *da.*, at 6%? At 8%?
26. $1235 for 2 *yr.* 5 *mo.* 5 *da.*, at 6%? At 4%?
27. $445.25 for 5 *mo.* 4 *da.*, at 6%? At 9%?
28. $2440.50 for 97 days, at 6%? At 7%? (See Ex. 1, Art. **455**.)
29. $3125 for 38 days, at 6%? At 7%?
30. $247.50 for 69 days, at 6%? At 5%?
31. $512.45 for 5 *mo.* 11 *da.*, at 6%? At 7%? (See Ex. 2, Art. **455**.)
32. $1478 for 1 *yr.* 2 *mo.* 13 *da.*, at 6%? At 8%?
33. $2810.60 for 9 *mo.* 24 *da.*, at 6%? At 5%?
34. $944.50 for 1 *yr.* 10 *mo.* 22 *da.*, at 6%? At 4½%?
35. $575 for 2 *yr.* 8 *mo.* 16 *da.*, at 6%? At 9%?

36. $1112 for 3 *mo.* 14 *da.*, at 6%? At 4½%?
37. $5285 for 1 *yr.* 6 *mo.* 21 *da.*, at 6%? At 3%?
38. $7218 for 11 *mo.* 18 *da.*, at 6%? At 12%?

Find the amount of

39. $416.75 for 8 *mo.* 17 *da.*, at 6%? At 7%?
40. $1235 for 2 *yr.* 1 *mo.* 19 *da.*, at 6%? At 8%?
41. $575.60 for 1 *yr.* 4 *mo.* 23 *da.*, at 6%? At 5%?
42. $2214 for 4 *mo.* 25 *da.*, at 6%? At 4%?
43. $6315 for 5 *mo.* 29 *da.*, at 6%? At 9%?
44. $4312 for 4 *mo.* 26 *da.*, at 6%? At 4½%?
45. $384.30 for 2 *mo.* 28 *da.*, at 6%? At 3%?
46. $1296 for 1 *yr.* 11 *mo.* 27 *da.*, at 6%? At 12%?
47. $4375 for 2 *yr.* 8 *mo.* 24 *da.*, at 6%? At 5½%?

Find the interest of the following at 6%. (See Art. **458.**)

48. $2000 for 6 *da.*
49. $1728 for 37 *da.*
50. $3485 for 92 *da.*
51. $1234 for 69 *da.*
52. $375.60 for 8 *da.*
53. $3748 for 53 *da.* (54—1.)
54. $4126 for 89 *da.*
55. $1289 for 39 *da.*
56. $4000 for 17 *da.*
57. $2000 for 28 *da.*

NOTE.—Find the time in the following examples both in months and days, and in exact days (**310**).

58. $1234 from May 10 to Dec. 4, at 5%? At 4¾%?
59. $444.40 from Jan. 13 to Nov. 2, at 4%? At 5¼%?
60. $575.20 from June 5, 1882, to Feb. 4, 1883, at 7%? At 5%?
61. $2375 from July 17, 1884, to Nov. 27, 1885, at 6%? At 3½%? (Exact time, 1 *yr.* 133 *da.*)
62. $3212 from Aug. 24, 1881, to Jan. 20, 1884, at 4%? At 4½%?
63. $475.80 from May 12, 1882, to Feb. 1, 1884, at 7%? At 10%?
64. Find the interest of $180 for 253 days, at 6%. At 8%.

NOTE.—In many examples, labor can be saved by having the time and principal exchange places. In the above example, the interest of $183 for 253 days is the same as $253 for 180 days ($2.53 × 3).

65. Find the interest of $600 for 173 days at 9%. At 4%.
66. Find the interest of $3000 for 111 days at 12%. At 3%.

Find the interest of

67. $1800 from Jan. 17 to Oct. 2, at 6%. At 4¾%.
68. $540 from May 11 to Dec. 18, at 5%. At 4½%.
69. $3000 from Feb. 4 to July 13, at 4%. At 4½%.
70. $2400 from July 13 to Dec. 1, at 5%. At 5¼%.
71. $600 from Aug. 16 to Nov. 24, at 6%. At 5½%.
72. $1200 from May 19 to July 3, at 7%. At 6½%.
73. $480 from March 13 to Sept. 3, at 8%. At 3½%.
74. $720 from Feb. 27 to May 15, at 9%. At 4¾%.
75. $2100 from Sept. 2 to Nov. 30, at 10%. At 7½%.

76. If $9200 is loaned Sept. 18, 1882, at 6%, what is due May 9, 1885? (Time by C. S.)

77. What is a banker's gain in 1 year on $10000 deposited at 6%, and loaned 11 times at 1½% a month?

78. A note for $1421, with interest after 4 months, at 7%, was given Dec. 1, 1881, and paid Aug. 12, 1883. What was the amount due? (C. S.)

79. Nov. 6, 1881, I bought a lot of grain for $753.20; Dec. 16, I sold a part of it for $375.60; and, Dec. 31, I sold the remainder for $411.40. Money being worth 6%, how much did I gain by the transaction?

80. A merchant marks his goods with two prices, the one for cash and the other for 4 months' credit. If the cash price is $28, what ought the credit price to be, money being worth 10%?

81. May 27, $328 is loaned at 6%, and Aug. 16, $1000 is loaned at 5½%. What is the total amount due Dec. 11?

82. A banker borrows $100000 at 3½%, and pays the interest at the end of the year; he loans it at 5% and receives the interest semi-annually. How much does he gain in one year, if he loans the semi-annual interest until the end of the year?

83. A buys a bill of goods amounting to $2776.40, on the following terms:—"4 months, or less 5% cash." He accepts the latter terms, and borrows the money at 6% to pay the bill. How much does he gain?

84. A person buying a building lot for $5400, agreed to pay for it in four equal semi-annual installments, with interest at 6%; what was the total amount of money paid, the first payment being made at the time of the purchase?

85. A banker borrows $10000 at 4½%, and lends half of it at 6% and half at 8%. What does he gain in 2 *yr.* 4 *mo.* 26 *da.*?

In reckoning accurate interest, on account of the many short methods of ordinary interest, many accountants prefer to calculate ordinary interest first, and then make the necessary deduction.

Since 1/73 is about 1⅓%, the following approximate method may be used in reducing ordinary interest to accurate interest: ***From the ordinary interest subtract 1% and ⅓% of itself.***

Ex. Reduce $32.70 ordinary interest to accurate interest.

OPERATION.

32.70
.327 1%
32.373
.109 ⅓%
32.264

NOTE.—The exact result should be $32.252. The results by this method are too great by 1 cent for each $27 interest; $.036 for each $100 interest; $.36 for each $1000 interest. Where greater accuracy is required, the necessary correction can be made.

EXAMPLES.

463. What is the accurate interest of

1. **$435.32, at 6%, for 25 days?** *5.* **$292, at 3½%, for 140 days?**
2. **$6030, at 5%, for 141 days?** *6.* **$438, at 6%, for 210 days?**
3. **$780, at 6%, for 90 days?** *7.* **$350, at 4%, for 150 days?**
4. **$437.80, at 7%, for 63 days?** *8.* **$500, at 4½%, for 100 days?**
9. **$3110.45, at 5½%, for 90 days?**
10. **$373.70, at 7%, from June 4 to Dec. 28?**
11. **$500, at 6%, from July 24, to Sept. 16?**
12. **$365, at 6%, from June 30 to Dec. 21?**
13. **$1080, at 5%, from May 9, 1878, to Jan. 30, 1879?**
14. **$1728, at 7%, from Jan. 6, 1878, to Jan. 21, 1880?**

15. Required the exact interest on three U. S bonds of $5000 each, at 3½%, from July 1 to Aug. 11.

16. What is the interest on three U. S. bonds of $1000 each, at 4½%, from Sept. 1 to Nov. 15?

17. What is the interest on a $5000 U. S. bond, at 4%, from Oct. 1 to Dec. 16?

18. What is the interest on a U. S. bond of $1000, bearing 3½% interest, from May 1 to July 19?

19. What is the interest on a $500 U. S. bond, at 4%, from Apr. 1 to May 10?

20. What is the interest on a $5000 U. S. bond from Nov. 1, 1881, to Jan. 3, 1882, at 3½%?

21. What is the difference between ordinary and accurate interest of $10000 for 219 days at 6%?

[illegible] OF INTEREST.

[illegible] principal, interest or amount, given.

[illegible] in 3 yr. 4 mo. 10 da., produce

[illegible]

Operation.—The interest on a given principal for a given time is in proportion to the rate per cent. At one per cent, [illegible] will, in 3 yr. 4 mo. 10 da., produce [illegible] interest. To produce [illegible] interest, the required rate must be as many times 1%, as [illegible] is contained times in $44.10, or 4½ times. Hence the answer is 4½%.

[illegible] 4½% Ans.

—Divide the given interest by the interest of [illegible]pal, for the given time, at 1%.

[illegible] the amount is given, find the interest by subtracting th [illegible]

EXAMPLES.

[illegible] rate will

[illegible] produce $42 interest?

[illegible] produce $51.25 interest?

[illegible] amount to $[illegible]?

[illegible] mo. 24 da. produce $[illegible] interest?

[illegible] mo. 21 da. amount to $[illegible]?

[illegible] produce $41.32 interest?

[illegible] mo. 18 da. amount to $[illegible]?

[illegible] mo. [illegible] da. produce $34.64 interest?

[illegible] 8 mo. 24 da. amount to $[illegible]?

[illegible] mo. 8 da. produce $[illegible] interest?

[illegible] 11 mo. 12 da. amount to $[illegible]?

[illegible] 8 mo. 22 da. produce $[illegible] interest?

[illegible] 3 mo. 28 da. amount to $[illegible]?

[illegible] yr. 11 mo. 18 da. produce $[illegible] interest?

[illegible]

467. **To find the time, the principal, interest or amount, and rate being given.**

Ex. In what time will $426, at 6%, produce $59.427 interest?

OPERATIONS.

```
 $426                                Or,  $426
  .06                                      .06
 ------                                   ------
$25.56 ) $59.427 ( yr. 2.325             $25.56 ) $59.427 ( 2 yr.
          51 12          12                         51 12
         ------       ------                       ------
          8 307 mo. 3.900                           8.307
          7 668          30                            12
         ------       ------                       ------
           6390 da. 27.000               $25.56 ) 99.684 ( 3 mo.
           5112                                   76.68
          ------                                 ------
          12780                                  23.004
          12780                                      30
          ------                                 ------
              0                          $25.56 ) 690.120 ( 27 da.
```

ANALYSIS.—The interest on a given principal at a given rate % is in proportion to the time. In one year $426, at 6%, will produce $25.56 interest. To produce $59.427 interest, it will require as many years as $25.56 is contained times in $59.427, or 2.325 *yr.* 2.325 *yr.* equal 2 *yr.* 3 *mo.* 27 *da.* (**289**).

468. RULE.—*Divide the given interest by the interest of the given principal, at the given rate, for one year.*

The integral part of the quotient will be years. Reduce the decimal, if any, to months and days (**289**).

EXAMPLES.

469. In what time will

1. $3000, at 7%, produce $108.50 interest?
2. $1728, at 6%, amount to $1872?
3. $3932, at 7%, produce $597.88 interest?
4. $735, at 5%, amount to $742.66?
5. $1222.25, at 6%, produce $39.52 interest?
6. $375.60, at 7%, amount to $425.41?
7. $1461.75, at 6%, produce $420.25 interest?
8. $1200, at 3¾%, amount to $1413?
9. $4500, at 5%, produce $181.25 interest?
10. $276.50, at 10%, amount to $303.46?

In what time will

11. $1020, at 6%, produce $89.25 interest ?
12. $6495, at 7%, amount to $7161.81 ?
13. $100, at 6%, produce $100 interest ?
14. $125, at 7%, amount to $375 ?

470. To find the principal, the interest, time, and rate being given.

Ex. What principal will produce $152.64 interest, in 1 *yr.* 5 *mo.* 20 *da.*, at 6% ?

OPERATION.

```
$.088⅛ ) $152.64 ( 1728
    3          3
------------------
.265   )  457.920
          265
          ----
          1929
          1855
          ----
            742
            530
            ----
            2120
            2120
            ----
```

ANALYSIS.—The interest on any principal is as many times greater than the interest of $1, as that principal is greater than $1. One dollar, in 1 *yr.* 5 *mo.* 20 *da.*, at 6% (**447**), will produce $.088⅛ interest. To produce $152.64, the principal must be as many times $1 as $.088⅛ is contained times in $152.64, or $1728.

471. RULE.—*Divide the given interest by the interest of $1 for the given time, at the given rate.*

EXAMPLES.

472. What principal will produce

1. $1235 interest, in 1 *yr.* 8 *mo.* 12 *da.*, at 6% ?
2. $49.81, in 9 *mo.* 24 *da.*, at 7% ?
3. $186.75, in 1 *yr.* 4 *mo.* 20 *da.*, at 6% ?
4. $244.44, in 7 *mo.* 18 *da.*, at 5% ?
5. $375.60, in 2 *yr.* 4 *mo.* 6 *da.*, at 8% ?
6. $54.25, in 3 *mo.* 3 *da.*, at 7% ?
7. $387.40, in 2 *yr.* 8 *mo.*, at 4½% ?
8. $456, in 93 *da.*, at 6% ?
9. $375, in 63 *da.*, at 7% ?
10. $1000, in 1 *yr.* 18 *da.*, at 3% ?
11. $538.80, in 10 *mo.* 24 *da.*, at 5% ?
12. $416.75, in 8 *mo.* 21 *da.*, at 4% ?
13. $645.39, in 4 *yr.* 8 *mo.* 10 *da.*, at 4% ?

473. **To find the principal, the amount, time, and rate, being given.**

Ex. What principal will amount to $1880.64, in 1 *yr.* 5 *mo* 20 *da.*, at 6%?

OPERATION.

```
$1.088⅓ ) $1880.64 ( 1728.
      3          3
---------------------
 3.265  )  5641.920
           3265
           ----
           23769
           22855
           -----
             9142
             6530
             ----
             26120
             26120
             -----
                 0
```

ANALYSIS.—The amounts of different principals for the same time and rate %, are to each other as the principals. One dollar, in 1 *yr.* 5 *mo.* 20 *da.*, at 6% will amount to $1.088⅓. To amount to $1880.64, the principal must be as many times $1 as $1.088⅓ are contained times in $1880.64, or $1728.

474. RULE.—*Divide the given amount by the amount of $1 for the given time, at the given rate.*

EXAMPLES.

475. What principal will amount to

1. $1272.254, in 6 *mo.* 6 *da.*, at 6%?
2. $5538.72, in 8 *mo.* 12 *da.*, at 7%?
3. $3695.04, in 1 *yr.* 4 *mo.* 18 *da.*, at 5%?
4. $442.71, in 2 *yr.* 2 *mo.* 24 *da.*, at 8%?
5. $14794.31, in 3 *yr.* 3 *mo.* 3 *da.*, at 6%?
6. $1793.38, in 7 *mo.* 17 *da.*, at 6%?
7. $1010.65, in 5 *yr.* 8 *mo.* 6 *da.*, at 7%?
8. $977.75, in 1 *yr.* 10 *mo.* 10 *da.*, at 6%?
9. $1716.75, in 3 *yr.* 4 *mo.* 21 *da.*, at 4%?
10. $2808.08, in 2 *yr.* 8 *mo.* 12 *da.*, at 8%?
11. $4312.22, in 1 *yr.* 2 *mo.* 11 *da.*, at 12%?
12. $6528.49, in 4 *yr.* 7 *mo.* 6 *da.*, at 5%?
13. $1763.02, in 1 *yr.* 2 *mo.* 21 *da.*, at 6%?
14. $2457.28, in 2 *yr.* 5 *mo.* 23 *da.*, at 6%?
15. $5375.34, in 1 *yr.* 6 *mo.* 15 *da.*, at 4%?
16. $3536.87, in 2 *yr.* 7 *mo.* 10 *da.*, at 9%?
17. $4221.50, in 3 *yr.* 10 *mo.* 27 *da.*, at 4½%?

PRESENT WORTH AND TRUE DISCOUNT.

476. The **Present Worth** of a debt due at some future time is its value now. Theoretically, it is a sum that, if placed at interest to-day for the given time, would amount to the face of the debt.

477. The **True Discount** is the difference between the face of the debt and the present worth.

This subject is an application of the principle illustrated in Art. **473,** the face of the debt being the amount, the present worth the principal, and the true discount the interest.

In actual business true discount is little used, banks and merchants generally using bank discount (**496**). True discount is the interest on the present worth for the given time, while bank discount is interest on the face of the debt. The difference is therefore equivalent to the interest on the true discount. For discount on bills, etc., when time does not enter as an element, see Art. **415.**

Ex. Mr. B owes me $212, payable one year from to-day without interest; what is the present worth of the debt, the current rate of interest being 6%?

ANALYSIS.—Since $1 in one year, at 6%, amounts to $1.06, it would require as many dollars to amount to $212, as $1.06 is contained times in $212, or $200. The true discount is $212−$200, or $12.

478. RULE.—*I. To find the present worth, divide the face of the debt by the amount of $1 for the given time, at the given rate.*

II. To find the true discount, subtract the present worth from the face of the debt.

EXAMPLES.

479. The current rate of interest being 6%, what is the present worth and true discount of

1. $1000, due 2 years hence? *3.* $600, due in 1 *yr.* 7 *mo.*?
2. $500, due in 2 *yr.* 4 *mo.*? *4.* $800, due in 9 *mo.* 24 *da.*?
5. $325, due in 2 *yr.* 5 *mo.* 12 *da.*?
6. $175, due in 1 *yr.* 4 *mo.* 16 *da.*?
7. $800, due in 5 *yr.* 8 *mo.* 22 *da.*?
8. $900, due in 6 *yr.* 8 *mo.* 14 *da.*?

9. Mr. C. desiring to pay a bill of $1728 4 months before it was due, was allowed a discount equivalent to the interest on the face of the bill for the unexpired time at 6% per annum (bank discount). How much greater was this discount than the true discount?

10. Goods to the amount of $3750 are sold on a credit of 4 months. For how much cash could the merchant afford to sell the same goods, money being worth 10% per annum?

11. If $10000 will be due me May 28, and $8000 May 16, what discount should I make on the two claims Apr. 1, money being worth 8%?

REVIEW EXAMPLES.

480. *1.* What is the interest of $375.60, for 1 *yr.* 10 *mo.* 16 *da.*, at 6%?

2. What is the amount of $1765 for 7 *mo.* 20 *da.*, at 7%?

3. At what rate will $1234, in 2 *yr.* 2 *mo.* 26 *da.*, produce $138.14 interest?

4. In what time will $585, at 6%, produce $67.08 interest?

5. What principal will, in 1 *yr.* 8 *mo.* 14 *da.*, at 6%, produce $176.22 interest?

6. The semi-annual interest on a mortgage at 7% is $350. What is the face of the mortgage?

7. Mr. B. invests $49500 in a business that pays him $594 per month. What annual rate of interest does he receive?

8. Which is the better investment, and what per cent., one of $8400, yielding $336 semi-annually, or one of $15000, producing $1425 annually?

9. May 18th, a speculator bought 1600 bushels of wheat, at $1.50 a bushel. He afterward sold the whole for $2472 cash, his profit being equivalent to 8% per annum on the amount invested. What was the date of the sale?

10. The par value of Mr. A.'s bank stock is $9000, and he receives a semi-annual dividend of $315. What per cent. is the dividend per annum?

11. Mrs. C.'s son is now 16 *yr.* old; how much must she invest for him at 6%, that, on arriving at age, he may have, with simple interest, $25000?

12. A bill of goods amounting to $4316.75 is due May 27; how much would settle it May 1 at 6%? How much July 3?

13. A gentleman loaned $15000, at 6%. Jan. 1, 1880, interest and principal together equalled $20000. When was the money loaned?

14. Find the interest on $3000, from Mar. 16 to Dec. 4, at 6%, by the following methods (**437**): 1, ordinary interest and compound subtraction; 2, ordinary interest and exact number of days; 3, accurate interest.

15. A man loaned another a sum of money, payable in 5 months, with interest at the rate of 6%, and at the end of that time received $666.25 in return. How much did he loan?

16. A speculator borrowed $10925 at 6%, May 16, 1882, with which he purchased flour at $6.25 per barrel. June 11, 1883, he sold the flour at $7.50 per barrel, cash. What did he gain?

17. B bought 225 *A.* 24 *sq. rd.* of land, Aug. 18, 1882, at $4 an acre, borrowing the money to pay for it, at 5%. He sold the land April 7, 1886, at an advance of $299.40 on cost. If meanwhile he paid $46.50 for taxes on the land, did he gain or lose, and how much?

18. A speculator bought 9000 *bu.* grain at $1.80 per bushel, Mar. 18, 1875, the money paid for it being borrowed at 5½%. Dec. 12, 1875, he sold ⅖ of the grain at $2.00 per bushel, and the remainder at $1.90 per bushel. What was gained or lost by the transaction?

19. A owes B £260 9*s.* 6*d.*, with interest at 5%, for 143 days. He pays 25% of the amount due; how much remains?

NOTE.—In England, interest is usually computed on the basis of 365 days to the year, when the time is given in days. The legal rate in England is 5%. To calculate interest on English money, reduce the shillings and pence to the decimal of a pound (see Art. **342**, Ex. 12, Note), apply any of the methods under Art. **481**, and reduce the resulting decimal to shillings and pence.

Find the accurate interest of

20. £425, from Aug. 4 to Dec. 28, at 5%.

21. £625 12*s.*, from Jan. 12 to Apr. 1, at 4%.

22. £717 16*s.* 10*d.*, from Mar. 3 to June 16, at 4½%.*

23. £429 10*s.* 8*d.*, from Sept. 16 to Nov. 30, at 3%.

24. £516 18*s.* 3*d.*, from Aug. 1 to Oct. 18, at 3½%.

25. £612 6*s.* 11*d.*, from July 1 to Nov. 3, at 5%.

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* When the time is less than 1 year, and the rate is 6% or less, reject the pence, if less than 6; add 1 shilling, if more than 6. The result will be sufficiently accurate.

COMPOUND INTEREST.*

481. Compound Interest is interest not only on the principal, but also on the interest after it becomes due (**433**).

1. Interest may be compounded annually, semi-annually, quarterly, etc.

2. Interest upon interest due, or compound interest, cannot be collected by law, that is, payment cannot be enforced; but such a payment is equitable, and the receiving of it, if the debtor is willing or can be induced to pay it, does not constitute usury in the legal sense of the word. In the State of Missouri, parties may contract in writing for the payment of interest upon interest, but it shall not be compounded oftener than once a year.

Ex. What is the compound interest of $1000 for 3 years, at 6%?

OPERATIONS.

$1000.00	Principal.	Or	$1000
60.00	Interest for 1 *yr.*		1.06
1060	Amount for 1 *yr.*, or 2d principal.		1060
63.60	Interest of $1060 for 1 *yr.*		1.06
1123.60	Amount for 2 *yr.*, or 3d principal.		1123.60
67.416	Interest of $1123.60 for 1 *yr.*		1.06
1191.016	Amount for 3 *yr.*		1191.016
1000	Original principal.		1000
191.016	Compound interest for 3 *yr.*		191.016

482. Rule.—*Find the amount of the given principal for the first period of time, and make it the principal for the second. Find the amount of the second principal for the second period of time, and make it the principal for the third; and so continue for the whole time. The last amount is the amount required.*

The last amount, less the given principal, will be the compound interest.

Notes.—1. When the time is not a multiple of the interest period, find the amount of the principal to the end of the last period; then compute the simple interest on this amount for the remaining time, and add it to the last amount. The sum will be the required amount.

2. The work of computing compound interest may be shortened by using the tables on pages 194 and 195.

---

* For Annual Interest, see page 319.

484. Table showing the sum to which $1, paid at the beginning of each year will increase at compound interest, in any number of years not exceeding 50.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	8%.	10%.	Yrs.
1	1.0300	1.0350	1.0400	1.0500	1.0600	1.0700	1.0800	1.1000	1
2	2.0909	2.1062	2.1216	2.1525	2.1836	2.2149	2.2464	2.3100	2
3	3.1836	3.2149	3.2465	3.3101	3.3746	3.4399	3.5061	3.6410	3
4	4.3091	4.3625	4.4163	4.5256	4.6371	4.7507	4.8666	5.1051	4
5	5.4684	5.5502	5.6330	5.8019	5.9753	6.1533	6.3359	6.7156	5
6	6.6625	6.7791	6.8983	7.1420	7.3938	7.6540	7.9228	8.4872	6
7	7.8923	8.0517	8.2142	8.5491	8.8975	9.2598	9.6366	10.4359	7
8	9.1591	9.3685	9.5828	10.0266	10.4913	10.9780	11.4876	12.5795	8
9	10.4639	10.7314	11.0061	11.5779	12.1808	12.8164	13.4866	14.9374	9
10	11.8078	12.1420	12.4864	13.2068	13.9716	14.7836	15.6455	17.5312	10
11	13.1920	13.6020	14.0258	14.9171	15.8699	16.8885	17.9771	20.3843	11
12	14.6178	15.1130	15.6268	16.7130	17.8821	19.1406	20.4952	23.5227	12
13	16.0863	16.6770	17.2919	18.5986	20.0151	21.5505	23.2149	26.9750	13
14	17.5989	18.2957	19.0236	20.5786	22.2760	24.1290	26.1521	30.7725	14
15	19.1569	19.9710	20.8245	22.6575	24.6725	26.8881	29.3243	34.9497	15
16	20.7616	21.7050	22.6975	24.8404	27.2129	29.8402	32.7502	39,5447	16
17	22.4144	23.4997	24.6454	27.1324	29.9057	32.9990	36.4502	44.5992	17
18	24.1169	25.3573	26.6712	29.5390	32.7600	36.3790	40.4463	50.1591	18
19	25.8704	27.2797	28.7781	32.0660	35.7856	39.9955	44.7620	56.2750	19
20	27.6765	29.2695	30.9692	34.7193	38.9927	43.8652	49.4229	63.0025	20
21	29.5368	31.3290	33.2480	37.5052	42.3923	48.0058	54.4568	70.4027	21
22	31.4529	33.4604	35.6179	40.4305	45.9958	52.4361	59.8963	78.5430	22
23	33.4265	35.6665	38.0826	43.5020	49.8156	57.1767	65.7648	87.4973	23
24	35.4593	37.9499	40.6459	46.7271	53.8645	62.2490	72.1059	97.3471	24
25	37.5530	40.3131	43.3117	50.1135	58.1564	67.6765	78.9544	108.1818	25
26	39.7096	42.7591	46.0842	53.6931	62.7058	73.4838	86.3508	120.0999	26
27	41.9309	45.2906	48.9676	57.4026	67.5281	79.6977	94.3388	133.2099	27
28	44.2188	47.9108	51.9663	61.3227	72.6398	86.3465	102.9659	147.6309	28
29	46.5754	50.6227	55.0849	65.4388	78.0582	93.4608	112.2332	163.4940	29
30	49.0027	53.4295	58.3283	69.7608	83.8017	101.0730	122.3459	180.9434	30
31	51.5028	56.3345	61.7015	74.2988	89.8898	109.2182	133.2135	200.1378	31
32	54.0778	59.3412	65.2095	79.0638	96.3432	117.9334	144.9506	221.2515	32
33	56.7302	62.4532	68.8579	84.0670	103.1838	127.2588	157.6267	244.4767	33
34	59.4621	65.6740	72.6522	89.3203	110.4348	137.2369	171.3168	270.0244	34
35	62.2719	69.0076	76.5983	94.8363	118.1209	147.9135	186.1021	298.1268	35
36	65.1742	72.4579	80.7022	100.6281	126.2681	159.3374	202.0703	329.0395	36
37	68.1594	76.0289	84.9703	106.7095	134.9042	171.5610	219.3159	363.0434	37
38	71.2312	79.7249	89.4091	113.0950	144.0585	184.6403	237.9412	400.4478	38
39	74.4013	83.5503	94.0255	119.7998	153.7620	198.6351	258.0565	441.5926	39
40	77.6633	87.5095	98.8265	126.8393	164.0477	213.6096	279.7810	486.8518	40
41	81.0232	91.6074	103.8196	134.2318	174.9506	229.6322	303.2435	536.6370	41
42	84.4839	95.8486	109.0124	141.9933	186.5076	246.7765	328.5830	591.4007	42
43	88.0484	100.2383	114.4129	150.1430	198.7580	265.1208	355.9496	651.6408	43
44	91.7199	104.7817	120.0294	158.7002	211.7435	284.7493	385.5056	717.9048	44
45	95.5015	109.4840	125.8706	167.6852	225.5081	305.7518	417.4261	790.7953	45
46	99.3965	114.3510	131.9454	177.1194	240.0986	328.2244	451.9002	870.9749	46
47	103.4084	119.3883	138.2632	187.0254	255.5645	352.2701	489.1322	959.1723	47
48	107.5406	124.6018	144.8337	197.4267	271.9584	377.9990	529.3427	1056·1896	48
49	111.7969	129.9979	151.6671	208.3480	289.3359	405.5289	572.7702	1162.9085	49
50	116.1807	135.5828	158.7738	219.8154	307.7561	434.9859	619.6718	1280.2993	50

To find the sum to which a given amount, per annum, will increase at compound interest, at any of the rates per cent. and number of years expressed in the above Table:

Multiply the given amount, per annum, by the sum to which one dollar per annum will increase at the rate and for the number of years required, marking off as many decimals from the product as there are decimals in the multiplier and multiplicand.

NOTE.—If the amount be payable semi-annually, and compound interest is to be allowed semi-annually, take the amount for double the number of years at one-half the rate per cent. Thus, for a semi-annual payment of $1 for 10 years at 10 per cent., take the amount of $1 for 20 years at 5 per cent. = $34.7193. For a quarterly payment, take the amount for four times the number of years at one-fourth the rate per cent.

EXAMPLES.

485. *1.* What will $450 amount to at compound interest, in 4 years, compounded annually at 4%? At 3%?

2. Find the compound interest of $360, for 2 years, interest compounded semi-annually at 6%. At 5%.

3. What is the compound interest of $800 for 1 *yr.* 3 *mo.* at 8%, interest compounded quarterly?

4. At compound interest, what is the amount of $1728 for 3 *yr.* 4 *mo.* 16 *da.*, interest compounded annually at 3%? At 6%?

NOTE.—First find the amount for 3 years, and use this amount as the principal for the remaining time.

5. B holds a mortgage against A's property dated Apr. 1, 1881, for $20000, interest payable annually at 6%. The interest due Apr. 1, 1882, is not paid until May 26, 1882. How much is then due, A having consented to pay interest upon interest? (See Note 2, Art. **481.**)

NOTE.—In solving the following examples, use the tables in Art. **483-484.**

6. A gentleman deposits in a savings bank $100 when his child is one year old. How much will this amount to when the child is 21 years old, interest being compounded semi-annually at 4%? At 5%?

7. If, at the age of 25 years, a person places $2000 on interest, compounded annually at 6%, what will be the amount due him when he is 50 years old?

8. What will $625 amount to at compound interest, in 36 years, compounded annually at 3%? At 4%?

9. At the age of 20, and every year thereafter, a young man places $200 at compound interest at 6%. How much will he have at the age of 30? At the age of 40? (See Art. **484.**)

10. How much will a gentleman have at the end of three years, if he places at compound interest at 5% $300 at the beginning of each year?

11. Mr. B., whose life is insured for $4000, pays an annual premium of $114. How much would this amount to at 6% compound interest in 20 years?

12. A lady deposits $50 in a savings bank Jan. 1 and July 1, of each year; how much will be placed to her credit in 15 years, money being worth 6%, compound interest?

13. **What sum must be placed at compound interest, at 6%, to mount to $1000 in 5 years?**

NOTE.—In compound interest, as in simple interest, the amounts are proportional to the principals; hence the amount of any **principal** is as **many times greater than** the amount of $1, as that principal is greater **than $1.**

To find the principal, divide the given amount by the amount of **$1 for the given time and rate.**

In simple interest, the interest on a given principal for a given time is in proportion to the rate per cent., and at a given rate, in proportion to the time; but, in compound interest, such is not the case. If the rate or time be doubled, the interest is more than doubled.

14. How much should a gentleman invest at compound interest, 6%, for his son who is now 6 years old, so that, when he becomes 21 years of age, he may have $10000?

15. In the above example, how much should be invested at the beginning of each year to produce the same sum?

16. A gentleman at his death left $7850 for the benefit of his only son, 12 years old, the money to be paid to him when he should be 21 years of age. How much did he receive, interest at 6%, compounded semi-annually?

17. How much must a person at the age of 25 years, place at compound interest at 6%, so that the amount due him, when he is 50 years old, will be $20000?

18. In the above example, how much should he invest annually to produce the same sum?

COMMERCIAL PAPER.

486. **Commercial Paper** embraces notes, drafts, bills of exchange, etc.

487. A **Note** (also called a Promissory Note) is a written promise to pay a certain sum of money on demand or at a specified time.

488. The **Maker** of a note is the person who signs it, and thus becomes responsible for its payment. The **Payee** is the person to whom, or to whose order, it is made payable. The **Face** of a note is the sum promised.

In Note 1, Art. **495**, Peter Cooper is the maker; George Peabody **is the payee; the face of** the note is **$1000.**

489. A **Negotiable Note** is a note which is made payable to bearer or to the order of some person. (See Notes, Art. **495.**)

1. A note is *non-negotiable* when it is payable only to the party named in the note.

2. A negotiable note made in New Jersey must contain the words "without defalcation or discount;" in Missouri, the words "negotiable and payable without defalcation or discount."

3. Negotiable notes payable to order may be sold or transferred by the payee writing his name upon the back of the note. He then becomes an indorser.

4. Negotiable securities are good in the hands of one who purchases in good faith and before maturity, although the seller may have found or stolen them.

5. Where no place of payment is specified, a promissory note is payable at the maker's place of business, or if none is known, at the residence of the maker.

6. A note or draft must be presented at the place where it is made payable. If at a bank, during banking hours; if at a place of business, during business hours; if at a residence, during family hours; and if the maker, or some one for him, is not ready with legal tender currency to pay it, the holder need not call again. A check, even if certified, is not a legal tender, and may be lawfully refused.

490. The **Indorser** of a note or draft is the person who writes his name on the back of it, and by so doing guarantees its payment.

If Mr. Erastus Corning desires to sell or transfer Note 3, Art. **495**, it will be necessary for him to indorse it. If he writes his name only, it is called an indorsement *in blank*, and the note is then payable without further indorsement to any person lawfully holding the same. He may indorse it *in full* by making it payable to a particular person, thus—"Pay to the order of Henry R. Pierson, Erastus Corning." Before it can be again transferred, it will require the indorsement of Henry R. Pierson. For greater security, checks, notes, drafts, etc., are indorsed in full when sent by mail.

If an indorser does not wish to guarantee the payment of a note, draft, etc., he writes "Without recourse" over his name at the time of the indorsement.

Sometimes notes and drafts are drawn to the order of the maker or the drawer (to the order of myself or ourselves) to facilitate their transfer without the indorsement of the holder.

491. A **Draft**, or **Bill of Exchange** is an order or request addressed by one person to another, directing the payment of a specified sum of money to a third person or to his order.

492. The **Drawer** of the draft is the person who signs it. The **Drawee** is the person on whom it is drawn. The **Payee** is the person to whom, or to whose order, it is made payable.

In Draft 5, Art. 495, C. P. Huntington is the drawer; Drexel, Morgan & Co. are the drawees; J. & W. Seligman & Co. are the payees.

1. The person in whose favor the bill is drawn is sometimes called the buyer, and becomes the "remitter." After the bill is presented and accepted, the drawee is called the acceptor, and the draft, an acceptance. The draft then has the same legal significance as a promissory note.

2. A person accepts or promises to pay a draft by writing the word "Accepted" and the date over his name across its face.

3. Drafts are sometimes accepted in the following form:—"Accepted August 20, 1881, and payable at the National Park Bank, New York, G. B. Horton & Co."

4. In the State of New York, both by law and custom, the drawee of a draft may demand 24 hours consideration from the time the draft is presented for acceptance.

When accepted, it must bear the date when first seen by him.

5. To "honor" a draft is to accept it or pay it on being presented.

493. A **Protest** is a formal statement made by a Notary Public, declaring that a draft or note has been presented for payment or acceptance, and was refused.

494. Days of Grace and Maturity.—The **day of maturity** is the day on which a note becomes legally due. According to the laws of some of the States, a note is not legally due until three days after the expiration of the time specified in the note, except the note contain the words "without grace." These days are called *days of grace*, but they are of no advantage to the payer, since interest is charged for them as for any others.

1. California, Illinois, Maryland, Massachusetts, Missouri, Nebraska, New York, Ohio, Pennsylvania, and many other States have abolished days of grace altogether.

2. The following is an analysis of the Holiday law of 1887 of the State of New York:

Paper due on a holiday is payable the following business day.

Paper due on Saturday, except when payable at sight or on demand, is payable the following business day.

Paper due on Sunday must be paid on the business day following it. If one of the mentioned holidays fall on Sunday, paper due on that day must be paid the following business day.

Paper due on Monday, where the preceding Sunday is a holiday, is not payable until the following business day.

3. In certain States (New York and Pennsylvania, exceptions) paper due on Sunday or on a legal holiday, must be paid the preceding business day. Thus, if a holiday falls on Thursday, all notes, etc., must be paid on Wednesday; if a holiday falls on Monday, all notes due Sunday or Monday would be payable on Saturday; if a holiday falls on Saturday, notes due Saturday or Sunday would be payable on Friday.

4. The legal holidays in the State of New York are New Year's Day (Jan. 1), Lincoln's Birthday (Feb. 12), Washington's Birthday (Feb. 22), Decoration Day (May 30), Independence Day (July 4), Labor Day (the first Monday of September), Election Day (the first Tuesday after the first Monday of November), Thanksgiving Day (the day appointed by the President of the United States and Governor of the State, usually the last Thursday of November), Christmas Day (Dec. 25), and every Saturday from 12 o'clock at noon until 12 o'clock at midnight (Saturday Half-Holiday).

When a legal holiday falls on Sunday, Monday is, by the statute of New York, made a legal holiday.

5. When the time of a note is expressed in months, calendar months are used to determine the day of maturity; when in days, the exact number of days is used.

Thus, a note dated July 16, and payable two months from date, would mature Sept. 16. A note having the same date, and payable sixty days from date, would mature Sept. 14.

6. A note due in one or more months from date, matures on the corresponding day of the month up to which it is reckoned, *if there are so many days in that month;* but if not so many, it then matures on the last day of said month. Thus, notes dated Jan. 28, 29, 30, or 31, and payable one month from date, would become due Feb. 28.

7. When drafts are payable a certain time after *sight*, the date of acceptance and the time of the draft determine the day of maturity. Thus, if a draft is dated May 16, accepted May 20, and payable sixty days after sight, it would mature or be due 60 days after May 20, or July 19. If payable 60 days after *date*, it would mature 60 days after May 16, or July 15. It is not necessary to present for acceptance drafts drawn a certain time after date, but as a courtesy to the drawee, it is usually done.

8. Days of grace are allowed on drafts according to the custom of the place where they are payable.

495. FORMS OF NOTES AND DRAFTS.

1. DEMAND NOTE.

$1000. NEW YORK, *August 19, 1887.*

On demand, I promise to pay GEORGE PEABODY, or bearer, One Thousand Dollars. Value received.

PETER COOPER.

The above note is payable on demand,—that is, whenever presented; is negotiable (payable to bearer): and bears interest from date at the legal rate of the State in which it is made. If the words "or bearer" were omitted the note would not be negotiable.

How much would be due on the above note, Jan. 1, 1888, finding the time by compound subtraction?

2. TIME NOTE—INTEREST-BEARING.

875\frac{44}{100}$. CINCINNATI, OHIO, *July 16, 1888.*

Six months after date, I promise to pay GEO. C. MILLER, or order, Eight Hundred Seventy-five and $\frac{44}{100}$ Dollars, with interest at eight per cent. Value received.

ALEX. MCDONALD.

The above note is payable 6 *mo.* after its date, or Jan. 16, 1889; is negotiable (payable to order); and draws interest from its date at 8% per annum. If the rate of interest was omitted, it would bear interest at the legal rate of the State for such cases, 6%. (See Art. **436**.)

How much would be due on the above note at its maturity? How much, March 1, 1889? Supposing the rate of interest to be omitted in the note, how much would be due May 4, 1889?

3. TIME NOTE—WITHOUT INTEREST—PAYABLE AT A BANK.

$6000. ALBANY, N. Y., *December 7, 1889.*

Sixty days after date, I promise to pay to the order of ERASTUS CORNING, Six Thousand Dollars, at the Second National Bank. Value received.

DAVID MURRAY.

The above note is payable 60 days from Dec. 7, 1889, or Feb. 5, 1890. It is payable at the Second National Bank. No interest will be due at maturity (Feb. 5). If the note is not paid at maturity, it will bear interest from that date.

Supposing the above note was payable 90 days from date, what would be its due date (**311**, Ex. 10)? The note as given not being paid at maturity, how much would be due Feb. 25, 1890, protest fees $2.10?

4. JOINT AND SEVERAL NOTE.

416\frac{32}{100}$. WORCESTER, MASS., *May 27, 1888.*

Four months after date, we jointly and severally promise to pay JOHN S. BALLARD & Co., or order, Four Hundred Sixteen $\frac{32}{100}$ Dollars, with interest from date, value received.

T. K. EARLE.
CHAS. W. SMITH.

If the above note were written "we jointly promise, etc.," it would be called a *joint note.* The makers of a joint note must be sued jointly, each being responsible for one-half of the amount of the note. The makers of a joint and several note may be sued separately, either being responsible for the full amount of the note.

How much would be due on the above note, Dec. 30, 1888? How much Sept. 27, 1888, the date of maturity?

5. SIGHT DRAFT.

$8000. SAN FRANCISCO, CAL., *May 1, 1882.*

At sight, pay to the order of J. & W. SELIGMAN & Co., Eight Thousand Dollars, value received.

C. P. HUNTINGTON.

To DREXEL, MORGAN & Co., New York.

6. TIME DRAFT.

$5000. BURLINGTON, IOWA, *June 18, 1887.*

At sixty days' sight, pay to the order of ADDISON BALLARD, Five Thousand Dollars, value received, and charge to account of

A. G. ADAMS.

To BARTON & JONES, Chicago, Ill.

Drafts are sometimes drawn a certain number of "days after date." (See Art. **494**, Note 8.)

For Foreign Bills of Exchange, see Art. **555**.

If the above draft was accepted June 19, 1887, what was the date of maturity?

7. A sixty-day note given on Monday will mature on what day?

8. A note payable 90 days from date and given on Thursday will fall due on what day of the week? If payable in 30 days and given on Monday, on what day will it become due?

9. A note dated July 25, and payable in 90 days, would mature on what date?

BANK DISCOUNT.

496. Bank Discount is simple interest of a note, paid in advance, for the number of days the note has to run. It may be computed by any of the methods given for simple interest.

On notes without interest (the usual case of notes discounted at banks), bank discount is reckoned on their face, the amount due at maturity; on notes with interest, it is reckoned on the amount due at maturity, or their face plus the interest for the full time of the note.

497. The **Proceeds** of a note is the amount received by the holder from the bank when the note is discounted. It is the amount on which the discount is reckoned less the discount.

498. Call Loans.—Banks in the large cities loan large amounts of money upon stocks, bonds, negotiable warehouse receipts for grain, cotton, petroleum, etc., as collateral security, payable on demand or on giving one day's notice. Such loans are termed "call" or demand loans, and interest on them is paid at the end of the time. (See Art. **436,** Note d.)

499. The time to be reckoned on a loan or note is exclusive of the day of date, but includes the day of maturity or payment. Thus, in discounting a note in the City of New York, Apr. 4, which would mature Apr. 24, the discount would be calculated for 20 days.

1. In Philadelphia, Baltimore and some other cities it is the custom of banks in finding time to include both the day of discount and the day of maturity. Thus, the discount on the above note would be reckoned for 21 days.

2. Banks of the City of New York reckon discount both on the basis of 360 and 365 days to the year, the greater number on the former basis.

3. When notes are payable in other cities or towns, some banks charge interest for the time required for the collecting bank to remit the money, in addition to the interest on the note from the day of discount to the legal day of maturity. Thus, if a note, maturing June 10 and payable at a Chicago bank, is discounted at a bank in New York, the remittance in settlement would not be received before June 12 in New York, and the New York bank in discounting the note would be justified in charging interest for two days beyond the day of maturity.

4. Some banks charge a small fee for collection and exchange in addition to the interest in discounting notes which are payable in other cities or towns.

EXAMPLES

500. Find the date of maturity and proceeds of the following notes:

(*1.*)

$10000. New York, *July 16, 1909.*

Four months after date, I promise to pay to the order of Fisk & Hatch, Ten Thousand Dollars, at the First National Bank, value received.

S. D. Babcock.

Discounted July 16, 1909, at 6%.

Analysis.—The note is due 4 months (**494,** 5) after July 16, or Nov. 16. From the day of discount (July 16) to the day of maturity (Nov. 16) there are 123 days.

The interest of $10000 for 123 days at 6%, if reckoned on the basis of 360 days to the year, is $205, and the proceeds are $10000 less $205, or $9795.

The interest on the basis of 365 days to the year would be $2.81 less, or $202.19, and the proceeds would be $9797.81.

If the note was discounted Sept. 1, the interest or discount would be reckoned for 76 days (Sept. 1 to Nov. 16).

(*2.*)

$8000. Brooklyn, N. Y., *July 16, 1907.*

Ninety days from date, I promise to pay S. B. Chittenden, or order, Eight Thousand Dollars, value received.

A. A. Low.

Discounted Aug. 31, 1907, at 6%.

Analysis.—The note is due 90 days after July 16, or Oct. 14. Compute the discount for 44 days (Aug. 31 to Oct. 14) on $8000.

If the note had been discounted July 16, the date of the note, the interest would have been computed for 90 days, the full time of the note.

Note.—The results of the following examples will be given on the basis of both 360 and 365 days to the year:

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount.
3	Jan. 24	90 days	$1200	Jan. 24..........	6%
4	May 18	3 mo.	$5280	May 18..........	6%
5	Aug. 31	60 days	$2560	Aug. 31..........	8%
6	June 4	4 mo.	$3756	June 4..........	7%
7	Oct. 16	30 days	$6425	Oct. 16..........	5%
8	Mar. 13	6 mo.	$8375	Mar. 13	5½%

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount.
9	May 29.........	3 mo.	$4500	July 7..........	10%
10	July 27.........	60 days	$8240	Sept. 2..........	6%
11	Mar. 28.........	90 days	$4324	Apr. 14.........	5¼%
12	May 27.........	6 mo.	$4885	Aug. 15........	8%
13	Jan. 3..........	120 days	$9000	Feb. 28.........	6%
14	Sept. 12........	4 mo.	$5000	Oct. 14.........	7%
15	Nov. 1..........	90 days	$6000	Nov. 28.........	5⅘%

16. What were the proceeds of Note 3, Art. **495**, if discounted Dec. 16, 1889, at the legal rate?

17. Find the date of maturity and proceeds of a note of $5000 payable 60 days from date, dated and discounted at a Philadelphia bank, Aug. 3. (See Art. **499**, 1.)

18. Find the date of maturity and proceeds of a note of $3750, payable 60 days from date, dated and discounted at a Maryland bank, Jan. 31, 1882.

NOTE.—In the following examples, the charge for collection and exchange is a certain per cent. of the face of the notes, without reference to the time they have to run (**499**, 4).

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount.	Rate of Collection.
19	May 5......	90 days	$7000	May 5......	6%	⅛%
20	March 1....	4 mo.	$9000	Apr. 30.....	5%	1/10%
21	June 18....	6 mo.	$5000	July 31.....	8%	1/20%
22	July 28....	60 days	$4500	Aug. 1.....	6%	1/10%
23	Sept. 3.....	90 days	$9000	Sept. 5......	5½%	1/20%
24	Aug. 5.....	4 mo.	$3000	Oct. 18.....	6%	⅛%

Required the proceeds and date of maturity of the following notes discounted (360 days to the year) through a broker, his commission being ¼% of the face of the notes.

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount.
25	Feb. 21........	4 mo.	$10000	Feb. 21........	4¾%
26	June 8..........	4 mo.	$6000	June 12........	4⅛%
27	Jan. 10.........	4 mo.	$6000	Jan. 10.........	4¼%
28	Mar. 3..........	6 mo.	$8775	Apr. 30........	4¾%

29. May 4, a New York bank discounts, at 6%, a note of $8000, payable in St. Louis, Sept. 1. What are the proceeds, if interest is charged for the two days required for the remittance of a draft in settlement?

30. A note dated Mar 27, for $9000, payable in 4 months at a bank in Omaha, is discounted the same date at a bank in Providence, R. I. What were the proceeds, if interest for 3 days beyond the day of maturity is charged?

31. A broker discounts a note payable in 4 months at 4¾%, and charges ¼% brokerage. This is equivalent to what rate of interest per annum, making no allowance for the days of grace?

32. A merchant can discount a note at his bank at 6%, 365 days to the year, or through a broker at 4¾%, 360 days to the year, broker's commission ¼%. How much better is the latter method on a note of $10000, payable in 4 months, dated and discounted May 21?

Find the date of maturity and proceeds of the following interest-bearing notes:

(*33.*)

$3000. ALBANY, N. Y., *September 16, 1881.*

Four months after date, I promise to pay W. J. KLINE or order, Three Thousand Dollars, with interest at 5%, value received.

J. M. THOMAS.

Discounted Nov. 3, 1881, at 6%.

NOTE.—Compute the discount at 6% for 74 days (Nov. 3 to Jan. 16) on the amount due at maturity ($3000 plus the interest of $3000 for 4 months at 5%).

34. A note dated May 27, 1909, payable in 3 months, for $3750, with interest at 7%; discounted May 27, 1909, at 8%.

35. A note dated Jan. 16, 1909, payable in 4 months, for $1632, with interest at 6%; discounted Mar. 5, 1909, at 7%.

36. A note dated Oct. 12, 1908, payable in 6 months, for $875, with interest at 7%; discounted Jan. 10, 1909, at 10%.

37. For what amount must a note be given for 60 days to afford $1000 proceeds, if discounted at 6%?

ANALYSIS.—The proceeds of any note is as many times greater than the proceeds of $1, as the face of the note is greater than $1. If a note of $1 is discounted for 60 days at 6%, it will afford $.99 proceeds; to afford $1000 proceeds, the face of the note must be as many times $1 as $.99 is contained times in $1000, or $1010.10.

The following approximate method is generally used by business men: *To the given proceeds, add the interest for the given time.*

The interest of $1000 for 60 days is $10. $1000 + $10 = $1010. Since the interest is reckoned on the proceeds instead of the face of the note, the error, 10 cents, is equivalent to the interest of the interest ($10) for the given time.

Where greater accuracy is required, the necessary correction may be made. The interest of $10 for 60 days is 10 cents. $1010 + $.10 = $1010.10.

38. A owes B $1500; how large a 90-day note must A give B that when discounted at a bank at 6%, the proceeds will be sufficient to pay the debt?

39. I hold a note of $3000 against Mr. C., which he pays by giving a new note at 90 days for $1500, and the balance, including the discount on the new note, in cash. Required the amount of cash paid.

40. A merchant having $8000 to pay, gets a note for $5000, that will mature in 40 days, discounted at a bank at 6%. How large a note must he draw, payable in 90 days, for discount at the same rate, that the proceeds of the two notes may enable him to meet his payment?

PARTIAL PAYMENTS.

501. Partial Payments are payments in part of a note, mortgage, or other debt, made at different times.

502. Indorsements are the acknowledgments of the payments, written on the back of the note, mortgage, etc., and stating the amount and date of the payment.

Special receipts are sometimes given for such payments.

UNITED STATES RULE.

503. Ex. How much would be due Sept. 1, 1882, on a note of $600, dated March 1, 1882, with interest at 6%? Suppose a payment of $100 be made Sept. 1, 1882, to pay the interest and part of the principal, how much would then be due? *Ans.* $518.

Ex. How much would be required to settle the above note Jan. 1, 1883, the balance of $518 remaining on interest at the same rate from Sept. 1, 1882? *Ans.* $528.36.

Ex. Find the amount due on the following note, Jan. 19, 1885:

$1000. Boston, Mass., *Aug. 1, 1881.*

One year after date, I promise to pay Jordan, Marsh & Co., or order, One Thousand Dollars, for value received, with interest from date, at 6 per cent.

Alexander H. Rice.

On this note are the following indorsements:

Received Apr. 21, 1882, $200. Received Aug. 1, 1883, $100.

Received Dec. 1, 1882, $25. Received July 1, 1884, $400.

Note. — The method given in the following operation, is that adopted by the Supreme Court of the United States, and has been made the legal method of nearly all the States. By the United States Rule, as this is generally called, settlements are made whenever the payments are equal to or exceed the interest due. If the payment exceeds the interest, it is applied first to discharge the interest, and the surplus is applied towards paying the principal. If the payment is less than the interest, it is not applied until the payments taken together are sufficient to pay all interest due; since no unpaid interest is added to the principal to draw interest, a new principal can never be greater than the preceding principal.

OPERATION.

Face of note, or principal, from Aug. 1, 1881		$1000
Interest from Aug. 1, 1881, to Apr. 21, 1882 (8 *mo.* 20 *da.*), added		43.33
Amount, Apr. 21, 1882		1043.33
First payment, Apr. 21, 1882		200.00
New principal from Apr. 21, 1882		843.33
Interest of $843.33 from Apr. 21, 1882, to Dec. 1, 1882, (7 *mo.* 10 *da.*)	$30.92	
(Interest exceeds the payment, and a new principal is not formed.)		
Interest of $843.33 from Dec. 1, 1882, to Aug. 1, 1883, (8 *mo.*)	33.73	64.65*
(Payments $125 ($25 + $100), now greater than the interest due ($64.65).)		
Amount, Aug. 1, 1883		907.98
Second and third payments, $25 + $100		125
New principal from Aug. 1, 1883		782.98

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* In many cases it can be determined mentally in advance whether the payment is greater or less than the interest. In this case the interest could be taken at once from Apr. 21, 1882, to Aug. 1, 1883 (1 *yr.* 3 *mo.* 10 *da.*), since it is evident that the payment ($25) is less than the interest of $843.33 for 7 *mo.* 10 *da.* (The interest of $800 for 7 mo. is 3½ × $8, or $28, and it would be more on $843.33 for 7 mo. 10 da.) If it is doubtful whether the payment is greater or less than the interest, perform all the work.

New principal from Aug. 1, 1883	$782.98
Interest of $782.98 from Aug. 1, 1883, to July 7, 1884 (11 *mo.* *da.*)	43.85
Amount, July 7, 1884, ...	826.83
Fourth payment, July 7, 1884, ...	400
New principal from July 7, 1884	426.83
Interest of $426.83 from July 7, 1884, to Jan. 19, 1885 (6 *mo.* 12 *da.*)	13.66
Amount due Jan. 19, 1885, the final day of settlement, . *Ans.*	$440.49

504. United States Rule.—*Find the amount of the given principal to the time when the payment or the sum of the payments exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before, to the time of settlement.*

EXAMPLES.

505. Notes.—1. In the following examples, find the time by compound subtraction.

2. In the first five examples, all the payments exceed the interest.

$1680. Trenton, N. J., *Oct. 9, 1880.*

1. On demand, I promise to pay Cooper, Hewitt & Co., or order, Sixteen Hundred Eighty Dollars. Value received.

John A. Roebling.

On this note were indorsed the following payments :

Dec. 21, 1881, received $289.12. June 9, 1883, received $991.50.

How much was due Jan. 30, 1884?

2. On a note dated May 11, 1877, for $2000, are the following indorsements : — Aug. 6, 1879, $361 ; Feb. 11, 1880, $901.60; Nov. 2, 1882, $1000. What remained due Feb. 2, 1883, at 6%? At 5%?

3. On a note dated July 11, 1878, for $2400, are the following indorsements : — Sept. 17, 1879, $200 ; Jan. 29, 1880, $400 ; Nov. 29, 1881, $1150. What is the amount due Jan. 11, 1882, the interest being at 6% ? At 7%?

4. On a mortgage for $1700, dated May 28, 1880, there was paid Nov. 12, 1880, $80; Sept. 20, 1881, $314; Jan. 2, 1882, $50; Apr. 17, 1882, $160. What was due Dec. 12, 1882, at 6%? At 8%?

5. On a note dated May 30, 1879, for $1666, are the following indorsements:—Apr. 9, 1880, $314; Nov. 4, 1880, $180; Aug. 25, 1881, $575. What was due June 30, 1882, at 6%? At 8%?

6. What was the amount due Oct. 17, 1881, upon a note for $1000, dated New York, Mar. 2, 1880, and on which the following payments were indorsed:—June 2, 1880, $80; Dec. 15, 1880, $20; May 2, 1881, $32; June 2, 1881, $60?

7. A note for $3600, dated May 12, 1880, bore the following indorsements:—Jan. 2, 1881, $255; Mar. 15, 1881, $225; June 3, 1881, $120; Aug. 6, 1881, $300; Feb. 3, 1882, $30. What was due June 2, 1882, at 6%? At 10%?

8. A note for $4000, dated Mar. 9, 1874, was indorsed as follows:—Jan. 18, 1876, $300; June 4, 1876, $400; Dec. 9, 1876, $1800; Sept. 1, 1879, $2000. How much had to be paid Jan. 1, 1880, to take up the note, at 6%? At 7%?

9. A mortgage of $6000 is dated May 9, 1877, on which there were the following payments:—July 15, 1878, $500; Nov. 27, 1878, $1000; June 1, 1879, $100; May 9, 1880, $275; Sept. 27, 1880, $2000. What was due Nov. 9, 1880, the interest being at 6%? At 12%?

MERCANTILE RULES.

506. The following methods are frequently used by merchants in finding the balance due on a note where partial payments have been made. They are similar to the methods in general use for finding the balance due on an open account (**592**).

507. When the note runs for one year only, or less.

508. RULE.—*Compute the interest on the principal from the time it commenced to draw interest, and on each payment from the time it was made until the time of settlement, and deduct the amount of all the payments, including interest, from the amount of the principal and interest.*

NOTES.—1. This rule is used by some merchants when the note runs more than one year, although it is greatly to the disadvantage of the creditor, or holder of the note.

2. In solving examples by this rule, the different methods for finding time and interest, given in Art. **437**, are used. The results of the following examples will be given for the first method (Compound Subtraction and 360 days to the year).

EXAMPLES.

509. *1.* According to the mercantile rule, find the balance due May 12, 1882, on a note for $2400, dated July 12, 1881, on which the following payments have been made: Dec. 16, 1881, $40; Jan. 2, 1882, $100; Mar. 15, 1882, $150.

OPERATION.

Face of note, or principal, July 12, 1881, ...		$2400.00
Interest on the same to May 12, 1882 (10 *mo.*)		120.00
Amount, May 12, 1882, ...		2520.00
First payment, Dec. 16, 1881, ...	$40.00	
Interest on the same to May 12, 1882 (4 *mo.* 26 *da.*) .	.97	
Second payment, Jan. 2, 1882, ...	100.00	
Interest on the same to May 12, 1882 (4 *mo.* 10 *da.*) .	2.17	
Third payment, Mar. 15, 1882, ...	150.00	
Interest on the same to May 12, 1882 (1 *mo.* 27 *da.*) .	1.42	294.56
Balance due May 12, 1882, ...		$2225.44

2. On a note dated Jan. 13, 1882, for $1234, are the following indorsements:—May 17, 1882, $234; June 16, 1882, $345; July 27, 1882, $123; Sept. 19, 1882, $135. What remained due Nov. 13, 1882, at 6%? At 7%?

3. A note for $1567, dated Jan. 14, 1881, bore the following indorsements:—Mar. 11, 1881, $50; May 13, 1881, $245; June 19, 1881, $374; Aug. 30, 1881, $412; Sept. 28, 1881, $316.40. What was due Jan. 1, 1882, at 6%? At 5%?

4. On a note dated Aug. 17, 1881, for $3300, were the following indorsements:—Dec. 18, 1881, $320; Feb. 5, 1882, $425; Apr. 13, 1882, $550; June 29, 1882, $630; July 16, 1882, $375; Aug. 1, 1882, $500. What amount was due Aug. 17, 1882, at 6%? At 10%?

510. When the note runs for more than one year.

511. Since it is the custom of merchants and bankers to balance their accounts annually, the following method is used by them in computing the balance due on a note when it runs more than one year.

It is equivalent to finding the balance due yearly by the previous rule, and treating the balance as a new principal. The periodical settlements are made annually, semi-annually, or quarterly, depending upon the custom of the merchant or banker in balancing his accounts. Some merchants make the end of the business year, Jan. 1 or July 1, the periodical rest, or date of settlement for notes and accounts.

512. RULE.—*Find the amount of the principal for one year; also of each payment made during the year from the time the payment was made to the end of the year (1 yr. from the date of the note). From the amount of the principal, subtract the sum of the payments, including interest. With the remainder as a new principal, proceed thus for each entire year that follows, and for the interval between the end of the last year and the final date of settlement.*

NOTE.— When payments are made yearly greater than the interest due, this rule is the same as the New Hampshire rule (**675**) for notes "with interest annually."

EXAMPLES.

513. *1.* By the above rule, find the balance due Jan. 19, 1885, at 6%, on a note for $2400 dated Aug 1, 1881, on which the following payments have been made:—Apr. 21, 1882, $200; Dec. 1, 1882, $25; Aug. 1, 1883, $100; July 7, 1884, $400. (Time by Compound Subtraction.)

OPERATION.

Face of note, or principal, Aug. 1, 1881,		$2400.00
Interest on the same for 1 year,		144.00
Amount, Aug. 1, 1882,		2544.00
First payment, Apr. 21, 1882,	$200.00	
Interest on the same to Aug. 1, 1882 (3 *mo.* 10 *da.*)	3.33	203.33
Balance and new principal, Aug. 1, 1882,		2340.67
Interest on the same for 1 year,		140.44
Amount, Aug. 1, 1883,		2481.11
Second payment, Dec. 1, 1882,	$25.00	
Interest on the same to Aug. 1, 1883 (8 *mo.*).	1.00	
Third payment, Aug. 1, 1883,	100.00	126.00
Balance and new principal, Aug. 1, 1883,		2355.11
Interest on the same for 1 year,		141.31
Amount, Aug. 1, 1884,		2496.42
Fourth payment, July 7, 1884,	$400.00	
Interest on the same to Aug. 1, 1884, (24 *da*)	1.60	401.60
Balance and new principal, Aug. 1, 1884,		2094.82
Interest on the same to date of settlement, Jan. 19, 1885 (5 *mo.* 18 *da.*)		58.65
Balance due Jan. 19, 1885,		$2153.47

2–9. Solve Examples 2–9, Art. **505**, according to the mercantile rule.

RATIO AND PROPORTION.

514. Ratio is the relation of two numbers as expressed by the quotient of the first divided by the second. Thus the ratio of 6 to 3 is 6÷3, or 2.

1. There is no ratio between quantities of different kinds; as 6 *bu.* and 8 *ft.* But a ratio exists between quantities of the same kind though of different denominations; as 6 *ft.* and 8 *in.* To express the ratio in such cases, the quantities must first be reduced to the same denomination. Thus, the ratio of 6 *ft.* to 8 *in.* is 72 *in.*÷8 *in.*, or 9.

2. The ratio between two numbers is denoted by placing a colon (the sign of division without the horizontal line) between them. Thus, the ratio of 6 to 3 is expressed 6:3.

515. A **Simple Ratio** is a ratio between two numbers; as 4:5.

516. A **Compound Ratio** is a ratio formed by the combination of two or more simple ratios.

Thus, $\begin{matrix}4:5\\3:2\end{matrix}$ is a compound ratio, and is equivalent to 4×3:5×2, or 12:10.

517. The numbers whose ratio is expressed are the **terms** of the ratio. The two terms of a ratio form a **couplet**, the first of which is the **antecedent**, and the second, the **consequent.**

518. Proportion is an equality of ratios.

The ratio of 6 *yd.* to 3 *yd.* is 2, and the ratio of $24 to $12 is 2; hence from the two equal ratios the following proportion can be formed—6 *yd.*:3 *yd.* =$24:$12. This expression is read, "The ratio of 6 *yd.* to 3 *yd.* equals the ratio of $24 to $12." In place of the sign of equality (=), four dots (::) are generally used; thus, 6 *yd.*:3 *yd.*:: $24:$12. The expression is also read, "6 *yd.* is to 3 *yd.* as $24 is to $12."

519. The first and fourth terms of a proportion are called the **extremes:** and the second and third, the **means.**

520. PRINCIPLES.—1. *The product of the means is equal to the product of the extremes.*

2. *A missing mean may be found by dividing the product of the extremes by the given mean.*

3. *A missing extreme may be found by dividing the product of the means by the given extreme.*

SIMPLE PROPORTION.

521. Simple Proportion is an equality of two simple ratios; as 9 *lb.*:16 *lb.*::$27:$48.

Ex. If 24 hats cost $27, what will 32 hats cost?

ANALYSIS.—For convenience, make the fourth term the missing term, or the required answer. Since the third and fourth terms must be of the same denomination and the denomination of the answer will be dollars, take $27 as the third term. From the nature of the example, the answer will be more than $27, the third term; therefore make 32 hats the second term, and 24 hats the first term. The proportion will then be stated as follows: 24 hats:32 hats::$27:$x$ (Let x represent the unknown term). Multiplying 32 by 27, and dividing the product by 24, the fourth or missing term will be $36.

522. RULE.—*For convenience, take for the third term the number that may form a ratio with, or is of the same denomination as, the answer. If from the nature of the example, the answer is to be greater than the third term, make the greater of the two remaining terms (which must be of the same denomination) the second term; when not, make the smaller the second term. Then multiply the means (the second and third) together, and divide their product by the given extreme (the first term).*

NOTES.—1. After the example is stated, any factor of the given extreme may be cancelled with an equal factor of either of the means.

2. The above rule is sometimes called the "Rule of Three."

EXAMPLES.

523. Find the missing term (represented by x) in each of the following proportions (See Principles, Art. 520).

1. $16 : x :: 24 : 18$.
2. $x : 27 :: 18 : 54$.
3. $32 : 27 :: x : 135$.
4. 24 *bu.*:32 *bu.*::$27:$x$.
5. $\$48 : \$75 :: \$32 : x$.
6. $375:$144::625 *lb.*:x.
7. $\$1728 : \$288 :: \$666 : x$.
8. 144 *yd.*:175 *yd.*::$18:$x$.

9. If 19 *yd.* of silk cost $28.50, what will 37 *yd.* cost ?

10. If 64 *yd.* of carpet 36 *in.* wide will cover a floor, how many yards 27 *in.* wide will be required to cover the same floor ?

11. A cane 3 *ft.* 3 *in.* high casts a shadow 5½ *ft.* long ; how long a shadow is cast by the steeple of a church which is 234 feet high ?

12. If the freight of a long ton (**336**, 3) is 70 shillings, what is the freight of 16375 pounds ?

13. The net assets of a bankrupt are $27675, and the liabilities $138375. How much must be paid to Mr. A, whom he owes $4800 ?

14. A building is insured in several companies for $28000. During a fire the building is damaged to the amount of $13500. What is the loss of company A, whose risk is $5000 ?

15. A invests in business $8450, and B $7200, and the gain or loss is divided according to the investments. What is each partner's share of gain, the total gain being $3474.30 ?

16. The U. S. gold dollar (**112, 113**) contains 23.22 (25.8 —$\frac{1}{10}$) grains of pure gold, and the standard silver dollar 371.25 (412.5—$\frac{1}{10}$) grains of pure silver. What is the relative value of pure gold to pure silver ?

17. The assessed value of the property of a certain town is $325000, and the total tax is $10238. How much is the tax of Mr. A, whose property is valued at $5700 ?

18. A company with a capital of $250000 divides $8750 among its stockholders. How much will be received by a stockholder who owns 36 100-dollar shares ?

19. If a long ton of coal is worth $4.25, what is the value of a short ton ?

20. If a farm valued at $4500 is taxed $26.24, what should be the tax on property valued at $23500 ?

21. A merchant gains $625 by selling $12000 worth of goods ; what amount must he sell to gain $8000 ?

22. How many feet of boards will be required for a fence 764 feet long, if 888 feet of boards are required for 288 feet ?

23. If one franc is worth $0.193, and one pound sterling, $4.8665, what is the value of the pound sterling expressed in francs ?

24. If 2175 yards of cloth are made from 458 pounds of yarn, how many pounds of yarn would be required to make 1200 yards of cloth ?

25. If a railway train goes 412 miles in 9 *hr.* 30 *min.*, how many hours would it require to go 900 miles?

26. The railway fare from A to B, a distance of 228 miles, is $6.75. What should be the fare from A to C, a distance of 375 miles?

27. A certain quantity of grain will last 92 horses 48 days. How long will it last 64 horses?

28. A house and lot are worth $9600, and the value of the lot is to the value of the house as 5 to 11. Find the value of the lot.

29. A merchant failing, owes $11375, and has property worth $4425. How much will he pay a creditor whom he owes $2345?

30. The distance between two poles was measured as 48 yards, but the yard measure was ½ of an inch too short. What was the actual distance?

31. From a field of wheat containing 375 acres, 4850 bushels are harvested. How many bushels would be harvested from a field containing 344 acres of similar wheat?

32. If the tax on property, valued at $6500, is $144, what should be the tax on property valued at $3800?

33. If a Troy ounce of standard silver is worth 85 cents, what is the intrinsic value of the standard silver dollar (**112**)?

34. If the freight on 575 pounds is $1.84, what should be the freight on 975 pounds?

35. If a railway company charges $13 for carrying one ton 480 miles, what should be the charge on one ton for 650 miles?

36. The through rate from A to C, a distance of 900 miles, is $48 per car. What should be the portion of the A. & B. R.R. (425 miles), and the portion of the B. & C. R.R. (475 miles)?

37. If 48 men can do a certain piece of work in 60 days, in what time can 64 men do the same work?

38. If a Troy ounce (**337**) of silver is worth $1.20, what is the value of an Avoirdupois ounce?

39. The ratio of the diameter of a circle to its circumference is 1:3.1416. What is the circumference of a circle whose diameter is 475 feet?

40. If 276 long tons of coal last a manufacturer 21 months, how long would 276 short tons last him?

41. A bankrupt can pay 48 cents on a dollar. If his assets were $1887 more, he could pay 65 cents. Find his debts and his assets.

COMPOUND PROPORTION.

524. Compound Proportion is an equality of a compound tio and a simple ratio, or of two compound ratios.

Thus, $\left.\begin{array}{l}4 \text{ men} : 6 \text{ men} \\ 8 \text{ days} : 16 \text{ days}\end{array}\right\} :: \$10 : \$30$, and $\left.\begin{array}{l}2 : 3 \\ 4 : 5\end{array}\right\} :: \left\{\begin{array}{l}8 : 5 \\ 4 : 12\end{array}\right.$ are compound oportions.

Ex. If 12 men earn $60 in 4 days, how much will 16 men rn in 2 days?

OPERATION.

$$\left.\begin{array}{l}12 : 16 \\ 4 : 2\end{array}\right\} :: 60 : x.$$

$$\begin{array}{r|l} {}^{3}\cancel{12} & \cancel{16}\,{}^{4} \\ \cancel{4} & 2 \\ & \cancel{60} \\ & 20 \\ \cline{2-2} & 40 \end{array}$$

ANALYSIS.—Since the answer (fourth term) is required in dollars, make $60 the third term of the proportion. If 12 men earn $60, 16 men will earn more; therefore make 12 the first term and 16 the second term of one ratio. If they earn $60 in 4 days, in 2 days they will earn less; therefore make 4 the first term and 2 the second term. The proportion will then be stated as in the operation. To find the fourth or unknown term, divide the continued product of the means by the continued product of the extremes. e quotient will be the answer. Apply cancellation as in the operation, by ing the means at the right of a vertical line, and the extremes at the left.

525. RULE.—*Take for the third term the number that y form a ratio with, or is of the same denomination as, e answer.*

With each pair of similar terms remaining form a ratio as if the result depended upon these terms alone.

Multiply the means together, and divide their product y the product of the extremes. The quotient will be the required answer.

NOTE.—Compound Proportion is sometimes called the "Double Rule of Three."

EXAMPLES.

526. *1.* If it costs $39 to carpet a floor 16 feet long and 12 feet wide, what will it cost to carpet a floor 26 feet long and 20 feet wide?

2. If a man can walk 360 miles in 12 days, walking 8 hours each day, how many hours a day must he walk at the same rate to complete 450 miles in 20 days?

3. If 4 men can cut 56 acres of grass in 8 days, how many acres can 6 men cut in 12 days?

4. If it costs $1728 to pave a street 800 feet long and 50 feet wide, what will it cost to pave a street 1200 feet long and 90 feet wide?

5. How many hours a day must 42 men work, to do in 45 days what 27 men can do in 28 days of 10 hours each?

6. If the gas for 5 burners, 5 hours every evening for 10 days, costs $2.55, how many burners may be lighted 4 hours every evening for 15 days at a cost of $76.50?

7. If 22 men can dig a ditch 4200 feet long, 5 feet wide, and 3 feet deep, in 35 days of 9 hours each, in how many days of 11 hours each will 252 men dig a ditch 2100 feet long, 3 feet wide, and 2 feet deep?

8. If 3 men working 11 hours a day can reap a field of 20 acres in 11 days, in how many days can 9 men working 12 hours a day reap a field 360 yards long and 320 yards wide?

9. A person can read a book containing 220 pages, each of which contains 28 lines, and each line an average of 12 words, in 5¼ hours. How long will it take him to read a book containing 400 pages, each of which contains 36 lines, and each line an average of 14 words?

10. Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 acres respectively. The first gang works 7 hours a day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second gang complete theirs?

11. A transportation company charges $20 for carrying 15000 pounds 400 miles. How much ought they to charge for carrying 60000 pounds 320 miles?

12. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, in what time will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

13. If the interest of $100 for 1 year (360 days), at 6%, is $6, what is the interest of $1200 for 248 days at 8%?

14. If the freight of 18 *hhd.* of sugar, each weighing 1200 *lb.*, for 200 miles, is $320, what must be paid for the freight of 50 *hhd.* each weighing 960 *lb.*, for 420 miles?

15. If 18 men, working 10 hours a day, can hoe 60 acres in 20 days, how long will it take 50 boys, working 6 hours a day, to hoe 96 acres, 6 men being equal to 10 boys?

16. If a block of marble 2 × 1 × 3 *ft.* weighs 1020 pounds, what is the weight of a block 3 × 4 × 8 *ft.*?

INSURANCE.

527. Insurance is a contract by which one party (The Insurer or Underwriter) engages for a stipulated consideration (The Premium) to make up a loss which another may sustain.

Insurance is effected on property against loss or damage by fire and water, and on lives of persons. (For Life Insurance, see Art. **651**.)

Insurance is also effected against accidents to persons, the breakage of plate-glass, the loss of live stock, and the dishonesty of employees.

528. An **Insurance Company** is a company or corporation which insures against loss or damage.

529. Insurance companies may be classified according to principles of organization as follows:—1, Stock; 2, Mutual; 3, Mixed, or Stock and Mutual.

530. A **Stock Insurance Company** is one in which the capital is owned by individuals, called stockholders. They alone share the profits and are liable for the losses.

The business of a stock company and also of a mixed company, is managed by directors chosen by the stockholders. No policyholder, unless a stockholder, has any voice in any way in the election of the officers, or in the management of its business.

531. A **Mutual Insurance Company** is one in which there are no stockholders, and the profits and losses are shared among those who are insured (the policyholders).

Non-participating policies, the holders of which do not share in the profits or losses, are issued by certain mutual and mixed companies.

532. A **Mixed Insurance Company** is one which is conducted upon a combination of the stock and mutual plan.

Usually in a mixed company, all profits above a limited dividend to the stockholders are divided among the participating policyholders.

533. The **Policy** is the contract between the **Insurance Company** (the **Insurer** or **Underwriter**) and the **Insured**. It contains a description of the property insured, the amount of the insurance, and the conditions under which the policy is issued.

534. The **Premium** is the amount paid for the insurance.

1. Premium rates are expressed by giving the cost in cents of $100 insurance. The rate is sometimes expressed as a certain per cent. of the amount of the risk. Thus, a rate of 75 cents per $100 is equivalent to ¾%.

2. The premium rates depend upon the nature of the risk, and the length of time for which the policy is issued. The rate for 3 years in many Fire Insurance Companies is twice the rate for one year.

535. **Short Rates** are rates for a term less than a year.

If an insurance policy is terminated at the request of the policyholder, the company retains the customary "short rates" for the time the policy has been in force; if terminated at the option of the company, a ratable proportion of the premium is refunded for the unexpired term of the policy.

536. An **Insurance Agent** is a person who represents an insurance company or several companies, and acts for them in soliciting business, collecting premiums, adjusting losses, etc.

537. An **Insurance Broker** is a person who effects insurance, for negotiating which he receives a commission or brokerage from the company taking the risk.

Brokers are regarded as agents of the insured, and not of the insurance company.

538. The **Surplus** of an insurance company is the excess of the assets over the liabilities (including capital and unearned premium). .

FIRE INSURANCE.

539. **Fire Insurance** refers to insurance against loss or damage by fire.

540. **Adjustment of Losses.**—In an ordinary fire insurance policy, a person who insures will be paid the extent of his loss up to the amount of his insurance; but in policies containing the "average clause," the payment is such proportion of the loss as the amount of the insurance bears to the total value of the property.

1. The following is the usual form of the "average clause" above referred to: "It is a condition of this insurance, that if the whole value of the above described property, contained in any or all of the above mentioned buildings and premises, shall exceed the whole amount of insurance thereon, then, in case of loss or damage by fire, this policy shall contribute to the payment of said loss or damage in the proportion only that the whole amount of insurance on said property shall bear to the whole value of said property, in all of said buildings, at the time said loss or damage may occur."

2. Under a policy containing the "average clause," a person who insures $5000 on property worth $10000, would receive only $2500 in case of an actual loss of $5000; $1500 in a loss of $3000.

3. Insurance companies usually reserve the privilege of replacing or repairing the damaged premises.

MARINE INSURANCE.

541. Marine Insurance refers to insurance of vessels and their cargoes against the dangers of navigation.

1. Inland and Transit Insurance refer to insurance of merchandise while being transported from place to place either by rail or water routes, or both.

2. Policies on cargoes are issued for a certain voyage, or from port to port, and on vessels for a specified time or for a certain voyage.

3. The particular average clause is the clause which exempts the insurance company from the payment of any partial loss or particular average, unless it exceeds a certain per cent. of the value of the property. The particular average clause is sometimes applied to the value of each parcel or series of parcels, according to invoice numbers.

4. Insurance Certificates, showing that certain property has been insured, and stating the amount of the insurance and the name of the party abroad who is authorized to make the settlement, are issued by marine companies. They are negotiable, and are usually sent to the consignee of the merchandise to make the loss payable at the port of destination, and to otherwise facilitate the adjustment of the insurance in case of loss.

542. Adjustment of Losses.—In marine insurance, in case of loss or damage, the insurance company contributes such proportion of the loss as the amount of the insurance bears to the total value of the property.

1. The adjustment of marine losses is on the same principle as the adjustment of fire policies containing the "average clause" (**540**, 1).

2. In the adjustment of marine losses, the pound sterling is usually estimated at $4.95.

pen Policy is one upon which additional insur-
ntered at different times. It covers merchandise
hipped on "Vessel or Vessels" from "Ports and
orts and Places," for amounts "as endorsed" and
reed."

f the shipment, name of vessel, ports of shipment and
nount of the insurance, rate, premium, and a description
entered on the policy or in a pass-book, which is regarded
y. (See Ex. 27, Art. **544**.)
ies with pass-books attached and insuring merchandise
nage by fire, are issued by fire insurance companies.
s, which cover all risks whether accepted and endorsed on
are issued to merchants who are receiving merchandise from
and who do not always have a definite knowledge of the
Such policies usually contain the following clause: "The
entitled to premiums at their usual rates on all shipments
t is warranted by the assured to report every shipment on
g advice thereof, or as soon thereafter as practicable, when
n shall be fixed by the President of the Company."
icies cover the invoice cost and 10% additional until the
is endorsed on the policy or pass-book.
es are sometimes issued which cover only such risks as
nd endorsed on the policy by the company.

EXAMPLES.

building was insured for \$2500 in one company at
)00 in another company at 125 cents. What was
m paid ?
f goods was insured for \$9000 at $\frac{3}{4}$%. What was
nsurance, \$1.25 being charged for the policy ?
the total premium of the following insurances:
7000 at 45¢, \$2000 at 5%, \$3500 at 45¢, \$2000 at
%, \$2000 at 60¢, \$4500 at 25¢, and \$3600 at 125¢?
paid for an insurance of \$2500; what was the

as paid for an insurance at the rate of 70¢ per
as the amount of the risk ?
was insured for \$7500 for 1 year at $2\frac{1}{2}$%, stock for
nd raw material for \$2500 at $1\frac{1}{4}$%. What was the

8. A cargo of merchandise was insured for $6500 at ⅞% including the risk of fire while on wharf awaiting shipment. What was the premium?

9. A building was insured Jan. 1, 1880, for $2000, for 7 years, at 5%; what was the value of the unearned premium, Jan. 1, 1882?

10. A shipment of goods was insured in the Pacific Mutual Insurance Co. for $9600 at 75¢ less 20% in lieu of scrip and interest. What was the net cost of the insurance?

11. A house was insured for $5000 for 4 years at 60¢ per annum. The house was destroyed by fire. What was the actual loss of the company, making no allowance for interest?

12. Find the cost of insuring a house for 3 years for $4000 at 60¢, and the furniture for $1200 at 80¢, less 15% on both premiums.

13. A cargo of hides having increased in value since the insurance was effected, the anticipated profits were insured for $3000 at 1¾% less 20%. What was the premium?

14. A factory (worth $3000) and its contents are insured for $10000 as follows: $2000 on building, $3000 on machinery (worth $5000), and $5000 on stock (worth $8000). The building is damaged by fire to the amount of $1000, the machinery $4000, and stock is a total loss. How much is the claim against the insurance company?

15. A cargo of goods valued at $20000 was insured for $12000. If the goods were damaged to the amount of $15000, how much of the loss would be paid by the insurance company? **(542.)**

16. A building is insured in several companies for $60000, and is damaged by fire to the extent of $24000. What per cent. of its risk is paid by each company?

17. A stock of goods was insured, May 1, for 1 year, for $6000, at 90¢. The policy was cancelled Nov. 1, at the request of the insured. How much was the return premium, the short rate for 6 months being 63¢? How much would have been returned by the company, if the policy had been cancelled at its request?

18. A quantity of merchandise valued at $6000 is insured for $5000. It is damaged by fire to the amount of $1728. How much of the loss is paid by the insurance company, the policy containing the "average clause"? **(540.)**

19. What was paid for insuring a cargo of merchandise for $8750 at ⅞% less 20%?

20. A factory and its contents are insured for $5000 in company M, $5000 in N, $5000 in O, $4000 in P, and $2500 in each of the following companies: Q, R, S, T, U, V, W, X, Y, and Z. What was the total premium, the rate being 2% less 10%?

21. The above insurance covered the following property: $4000 on building marked A on plan, $4000 on B, $5000 on C, $500 on D, $500 on E, $3500 on stock and materials in building marked A on plan, $8000 on machinery, etc., in A, $11500 on stock and materials in B and C, $4000 on machinery, etc., in B and C, $2500 on horses in D, $500 on harness, hay, feed, etc., in D. Suppose building A and its contents were totally destroyed by fire, what would be the loss of company M? Of P? Of T?

NOTE.—The above insurance was divided pro rata among the several companies, each policy designating the exact amount on each building, etc.

22. In the above example, what is the amount of the risk of company M on the building marked A on plan? On C?

23. The net invoice value of a quantity of goods is $6325, and the insured value $6500. The insured value is what per cent. greater than the invoice value?

24. A quantity of merchandise valued at $9035, is insured for $9000. What is the insurance on part of the same, the estimated value being $2638?

25. If 500 packages of merchandise are insured for $2627.78, what is the insurance on 60 packages?

26. The estimated sound value of a quantity of merchandise, damaged at sea, was $328.55, and the proceeds when sold at auction, $299.35. How much of the loss was shared by the Insurance Co., the insurance having been $315.33?

27. Make the extensions of the following "open policy" and find the total amount.

Date.	Name of vessel.	From.	To.	On.	Amount insured.	Rate.	Amount of premium.
1881.							
Sept. 2	Othello.	N. Y. via Hull.	Stockholm.	50 Ba. Mdse.	5100	1⅛	**.**
" 7	Algeria.	New York.	Liverpool.	68 " "	6675	½	**.**
" 16	Germanic.	New York.	Liverpool.	92 " "	13500	½	**.**
" 17	Rialto.	N. Y. via Hull.	Christiania.	6 " "	600	1	*.
" 23	Otranto.	N. Y. via Hull.	Orebro.	30 " "	2700	1⅛	**.**
							$***.**
					Less 20%		**.**
							***.**

EXCHANGE.

545. Exchange is the system by which merchants in distant places discharge their debts to each other without the transmission of money.

Suppose, for example, A of New York owes B of Chicago $1000 for grain, and C of Chicago owes D of New York $1000 for dry goods. The two debts may be discharged by means of one draft or bill of exchange without the transmission of money. Thus, B of Chicago draws on A of New York for $1000, and sells the draft to C of Chicago, who remits it to D of New York. D of New York presents the draft to A of New York for acceptance or payment, and thus both debts are cancelled. There is in effect a setting-off or exchange of one debt for the other.

The business of exchange is usually conducted through the medium of banks and bankers, who buy commercial bills and transmit them for credit to the places on which they are drawn. They also sell their own drafts on their correspondents in any amounts demanded.

The greater part of the exchange in the United States is effected through the banks of New York, Boston, Philadelphia, Chicago, St. Louis, Baltimore, and San Francisco. The financial centres of Europe are London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Bremen, Berlin, and Vienna.

546. A **Bill of Exchange,** or **Draft,** is an order or request addressed by one person (the Drawer) to another (the Drawee), directing the payment of a specified sum of money to a third person (the Payee) or to his order. It is issued at one place and payable at another. (See Art. **495**, 5–6.)

For brevity, bills of exchange are frequently called "exchange."

According to the laws of most States, drafts drawn in one State and payable in another, are termed *foreign* bills of exchange. For the purposes of this book, the term "domestic exchange" will be applied to bills drawn and payable in the United States.

547. Bills of exchange are of two kinds, Inland or Domestic, and Foreign.

548. A **Domestic** or **Inland Bill of Exchange** is one which is payable in the same country in which it is drawn.

549. A Foreign Bill of Exchange is one which is payable in a different country from the one in which it is drawn; as a draft drawn in the United States and payable in England.

550. When drafts sell for more than their face value, exchange is above par or at a premium; when for less than their face, below par or at a discount.

When Chicago owes New York the same amount that New York owes Chicago, exchange will be at par; that is, drafts will sell at their face value. When Chicago owes New York more than New York owes Chicago, drafts on New York will sell at a premium; there will be more buyers of exchange than sellers, and drafts will sell for more than their face value. When Chicago owes New York less than New York owes Chicago, the demand in Chicago for drafts on New York will be less than the supply, and drafts will sell for less than their face value, or at a discount.

DOMESTIC EXCHANGE.

551. Domestic or **Inland Exchange** relates to drafts drawn at one place on another in the same country.

552. The domestic exchanges on New York at the places named were quoted as follows, May 7, 1881: Savannah, ⅛ @ ⅜ premium; Charleston, ⅛ @ ¼ premium; New Orleans, $1.50 @ $2.50 premium; St. Louis, 25 cents premium; Chicago, 50 @ 75 cents premium; and Boston, 25 cents discount.

1. At Savannah and Charleston the rates per cent. of the premium or discount are given. Thus, when exchange is quoted at ¼ premium, a draft of $100 may be purchased for $100¼ ($100.25).

2. At New Orleans, St. Louis, Chicago, and Boston, the premium or discount per $1000 is given. Thus, a draft of $1000 at $2.50 premium may be purchased for $1002.50. $2.50 per $1000 premium is equivalent to ¼% premium.

3. The selling rates are about ⅛% ($1.25) higher than the buying rates, and bankers' exchange is usually higher than commercial.

4. The rate of domestic exchange is limited by the cost of shipping gold or currency by express, and the premium or discount will not exceed this cost. Thus, if a merchant in Chicago is charged a premium of $10 for a draft of $10000, and he can send the currency by express for $7.50, it will be to his advantage to remit by the latter method.

The following appeared in a New York financial paper, May 8, 1881, the date of the above quotations: — "The domestic exchanges at the West are sufficiently high to permit of a movement of funds Eastward, but at the East,

New York funds are still at a discount and some shipments of gold and currency continue to be made to the Eastern cities."

5. The preceding quotations refer to sight exchange. Time drafts are discounted in the same manner as promissory notes. In certain cases bankers in discounting notes and drafts payable in distant places, charge interest for the time required for the return of the money when the note or draft is paid; and in the case of drafts drawn a certain number of days after sight, bankers sometimes charge interest for the time required for the acceptance of the drafts. Thus, if a draft was drawn in New York on St. Louis and payable 60 days after sight, it would require, in the ordinary course of the mails, 3 days for the acceptance of the draft. The draft would be paid in 60 days, and 3 days would elapse before the money would be returned to New York. The banker would be justified in charging interest for 66 days, the interval between the day he advanced the money in New York, and the day it was returned to him again. If the draft was drawn on San Francisco, fully 16 days (8 days for the acceptance and 8 days for the return of the money) would be added to the time of the draft. Between New York and San Francisco and other distant places, money is frequently transferred by telegraph. (See Art. **499**, 3.)

EXAMPLES.

553. *1.* What is the value in Savannah of a draft on New York for $8750 at $\frac{3}{8}$% premium?

2. Find the cost in New Orleans of a draft on New York for $8375 at $2.50 premium.

Find the value of the following drafts:

	Face.	Exchange.		Face.	Exchange.
3.	$5000,	$\frac{1}{8}$% premium.	*8.*	$4287.75,	15¢ discount.
4.	$4375,	$\frac{3}{8}$% discount.	*9.*	$3416.33,	25¢ premium.
5.	$8417,	$\frac{1}{8}$% premium.	*10.*	$2825.49,	$1.25 discount.
6.	$9873,	$\frac{1}{2}$% premium.	*11.*	$9873.62,	$2.50 premium.
7.	$5284,	$\frac{1}{4}$% discount.	*12.*	$8412.75,	75¢ discount.

13. A of Chicago buys cattle for B of New York to the amount of $9858.07. How large a draft should be drawn on B, so that when sold at a discount of 50¢ ($\frac{1}{20}$%), the proceeds would be sufficient to pay the bill?

NOTE.—To find the face of a draft, instead of dividing the value of the draft by the rate of exchange (in the above example, .99$\frac{19}{20}$ or .9995), business men and bankers calculate the premium or discount on the value of the draft, and subtract or add it to the value as the case requires. Thus, in the above example, the discount would be $\frac{1}{2}$ of $\frac{1}{10}$% of $9858.07, or $4.93, which added to the given proceeds would produce the face $9863. This method produces too small a result in all cases, the error being equivalent to the percentage of the premium or discount. In this example the error is less than $\frac{1}{4}$ cent.

[illegible] examples in business the foregoing method is sufficiently [illegible] of $4.50 (a very high rate for domestic exchange) on a draft [illegible] $1000, the error would be only 25 cents. If greater accuracy [illegible] necessary, correction can be made by adding the percentage of discount. Thus, if the value of the draft is $1000, and [illegible] discount, the face would be $1000 + [illegible] of $1000 [illegible] = $1000.25. If at [illegible] premium, the face would be [illegible]

[illegible] find the face of the following drafts:

	Premium.	Value.	Exchange.
[illegible]	[illegible] premium.	[illegible]	[illegible] premium.
[illegible]	[illegible] discount.	[illegible]	50¢ discount.
[illegible]	[illegible] discount.	[illegible]	$1.00 premium.
[illegible]	[illegible] premium.	[illegible]	$1.25 premium.
[illegible]	[illegible] premium.	[illegible]	75¢ discount.

[illegible] New Orleans being indebted to B of New York [illegible] to him a check on a New Orleans bank for [illegible] which B is obliged to allow a discount of [illegible] does A still owe B, and for what amount should [illegible] drawn to pay the amount due?

[illegible] draft on New York for $3000, [illegible] premium, and

[illegible]

[illegible] on New [illegible] exchange [illegible] for the

[illegible] $3900, pay-

[illegible] would be the [illegible] allowing [illegible]

[illegible] sight draft

FOREIGN EXCHANGE.

554. Foreign Exchange relates to drafts or bills of exchange drawn in one country and payable in another.

555. To secure safety and speed in the transmission of foreign bills of exchange, they are drawn in sets of two or three of the same tenor and date. The separate bills are sent by different steamers, and when any one of them is paid, the others become void. Some merchants send only the first and second, and preserve the third.

SET OF EXCHANGE.

(1.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1889.*

Sixty days after sight of this FIRST of Exchange (Second and Third unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

(2.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1889.*

Sixty days after sight of this SECOND of Exchange (First and Third unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

(3.)

EXCHANGE FOR £1000. NEW YORK, *May 16, 1889.*

Sixty days after sight of this THIRD of Exchange (First and Second unpaid), pay to the order of A. T. STEWART & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738. BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

Foreign bills of exchange are usually drawn in the moneys of account of the countries in which they are payable. Thus, drafts on England are usually drawn in pounds, shillings, and pence; on France, Belgium, and Switzerland, in francs; on Germany, in marks; on the Netherlands (Holland), in guilders.

Foreign bills of exchange are usually drawn at sight (3 days) or at sixty (63 days) days' sight. Sight drafts are frequently called "short" exchange, and 60 day drafts, "long" exchange. "Long" exchange is sold at a rate below that for "short" exchange, sufficient to equalize the difference in interest between the dates of maturity of the two classes of bills.

556. A **Letter of Credit** is an instrument issued by a banker and addressed to bankers generally, by which the holder may draw funds at different places and in amounts to suit his convenience, the total amount drawn not exceeding the limit of the letter of credit.

A *bill of exchange* is payable at a certain place, at a certain fixed time, and for a certain amount, while a *letter of credit* is payable at different places, at different times, and in different amounts.

A person who intends to travel in foreign countries, may procure a letter of credit by depositing either cash or securities with a foreign exchange banker for the amount of the letter. When the American banker is notified of the payment of the traveler's drafts in London, he debits the account of the holder of the letter of credit with the amount drawn and the charges, at the current rate of exchange. A small rate of interest is sometimes allowed on the account, and a settlement is made on the return of the traveler.

557. The **Intrinsic Par of Exchange** is the value of the monetary unit of one country expressed in that of another, and is based on the comparative fineness and weight of the coins, as determined by assay. (See Art. **566.**)

558. The **Commercial Par of Exchange** is the market value in one country of the *coins* of another.

559. The **Commercial Rate of Exchange** is the market or buying and selling value in one country of the *drafts* on another.

1. In giving quotations of foreign exchange, no reference is made to the par value, the quotations being given by means of equivalents.

2. Premium or discount for exchange cannot long exceed the transportation charges and insurance of shipping coin; for, if a merchant can ship gold cheaper than he can buy a bill of exchange, he will choose the former method of paying his indebtedness. When sight exchange is 4.84, gold can be imported at a small profit; and when sight exchange is 4.89¼, gold can be exported at a profit.

3. When exchange is above par, we are exporters of gold; when below par, we are importers of gold.

560. Exchange on England (Sterling exchange) is quoted by giving the value of £1 in dollars and cents.

Thus, when exchange is 4.84, a draft of £1 will cost $4.84; of £100, $484. The intrinsic par value of £1 is $4.8665 (**566**).

561. Exchange on France, Belgium, and Switzerland is quoted by giving the value of $1 in francs and centimes (hundredths of a franc).

Thus, when exchange is $5.27\frac{1}{2}$, $1 will buy a bill of 5 francs and $27\frac{1}{2}$ centimes; a draft of 1000 francs will cost $189.57 ($1000 \div 5.27\frac{1}{2}$). The intrinsic par value of 1 franc is $19\frac{3}{10}$ cents (**566**); of the equivalent exchange, $5.18\frac{1}{8}$ ($1.00 \div .193$).

In French, Belgian, and Swiss exchange, the higher the apparent rate, the less the value of the draft. Thus, when exchange is 5.13, a draft of 1000 francs is worth $194.93, and each franc is worth $19\frac{49}{100}$ cents. When exchange is $5.26\frac{3}{8}$, the same draft would be worth $189.98, and each franc 19 cents.

562. Exchange on Amsterdam (Netherlands) is quoted by giving the value of one guilder (gülden) or florin in U. S. cents.

The intrinsic par value of 1 guilder is $40\frac{2}{10}$ cents (**566**).

563. Exchange on Germany is quoted by giving the value of 4 reichsmarks in cents.

The intrinsic par value of 1 mark is $23\frac{8}{10}$ cents (**566**); of 4 marks $95\frac{2}{10}$ cents.

564. **Documentary Exchange** is a bill drawn by a shipper upon his consignee against merchandise shipped, accompanied by the letter of hypothecation, the bill of lading "to order," and the insurance certificates covering the property against which the bill is drawn.

565. Exchange on London in the countries named, and at London on the same countries, is quoted as follows:

United States, by giving the value of £1 in dollars and cents.
France and Belgium, by giving the value of £1 in francs and centimes.
Germany, by giving the value of £1 in marks and pfenniges.
Austria, by giving the value of £1 in florins and kreutzers.
Netherlands, by giving the value of £1 in guilders and cents.
India, by giving the value of 1 rupee in shillings and pence.

566. FOREIGN MONEYS OF ACCOUNT.

Country.	Standard.	Monetary Unit.	Value in U. S. Gold.
Argentine Republic.	Gold and silver.	Peso of 100 centavos...	.96,5
Austria	Silver	Florin of 100 kreutzers.	.35,9
Belgium	Gold and silver.	[a]Franc of 100 centimes..	.19,3
Bolivia	Silver	[b]Boliviano, 100 centavos.	.72,7
Brazil	Gold	Milreis of 1000 reis....	.54,6
British America....	Gold	Dollar of 100 cents	$1.00
Chili.	Gold and silver.	Peso of 100 centavos...	.91,2
Cuba	Gold and silver.	Peso of 100 centavos...	.93,2
Denmark	Gold	[c]Crown of 100 öre......	.26,8
Ecuador	Silver	[b]Sucre of 100 centavos..	.72,7
Egypt	Gold	Pound of 100 piasters..	4.94,3
France	Gold and silver.	[a]Franc of 100 centimes..	.19,3
German Empire....	Gold	Mark of 100 pfennige..	.23,8
Great Britain	Gold	Pound of 20 shillings..	4.86,6½
Greece	Gold and silver.	[a]Drachma of 100 lepta..	.19,3
Hayti	Gold and silver.	[d]Gourde of 100 centavos.	.96,5
India	Silver	Rupee of 16 annas[e]....	.34,6
Italy	Gold and silver.	[a]Lira of 100 centesimi ..	.19,3
Japan	Gold and silver.	Yen of 100 sen { Gold..	.99,7
		Silver.	.78,4
Liberia	Gold	Dollar of 100 cents	1.00
Mexico	Silver	Dollar of 100 centavos.	.79
Netherlands	Gold and silver.	Florin of 100 cents....	.40,2
Norway	Gold	[c]Crown of 100 öre......	.26,8
Peru	Silver	[b]Sol of 100 centavos....	.72,7
Portugal	Gold	Milreis of 1000 reis....	1.08
Russia	Silver	Rouble of 100 copecks .	.58,2
Spain	Gold and silver.	[a]Peseta of 100 centimes.	.19,3
Sweden	Gold	[c]Crown of 100 öre......	.26,8
Switzerland	Gold and silver.	[a]Franc of 100 centimes..	.19,3
Tripoli	Silver	Mahbub of 20 piasters .	.65,6
Turkey	Gold	Piaster of 40 paras....	.04,4
U. S. of Colombia..	Silver	[b]Peso of 100 centavos...	.72,7
Venezuela	Gold and silver.	[a]Bolivar of 100 centavos.	.19,3

The above rates, proclaimed by the Secretary of the Treasury, Jan. 1, 1887, are used in estimating, for Custom-House purposes, the values of all foreign merchandise made out in any of said currencies.

(a) The *franc* of France, Belgium, and Switzerland, the *peseta* of Spain, the *drachma* of Greece, the *lira* of Italy, and the *bolivar* of Venezuela have the same value.

(b) The *sucre* of Ecuador, the *peso* of United States of Colombia, the *boliviano* of Bolivia, and the *sol* of Peru have the same value.

(c) The *crowns* of Norway, Sweden, and Denmark have the same value.

(d) The *gourde* of Hayti and the *peso* of the Argentine Republic have the same value.

(e) The *anna* contains 12 *pies*

EXAMPLES.

567. *1.* Find the cost of a bill of exchange on London for £225 at 4.81¾. **(560)**

Analysis.—If £1 costs $4.81¾, £225 will cost 225 times $4.81¾.

2. What is the value of a draft for £324 16*s.* at 4.87½?

Analysis.—Write one-half of the greatest even number of shillings as tenths of a pound, and if there be an odd shilling write 5 hundredths. £324 16*s.* = £324.8. (See Art. **342**, Ex. 12, Note.) The value of £324 16*s.* at 4.87½ is found by multiplying $4.87½ by 324.8.

3. Find the value of a draft on London for £379 12*s.* 7*d.*, at 4.86⅜.

OPERATION.

```
   379.6
   4.86⅜
   -----
     949
     475
   22776
  30368
 15184
      14
--------
1846.420
```

Analysis.—If each penny be regarded as 2 cents, the result will be sufficiently accurate. For 11*d.* the maximum number of pence in any example, and exchange at 4.91, the error would be only ½ cent. $4.86⅜ × 379.6 = $1846.28. $1846.28 + $0.14 = $1846.42. To save one addition, add the 14 cents to the partial products as in the operation.

Find the value of

4. £500 at 4.81½.
5. £775 at 4.85¼.
6. £837 at 4.83⅞.
7. £84 8*s.* at 4.85.

Find the value of

8. £512 13*s.* at 4.84⅜.
9. £834 6*s.* 6*d.* at 4.88⅝.
10. £675 11*s.* 8*d.* at 4.87⅛.
11. £225 7*s.* 5*d.* at 4.82¾.

12. Find the cost of a bill of exchange on Liverpool, for £875 12*s.* 6*d.* at the par value. **(560)**

13. What are the proceeds of a draft of £959 5*s.* 4*d.*, sold through a broker, at 4.79½, brokerage ⅛%?

14. An exporter sold a draft for £540 3*s.* on Manchester, payable in London, at 4.84, brokerage ⅛%. What were the proceeds?

15. Find the proceeds of a draft on Newcastle-on-Tyne, at 60 days' sight for £1764 15*s.*, payable in London, at 4.82, brokerage on exchange ⅛%.

16. An importer purchased a bill of exchange on London, at 3 days' sight, for £488 16*s.* 6*d.*, at 4.85½. What was the cost?

17. How much exchange on London at 4.81¾ will $821.99 buy ?

ANALYSIS.—$4.81¾ will buy exchange for £1 : hence, $821.99 will buy as many pounds as $4.81¾ are contained in $821.99, or £170.625. £170.625 = £170 12*s.* 6*d.* (See Art. **289**, and Art. **342**, Ex. 19, Note.)

18. What will be the face of a 3 days' bill of exchange on London that can be bought for $5964.13, exchange 4.86½ ?

19. The face of a bill of exchange was £875, and its cost was $4233.91. What was the rate of exchange ?

20. An exporter received $9063.22 for a bill of exchange that was sold through a broker at $4.86¾ ; what was the face of the bill, the broker's commission being ⅛% ?

21. Find the cost of a bill of exchange on Paris for 7000 francs at 5.21⅞.

OPERATION.

$$\begin{array}{rl} 5.21\tfrac{7}{8}\ ) & 7000 \\ 8 & 8 \\ \hline 41.75\ ) & 56000.0000\ (\end{array}$$

ANALYSIS.—Since 5.21⅞ francs cost $1, 7000 francs will cost as many dollars as 5.21⅞ francs are contained times in 7000 francs.

Find the value of

22. 6000 francs at 5.16.

23. 5000 francs at 5.18⅛.

24. 4000 francs at 5.21⅜.

Find the value of

25. 8475 francs at 5.19½.

26. 7216 francs at 5.17¾.

27. 987.60 francs at 5.20⅞.

28. Find the cost of a draft on Antwerp at 3 days' sight, for 9640 francs, at 5.19⅜.

29. What is the value of a draft on London for £416 16*s.* 3*d.*, at 4.85⅜ ?

30. Sold exchange on Geneva, through a broker, for 8000 francs at 60 days' sight ; what were the proceeds of the draft, exchange being 5.20⅝, brokerage ⅛% ?

31. What are the proceeds of a draft on Paris for 12420 francs, at 5.19¾, brokerage on exchange ⅛% ?

32. What will it cost to remit to Antwerp 8750 francs at the par value ? **(561)**

33. Sold through a broker a draft on Geneva for 7324 francs. What were the proceeds, exchange being 5.18⅜, brokerage ⅛% ?

34. What will be the face of a bill of exchange on Geneva that can be bought for $15372, exchange selling at 5.22½ ?

35. Paid for a draft on Paris $3460.32 ; what was the face of the draft, exchange being 5.19⅜ ?

36. A merchant paid $6272 for a bill of exchange of 32512.48 francs; what was the rate of exchange?

37. Find the cost of a bill of exchange on Hamburg for 14400 marks (Reichsmarks) at 94⅛.

OPERATION.

$$\begin{array}{r} 4\ \underline{)\ 14400} \\ 3600 \\ \underline{.94\tfrac{1}{8}} \\ 3388.50 \end{array}$$

ANALYSIS.—Since 4 marks cost $0.94⅛, 14400 marks will cost 3600 (14400 ÷ 4) times $0.94⅛, or $3388.50.

Find the value of

38. 7200 marks at 94.

39. 8416 marks at 93½.

40. 3456 marks at 95¼.

41. 1237 marks at 93⅝.

42. 9894 marks at 95⅜.

43. 6515 marks at 94¾.

44. What is the cost of a bill of exchange on Frankfort for 16200 marks at 95½?

45. Sold a bill of exchange on Hamburg for 13200 marks, at 94⅛; what was the amount received, brokerage ⅛%?

46. An importer purchased a bill of exchange on London for £318 10*s.* 7*d.*, at 4.85¾; what did it cost?

47. What were the proceeds of a draft, sold through a broker, for 8748 marks, at 94⅜, brokerage ⅛%?

48. An exporter sold a draft on Paris for 12275 francs, at 5.19⅝; what were the proceeds, brokerage ⅛%?

49. What is the face of a bill on Hamburg that cost $816, exchange 94⅛?

ANALYSIS.—Since $.94⅛ will buy 4 marks, $816 will buy 4 times as many marks as $0.94⅛ is contained times in $816.

50. What is the face of a 3 days' draft on Bremen, that was purchased in New York for $3261.60, exchange 94⅜?

51. The cost of a draft of 12320 marks was $2922.15; what was the rate of exchange?

52. Find the cost of a bill of exchange on Amsterdam, for 7240 guilders, at 40⅛.

53. Find the cost of a bill of exchange on Amsterdam, at 60 days' sight, for 12480 guilders, exchange 39⅞.

54. An exporter received $1890.86 for a bill of exchange on Amsterdam; what was its face, exchange being 41⅛, brokerage ⅛%?

55. At 40⅜, how much exchange on Amsterdam will $2877.93 buy?

56. The value of a draft of 5280 guilders is $2145; what is the quotation?

57. The dividends of the N. Y. C. and H. R. R. Co., are paid in London at the rate of 49⅛ pence to the dollar. What is the equivalent rate of exchange?

58. Find the value in U. S. money of 16319 bushels of wheat at 4*s.* 4½*d.* per bushel, exchange 4.86½.

59. A merchant sent a messenger with a bill of exchange of 20000 francs to two bankers, A and B, with instructions to sell it to the best advantage. A offered 5.27 and B 5.27½. The messenger imprudently accepted the latter offer. How much did the merchant lose by the ignorance of the messenger?

60. When United States 4 per cent. consols are quoted in New York at 114½, and sterling exchange at 4.83½, what should be the London quotation of the bonds? What should be the London quotation of 4½ per cent. bonds, the New York quotation being 113¼?

NOTE.—In London, all American securities are quoted on an assumed value of the pound sterling of $5, instead of the actual value of $4.8665, or, more definitely speaking, its commercial value determined by the rate of exchange. Multiplying the New York quotation by 5 and dividing by the rate of exchange, the result will be the equivalent London quotation.

61. When American railway stocks are quoted in London at 88, what is the equivalent New York quotation, sterling exchange being quoted in New York at 4.88½?

62. What is the London equivalent of a New York quotation of 142, exchange being 4.83?

63. At Paris, what is the value of a draft on London of £550, exchange being 25.36½?

64. At London, what is the cost of a draft on Hamburg of 8000 marks, exchange being 20.45?

65. At Vienna, what is the cost of a draft on London of £625, exchange being 11.75?

66. At London, what is the value of a draft on Calcutta of 12000 rupees, exchange being quoted at 1*s.* 8 3/16*d.*?

67. A commission merchant wishes to remit $2475 to his principal in England. How large a draft must he purchase, exchange being 4.83¼?

EQUATION OF ACCOUNTS.

568. Equation of Accounts (called also Equation of Payments and Averaging Accounts) is the process of finding the time when several debts due at different dates may be paid in one amount without loss of interest to either party. It is also the process of finding the time when the balance of an account having both debits and credits may be paid without loss of interest to either party. This time is called the *equated* or *average time.*

NOTE.—It is important that the commercial student be thoroughly drilled in the theory and practice of Equation of Accounts, as examples in this subject are of frequent occurrence in many wholesale and commission houses.

569. To find the equated time when the items of the account are all on the same side, i. e., all debits or all credits.

ANALYTICAL STEPS.—By assuming a certain date as the time of settlement, we find what the loss or gain of interest would be to the payer if all the bills were paid by him on that date. We next find in how many days the total amount of the bills would produce a sum equivalent to this loss or gain of interest, and find the true day of settlement by counting forward or backward this number of days from the assumed date. Thus, if the sum of the several bills is $1000, and the loss of interest to the payer at the assumed date of settlement is $10 (the interest of $1000 at 60 days at 6%), it is evident that the true date of settlement, or the time when there would be no loss of interest to either party, must be 60 days after the assumed date.

NOTES.—1. The interest on the bills paid after they became due would equal the interest on the bills paid in advance, the former being a gain to the payer, and the latter, a loss.

2. Any date may be assumed as the time of settlement. For convenience, the earliest or latest date is generally used. If the earliest date is taken, the estimated interest is a loss to the payer; if the latest is taken, the interest is a gain.

When the time is found by Compound Subtraction, or each month is regarded as 30 days, the last day of the month preceding the earliest item is the most convenient. (See second interest method.)

In Equation Tables, Dec. 31 or Jan. 1 is taken for all examples.

The assumed date is sometimes called the *focal date.*

3. Any rate of interest may be used in making the computations, 6 and 12 being the most convenient rates.

570. Ex. At what date may the following bills of merchandise be paid in one amount without loss of interest to either party? Due Apr. 10, $114; due Apr. 26, $140; due May 22, $320; due June 6, $976.

OPERATION.—PRODUCT METHOD.

Due Apr. 10,	$114	×	0	=	0
" " 26,	140	×	16	=	2240
" May 22,	320	×	42	=	13440
" June 6,	976	×	57	=	55632
	1550			)	71312 (46 days after Apr. 10, or May 26.

ANALYSIS.—For convenience, assume Apr. 10, the earliest due date, as the time of settlement. If the first bill, which is due Apr. 10, is paid on that date, there will be no loss or gain of interest to either party. If the second bill, which is due Apr. 26, is paid Apr. 10, 16 days before it is due, there will be a loss to the payer of the interest or the use of $140 for 16 days, or $2240 for 1 day. On the third bill, there will be a loss of the interest of $320 for 42 days, or $13440 for 1 day. On the fourth bill, there will be a loss of the interest of $976 for 57 days, or $55632 for 1 day. If all the bills are paid Apr. 10, there will be a loss to the payer of the interest of $71312 for 1 day, or of $1550 for 46 days. Since the loss of interest to the payer is equivalent to the interest of the total amount of the bills for 46 days, it is evident that the day when there would be no loss of interest must be 46 days after Apr. 10, or May 26. The payer is entitled to defer payment 46 days after the assumed date as a compensation for the estimated loss.

The gain of interest to the payer on the first three bills, which are paid after they are due, equals the loss of interest on the fourth bill, which is paid before it is due.

PROOF.

The interest of $114 for 46 days at 6% is	$0.874
" " " 140 " 30 " "	.70
" " " 320 " 4 " "	.213
Total gain of interest to the payer	1.787
The interest (a loss to the payer) of $976 for 11 days is .	**1.789**

NOTES.—1. In finding the number of days from the assumed date to the other dates, instead of calculating from the assumed date each time, find the interval from one date to the next and add it to the last number of days. Thus, from Apr. 10 to May 22 is 42 days, and from May 23 to June 6, 15 days; hence, from Apr. 10 to June 6 is 57 (42+15) days. (See Art. **310**, Ex. 3.)

2. To determine the due date, find the number of days in the operation nearest to the quotient, and add or subtract, as may be necessary, the difference between it and the quotient, to its corresponding date. Thus, in the above example, the number of days in the operation nearest to the quotient is 42; hence the due date is 4 (46−42) days after May 22, or May 26. (See Art. **311**, Ex. 10.)

3. If the fraction of the quotient is less than ½, disregard it; if more than ½, add 1 day to the integral number of days in the quotient.

571. RULE FOR THE PRODUCT METHOD.—*Assume the earliest due date as the day of settlement for all the items. Multiply each item by the number of days intervening between the assumed date of settlement and the date of the item; and divide the sum of the several products by the sum of the account. Count forward from the assumed date the number of days obtained in the quotient. The result will be the equated time.*

572. OPERATION.—FIRST INTEREST METHOD.

		Days.	Interest.
Due Apr. 10,	$114	0	$.00
" " 26,	140	16	.233 for 10 days. .14 " 6 "
" May 22,	320	42	1.60 " 30 " .64 " 12 "
" June 6,	976	57	4.88 " 30 " 2.44 " 15 " 1.952 " 12 "
	60) 15.50		
	.258		) 11.885 (46 days

after Apr. 10, or May 26.

ANALYSIS.—Assume Apr. 10, the earliest due date, as the time of settlement. If the total amount ($1550) of the bills is paid Apr. 10, the assumed date of settlement, there will be a loss of interest to the payer of $11.885. The interest of $1550 for 60 days at 6% is $15.50, and for 1 day, $0.258. It will take $1550 to produce $11.885 interest as many days as $0.258 is contained times in $11.885, or 46 days. If, at the assumed date of settlement, there is a loss to the payer of the interest of $1550 for 46 days, the true day of settlement must be 46 days later, or May 26.

5. **Average the following account:**

NEW YORK, *July 1, 1882.*

MESSRS. RICE, STIX & CO.,

To LORD & TAYLOR, *Dr.*

1882. Apr. 4	Mdse. 30 days per bill rendered. . .	$816	37
" 21	" 30 " " " . .	724	25
May 13	" 30 " " " . .	342	46
" 25	" 30 " " " . .	535	84
June 16	" 30 " " " . .	628	62
	Due by equation June *, 1882.	****	**

NOTE.—When several bills are sold on a common term of credit, first find the average date of purchase, and to the result add the common term of credit.

Certain merchants sell uniformly on the same term of credit, while others sell on different credits, depending upon the class of goods, the standing of the customer, the state of the market, etc. (See Art. **416**.)

6. A. Hamilton bought of F. A. Leggett & Co., several bills of goods, as follows:

May 16,	a bill of	$212.46	on 60	days' credit.
" 28,	"	318.40	" 60	" "
June 6,	"	275.64	" 60	" "
" 21,	"	187.83	" 60	" "
July 13,	"	835.60	" 60	" "

A 60-day note for the whole amount is given in settlement. What must be its date, no allowance being made for the days of grace?

7. Sold on a credit of 90 days the following bills of goods: Mar. 4, $194.13; Mar. 27, $222.36; Apr. 12, $538.72; May 3, $432.64; May 28, $303.10. What is the equated time of payment? How much will settle the account Aug. 1, at 6%? How much July 1?

NOTE.—When monthly statements are sent to customers the accounts are frequently averaged. (See Ex. 5.) When the account is averaged, the simplest method of finding the cash balance due at a certain date, is to calculate the interest on the total amount from the average date to the time of payment, and add it, if the time of settlement is after the average date, and subtract it, if before.

nce a fraction of a day is not considered in determining the average this method of finding the cash balance is not as accurate as that of 589, in which the interest is reckoned on each item separately.

8. A commission merchant sold several bills of goods, on a it of 4 months, as follows: Aug. 16, 1881, $387; Sept. 4, 1, $243.60; Sept. 18, 1881, $637.75; Oct. 28, 1881, $165.50; c. 10, 1881, $856.45. What is the equated time of payment?

Note.—The above account may be averaged by first finding the average e of purchase, and adding the common term of credit, or by finding the e date of each bill separately, and determining the average due date from e dates thus found. Since the months have not uniformly the same numr of days, the results by the two methods sometimes differ by one or more ys, when the common term of credit is expressed in months.

9. Bought goods on 6 months' credit as follows: Feb. 16, 1881, $376.50; Mar. 12, 1881, $287.40; Mar. 19, 1881, $612.87; Apr. 5, 1881, $345.60; Apr. 26, 1881, $134.80; June 1, 1881, $612.35. What is the average time of maturity? How much would balance the account Jan. 1, 1882? How much Oct. 1, 1881?

10. Park and Tilford sold to R. M. Bishop & Co. the following bills of merchandise on 60 days' credit: Feb. 24, $176.82; Feb. 28, $327.49; Mar. 16, $282.75; Mar. 28, $512.14; Apr. 7, $438.36; Apr. 14, $109.70; May 1, $632.65. What is the equated time of payment, and how much would be required to balance the account June 1? How much July 1?

11. The following bills of merchandise were purchased on 4 months' credit: June 1, $237.16; June 18, $146.75; June 30, $333.84; July 5, $416; July 16, $535.62; July 27, $912.33; Aug. 13, $345.60. A note payable in 4 months was given in settlement. What was its date, no allowance being made for the days of grace?

12. Bought goods on 60 days' credit as follows: Aug. 11, $487.60; Aug. 20, $398.30; Sept. 1, $411.26; Sept. 13, $283.36: Sept. 22, $112.43; Sept. 30, $555.55; Oct. 20, $342.48; Nov. 4, $337.64. What is the average due date?

13. What is the average time for the payment of the following bills, each being sold on a credit of 4 months? Feb. 29, $224.37; Mar. 13, $642.50; Mar. 31, $377.65; May 4, $510.10; May 19, $388.84; June 3, $476.25; June 19, $227.30; June 30, $562.75.

14. Bought several bills of goods as stated below:

June 3,	a bill of	$375	on 30 days'	credit.
" 28,	"	420	" 60 "	"
July 16,	"	560	" 4 months'	"
Sept 4,	"	228	" 90 days'	"

What is the equated time of payment?

NOTE.—When the bills are sold on different terms of credit, first find the due date of each bill separately as in the following operation.

OPERATION.—PRODUCT METHOD.

Date of purchase.	Credit.	Due date.	Amount.		Days.		Products.
June 3,	30 days,	July 3,	$375	×	0	=	0
" 28,	60 "	Aug. 27,	420	×	55	=	*****
July 16,	4 *mo.*,	Nov. 16,	560	×	***	=	*****
Sept. 4,	90 days,	Dec. 3,	228	×	***	=	*****
			****			)	****** (** days.

OPERATION.—APPROXIMATE INTEREST METHOD.*

Mo.	Days.		Credit.	Interest.
0	June 3,	$375,	30 days,	$1.875 for 1 *mo.* .187 " 3 days.
0	" 28,	420,	60 "	4.20 " 2 *mo.* 1.68 " 24 days. .28 " 4 "
1	July 16,	560,	4 *mo.*,	11.20 " 4 *mo.* 2.80 " 1 " .933 " 10 days. .56 " 6 "
3	Sept. 4,	228,	90 days,	6.84 " 6 *mo.* .153 " 4 days.
	2)	15.83		
		7.915	7.915)	30.708 (3 *mo.* 26 *da.* after May 31, or Sept. 26.
				23.745
				6.963
				30
			7.915)	208.890 (26 days.

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* See second interest method, Art. 573.

15. **What is the equated time for the payment of the following ills?**

July	5,	1882,	$516.60	on	4 months'	credit.
	28,	"	327.35	"	60 days'	"
Aug.	15,	"	147.84	"	4 months'	"
Sept.	8,	"	485.42	"	30 days'	"
"	25,	"	230.39	"	60 "	"

16. Sold several bills of goods as follows:

May	4,	a bill of	$418.75	on	30 days'	credit.
"	16,	"	322.86	"	60 "	"
June	1,	"	513.44	"	4 months'	"
"	12,	"	118.70	"	60 days'	"
"	30,	"	786.30	"	6 months'	"
July	16,	"	274.85	"	60 days'	"

What is the average time of payment, and how much would alance the account Sept. 1? How much Oct. 1?

17. What is the average time of maturity for the payment of he following bills?

Mar.	4,	1883,	$117.26	on	4 months'	credit.
"	21,	"	97.43	"	30 days'	"
"	29,	"	243.84	"	60 "	"
Apr.	16,	"	376.14	"	4 months'	"
"	30,	"	182.75	"	90 days'	"
May	18,	"	412.50	"	60 "	"
June	1,	"	518.65	"	30 "	"

18. Bought goods of Henry Welsh as follows:

Nov.	13,	1881,	a bill of	$138.42	on	30 days'	credit
"	30,	"	"	416.10	"	60 "	"
Dec.	16,	"	"	324.70	"	30 "	"
Jan.	5,	1882,	"	586.85	"	4 months'	"
"	26,	"	"	234.38	"	60 days'	"
Feb.	12,	"	"	93.60	"	4 months'	"
"	23,	"	"	618.75	"	30 days'	"
Mar.	5,	"	"	374.36	"	60 "	"

What is the equated time for the payment of the whole?

19. Average the following sales:

Sept.	4,	1881,	$187.16	on	6 months'	credit.
"	16,	"	332.40	"	30 days'	"
"	24,	"	512.75	"	6 months'	"
Oct.	5,	"	164.60	"	6 "	"
"	27,	"	187.30	"	6 "	"
Nov.	5,	"	436.75	"	60 days'	"
"	16,	"	126.00	"	6 months'	"

20. Average the following account:

Dec.	1,	1882,	$246.75	on	30 days'	credit.
"	12,	"	312.40	"	60 "	"
"	26,	"	819.46	"	4 months'	"
Jan.	2,	1883,	674.32	"	4 "	"
"	10,	"	126.60	"	60 days'	"
Feb.	4,	"	434.50	"	4 months'	"

575. To find the equated time for the payment of the balance of an account having both debit and credit items.

576. Ex. At what date may the balance of the following account be paid without loss of interest to either party?

Dr. John Roach in account with Geo. H. Stuart. *Cr.*

1882.				1882.			
June 6	Mdse. 30 *da.*	456	00	July 26	Cash.	400	00
" 20	" 60 *da.*	384	00	Aug. 10	"	375	00
July 5	" 3 *mo.*	216	00	" 10	Mdse. 60 *da.*	288	00
" 26	" 3 *mo.*	552	00				

577. Operation.—Product Method.

Due	*Dr.*					Due	*Cr.*				
July 6,	$456	×	0	=	0	July 26,	$400	×	20	=	8000
Aug. 19,	384	×	44	=	16896	Aug. 10,	375	×	35	=	13125
Oct. 5,	216	×	91	=	19656	Oct. 9,	288	×	95	=	27360
" 26,	552	×	112	=	61824		1063				48485
	1608				98376						
	1063				48485						
	545			)	49891 (**92 days after July 6, or Oct. 6**						

ANALYSIS.—First find the due date of each item. For convenience, assume July 6, the earliest due date, as the day of settlement for all the items on each side of the account. (See Art. **569**, Note 2.) If the balance of the account is paid July 6, the assumed date of settlement, there would be a loss to the payer, on the debit side of the account, equivalent to the interest of $376 for 1 day, and a gain on the credit side, equivalent to the interest of $485 for 1 day; or a net loss of $49891 for 1 day, or of $545 for 92 days. Since the loss of interest to the payer by settling the account July 6, is equivalent to the interest of the balance, or the amount paid, for 92 days, it is evident that the day when there would be no loss of interest must be 92 days after July 6, 1882, or Oct. 6, 1882.

If the greater sum of the products had been on the credit side, there would have been a gain to the payer by settling the account July 6, and the day that the balance of the account would commence to draw interest would have been 92 days before July 6, or Apr. 5, 1882.

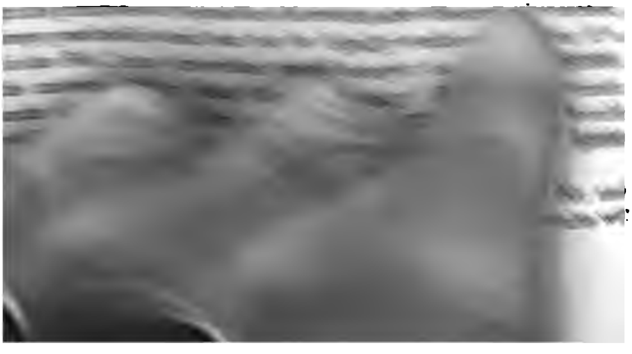
578. RULE FOR THE PRODUCT METHOD.—*First find the due date of each item. Assume the earliest due date as the day of settlement for all the items on both sides of the account. Multiply each item by the number of days intervening between the assumed date of settlement and the due date of the item, and find the sum of the products on each side of the account. Divide the balance (the difference between the sums of the debit and credit products) of the products by the balance of the account. The quotient will be the number of days intervening between the assumed date and the true date of settlement.*

To find the true date of settlement, count forward from the assumed date, when the balance of the account and the balance of the products are on the same side (both debits or both credits); and count backward, when on opposite sides.

NOTES.—1. The rule for counting backward and forward is the reverse of the above, when the latest date or a date after the latest date is taken as the assumed date of settlement.

2. Although the principles of equation of accounts are theoretically correct, they are not always practicable and can not be legally enforced. Thus, if a debt of $4000 is due Feb. 1, no merchant would accept a payment of $3600, Jan. 1, with the understanding that the remaining $400 would remain unsettled 9 months after Feb. 1, or until Nov. 1. The merchant would undoubtedly be willing to allow a discount equivalent to the interest of $3600 for the unexpired time, or 1 month.

3. In finding the equated time, reject the cents when less than 50; and add 1 dollar to the dollars when the cents are more than 50. The results will be sufficiently accurate.



579. OPERATION.—FIRST INTEREST METHOD.*

Dr. Due		Days.	Interest.	*Cr.* Due		Days.	Interest.
July 6,	$456	0	$0.00	July 26,	$400	20	$1.333
Aug. 19,	384	44	2.816	Aug. 10,	375	35	2.187
Oct. 5,	216	91	3.276	Oct. 9,	288	95	4.56
" 26,	552	112	10.304		1063		8.080
	1608		16.396				
	1063		8.08				

60) 5.45) 8.3160 (92 days after July 6, or

.0908 Oct. 6, 1882.

ANALYSIS.—If the account is settled July 6, the assumed date of settlement, Mr. R. would be entitled to a discount on the debit side of $16.396, and Mr. S. on the credit side of $8.08; or, Mr. R. would be entitled to a net discount of $8.316. If, by paying the balance of the account, July 6, Mr. R. is entitled to a discount of $8.316, it is evident that he should be allowed to defer payment until the balance would produce an equivalent interest, or 92 days. Hence, the true date of settlement is 92 days after July 6, 1882, or Oct. 6, 1882.

When the balance of the account and the balance of interest are both due the same party, the equated time is previous to the assumed date of settlement; and, when the balance of the account and the balance of interest are due different parties, the equated time is after the assumed date.

580. In the following operation, the latest due date is assumed as the date of settlement for all the items:

OPERATION.

Due		Days.	Interest.	Due		Days.	Interest
July 6,	$456	112	$8.512	July 26,	$400	92	$6.133
Aug. 19,	384	68	4.352	Aug. 10,	375	77	4.812
Oct. 5,	216	21	.756	Oct. 9,	288	17	.816
" 26,	552	0	.00		1063		11.761
	1608		13.620				
	1063		11.761				

60) 5.45 .0908) 1.8590 (20 days before Oct. 26, or

.0908 Oct. 6, 1882.

ANALYSIS.—If the account is settled Oct. 26, the assumed date of settlement, the payer will be obliged to pay $1.859 interest in addition to the balance of the account. Hence, the date when the balance only may be paid without loss to either party must be 20 days before Oct. 26, 1882, or Oct. 6, 1882.

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[*] See Art. 572.

581. OPERATION.—APPROXIMATE INTEREST METHOD.*

Dr.

Mo.	Days.	Credit.	Interest.
0	June 6,	$456 30 *da.*	$2.28 .456
0	" 20,	384 60 *da.*	3.84 1.28
1	July 5,	216 3 *mo.*	4.32 .18
1	" 26,	552 3 *mo.*	11.04 1.84 .552
		1608	25.788

Cr.

Mo.	Days.	Credit.	Interest.
1	July 26,	$400	$2.00 1.333 .40
2	Aug. 10,	375	3.75 .625
2	" 10,	288 60 *da.*	5.76 .48
		1063	14.348

1608 25.788
1063 14.348
2) 5.45 2.725) 11.440 (4 *mo.* 6 *da.* after May 31, or
2.725 10.900 Oct 6.
.540
30
2.725) 16.200 (6 days.

EXAMPLES.

582. *1.* At what date may the balance of the following account be paid without loss to either party?

Dr. ISAIAH B. PRICE. *Cr.*

1889.				1889.			
May 16	To Mdse.	437	00	May 23	By Cash.	400	00
" 31	" "	324	00	June 16	" "	300	00

2. **Find the average date of maturity for the balance of the following account:**

Dr. WILLIAM C. DOUGLAS. *Cr.*

1907.				1907.			
Jan. 4	Mdse. 30 *da.*	516	[illegible]	Feb. 1	Cash. . .	500	00
" 28	" 60 *da.*	[illegible]	[illegible]	" 4	Note 60 *da.*	300	00
Feb. 4	" [illegible] *mo.*	[illegible]	[illegible]				

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* Se[illegible] 573, and se[illegible] d, Ex. 14, page 244.

3. Average the following account:

Dr. JOSEPH H. WRIGHT. *Cr.*

1882.				1882.			
Mar. 27	Mdse, 4 *mo.*	716	48	Apr. 16	Cash. . .	300	
Apr. 16	" 60 *da.*	325	75	May 2	" . .	400	
May 1	" 4 *mo.*	413	40	July 8	" . .	500	
June 4	" 4 *mo.*	716	87				

4. What is the equated time for the payment of the balance of the following account?

Dr. A in account with B. *Cr.*

1882.				1882.			
Mar. 16	Mdse. 4 *mo.*	444	57	July 1	Cash. . .	400	
" 30	" 60 *da.*	376	82	" 20	" . .	375	
Apr. 20	" 30 *da.*	712	19	Aug. 16	" . .	700	
May 17	" 4 *mo.*	628	75	" 30	" . .	600	
" 28	" 4 *mo.*	419	31				

5. Average the following account. What will be the amount due Jan. 1, 1882?

Dr. C in account with D. *Cr.*

1881.				1881.			
June 16	Mdse. 30 *da.*	517	25	June 19	Note 60 *da.*	1000	
" 28	" 60 *da.*	487	50	July 30	Cash. . .	375	
July 5	" 4 *mo.*	816	75	Aug. 13	Mdse. 4 *mo.*	900	
" 21	" 6 *mo.*	924	30	Oct. 5	Cash. . .	500	
Aug. 12	" 4 *mo.*	317	65				

6. When will the balance of the following account commence drawing interest? How much would be due Mar. 1, 1883.

Dr. ANDREW CARNEGIE, Pittsburg, Pa. *Cr.*

1882.				1882.			
Sept. 4	Cash	100		Aug. 16	Mdse. 4 *mo.*	647	13
" 7	Note 4 *mo.*	900		" 29	" 4 *mo.*	322	85
Oct. 31	Cash	250		Sept. 4	" 4 *mo.*	412	90
Dec. 28	"	600		" 17	" 4 *mo.*	588	33
				" 17	" 30 *da.*	246	12
				Nov. 4	" 4 *mo.*	683	45

7. Find the equated time for the payment of the balance of the following account.

Dr. JAMES B. FARWELL, Chicago, Ill. *Cr.*

1881.					1881.				
Jan.	4	Mdse. 4 *mo.*	637	20	Mar. 16	Cash.	300	00
"	14	" 4 *mo.*	412	87	Apr. 20	"	400	00
"	14	" 60 *da.*	214	35	May 3	"	200	00
Mar.	16	" 4 *mo.*	298	60	" 6	Note 4 *mo.*	800	00
"	28	" 30 *da.*	973	25				

8. Average the following account:

Dr. ARNOLD, CONSTABLE, & Co. *Cr.*

1882.					1882.			
Apr.	4	Mdse. 4 *mo.*	426	32	Apr. 25	Cash.	375	
"	20	" Cash.	387	40	June 30	"	600	
May	13	" 60 *da.*	622	39	Aug. 3	Note 60 *da.*	600	
"	27	" 30 *da.*	584	75	" 15	Cash.	500	
July	5	" 4 *mo.*	224	50	Oct. 31	"	400	
"	16	" 4 *mo.*	838	95				

583. To find the equated time for the payment of the net proceeds (423) of an account sales (424).

584. 1. The sales form the credit side of the account, and the charges and advances the debit side.

2. The charges for transportation, cartage, and other items paid by the commission merchant are considered due at the time of the payment of the same.

3. The commission and other after-charges of the commission merchant are considered due by some at the average *due* date of the sales; and by others, at the average date of the sales. Since the commission is so small compared with the gross sales, in many examples, it makes no difference which date the commission is considered due. Certain merchants enter the commission at the date the account sales is rendered, and, by so doing, produce a result sufficiently accurate.

4. Many commission merchants, when the consignments are not separated and numbered, enter the sales and commission only on the account sales (See Ex. 4, Art. 586), and enter the advances

[illegible]
edit items.

What is the [illegible] time for the payment of the following account sales?

New York, Dec. 1, 1881.

of Seed
For account of William [illegible] & Co.
By [illegible] & Co.

Timothy Seed . 30 da.	[illegible]	79	[illegible]		
Mammoth Cl. Seed 60 da.	[illegible]	450			
Clover Seed , . Cash.	[illegible]	418	[illegible]	947	[illegible]
Charges.					
...portation		60	00		
...ission ... as Dec. 22, 1881. .		47	39	107	39
...ceeds due Dec. 26, 1881. . .				840	46

...average ... date of the sales is Dec. 22, 1881, which is ... the commission.

... to be averaged will now be as follows:

		Cr.			
...1, $60.00	Due	Dec.	4,	1881,	$79.53
" 47.39	"	Jan.	17,	1882,	450.00
	"	Nov.	28,	1881,	418.32

...e above, we find the net proceeds, $840.46, are due Dec.

...on is considered due Nov. 21, 1881, the average date of ...ceeds will be due Dec. 28, 1881.

...ame assumed date, or focal date, be taken in finding the ... the sales as in finding the average due date of the net

EXAMPLES.

586. Find the net proceeds and equated time of the following accounts sales. (Unless otherwise stated, the commission is considered due at the average due date of the sales.)

1. Sales of 400 bbls. flour received per N. Y. C. & H. R. R. R., for account of A. W. ARCHIBALD, Ottumwa, Iowa.

1881.							
May	11	125 bbls. "Kirkwood" cash, . .	$6^{\underline{15}}$	***	**		
"	12	150 " "Iowa" 4 *mo.*, .	$6^{\underline{50}}$	***			
"	18	125 " "Kirkwood" 4 *mo.*, .	$7^{\underline{00}}$	***		****	**
		CHARGES.					
May	3	Transportation and Cartage, . . .		425			
"	4	Inspection, ...		15			
"	18	Storage, ...		45			
		Commission and Guaranty 5%, . . .		***	**	***	**
		Net proceeds due per average, ——, 1881,				****	**

E. & O. E. E. R. LIVERMORE.

NEW YORK, May 20, 1881.

What would be the equated time for the payment of the above proceeds, if the commission and guaranty were considered due at the average due date of the sales? At the average date of the sales? If considered due May 18, the date of the last sale?

2. Account sales of 900 sides hemlock sole leather by MASSEY & JANNEY, for account of GRANT & HORTON, Ridgway, Pa.

1881.		Sides.	Description.	Terms.	Weight.	Price.				
Aug.	14	400	"Ridgway" #7	4 *mo.*	9407	27	****	**		
"	18	300	" #7	4 *mo.*	6875	$27\frac{1}{4}$	****	**		
"	21	200	" #8	30 *da.*	4712	$27\frac{1}{4}$	****	**	****	**
			CHARGES.							
Aug.	2	Transportation $70, Cartage $9, . .					**			
"	3	Inspection, ...					9			
		Commission and Guaranty 5%, . .					***	**	***	**
		Proceeds due ——, 1881, ...							****	**

E. & O. E. MASSEY & JANNEY.

PHILADELPHIA, PA., Aug. 22, 1881.

3. Find the equated time for the payment of the net proceeds of Ex. *25*, Art. **427**, supposing that the merchandise was sold for cash, and that the commission was due at the date given.

4. Sales by JAMES TALCOTT, New York, for account of Phenix Mills, Cohoes, N. Y. March 31, 1882.*

Date.		Cases.	No.	Description.	Time.	Yards.	Price.	Amount.
Mar.	1	2	7619	Fancy Cassimere.	30 *da.*	966^2	1.35	****.**
"	10	4	3475	" "	10 *da.*	1994	1.70	****.**
"	13	3	4157	" "	30 *da.*	1506^1	2.30	****.**
"	17	4	6283	" "	4 *mo.*	1936^3	1.65	****.**
"	26	2	3971	" "	Cash.	978	1.85	****.**
								*****.**
				Less Commission 5%, ...				***.**
				Proceeds due ——, 1882, . .				*****.**

5. Account Sales of merchandise by JOHN F. COOK, for account of Excelsior Packing Co., Cincinnati, Ohio.

1881.									
Oct.	16	50 Bbls. Mess Beef, . .	Cash.	11^{25}	***	**		
"	24	100 " N. M. Pork, . .	"	17^{50}	****			
"	31	25 " Hams 6376 lbs., .	10 *da.*	13½¢	***	**		
Nov.	9	25 " Shoulders 5717 lbs.,	60 *da.*	9¢	***	**		
"	18	75 " C. M. Pork, . .	4 *mo.*	13^{75}	****	**	****	**
		CHARGES.						
Oct.	13	Transportation, ...			325			
"	15	Cartage, ...			37	50		
"	15	Cooperage, ...			15			
"	15	Inspection, ...			13	75		
Nov.	18	Storage, ...			48	75		
		Commission 5%, ...			***	**	***	**
		Net proceeds due ——, 1881, . . .					****	**

E. & O. E. JOHN F. COOK.

NEW YORK, N. Y., NOV. 20, 1881.

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* If the commission is considered due at the average due date of the sales, and since there are no other charges, the net proceeds will be due at the same date.

ACCOUNTS CURRENT.

587. An Account Current is an itemized account of the business transactions between two houses, showing the balance or amount due at the current date. The amount due is sometimes called the *cash balance.*

1. An account current is a transcript of the ledger account with the addition of certain details taken from the books of original entry, and is arranged in a different form.

2. Interest is charged, or not, according to the custom of the business, or the agreement between the parties. This chapter treats only of accounts in which interest is charged. When interest is not charged, the balance due is the difference between the two sides of the account as originally entered in the ledger. The interest may be reckoned according to any of the methods of Art. **437.** In the illustrative example the exact time in days is found, and the days are regarded as 360ths of a year. In the examples for practice, unless otherwise stated, the interest is reckoned on the same basis.

3. Accounts current are rendered by merchants, bankers, and brokers annually (Ex. 2), semi-annually (Ex. 1), quarterly (Ex. 3), or monthly (Ex. 6). Since the interest draws interest after the account is balanced, the oftener the account is balanced, or the interest is added to the account, the greater the amount due. Some merchants render partial accounts current monthly, but do not carry the interest to the main column until the end of the year (Ex. 11). The twelve partial accounts current make, when combined, the complete account current for the whole year.

4. There are three methods in common use for finding the amount due on an account, including interest, at a certain date, all of which are presented in the following illustrative example: 1. By interest; 2. By products; 3. By daily balances.

588. Ex. Find the amount due, including interest at 6%, on the following account Jan. 1, 1882.

Dr. Geo. W. Childs in account with A. A. Low. *Cr.*

1881.			1881.		
Oct. 1	Balance.	1800	Oct. 31	Cash.	1000
" 16	Mdse. 30 *da.*	360	Nov. 19	Note 30 *da.*	600
Nov. 27	" 30 *da.*	432	Dec. 4	Cash.	240
Dec. 18	Bill of H.C. & Co.	420	" 26	"	300

589. Operation.—Interest Method.

	Dr.				*Cr.*		
Due.	Amount.	Days.	Interest.	Due.	Amount.	Days.	Interest.
Oct. 1,	$1800	92	$27.60	Oct. 31,	$1000	62	$10.33
Nov. 15,	360	47	2.82	Dec. 19,	600	13	1.30
Dec. 27,	432	5	.36	" 4,	240	28	1.12
" 18,	420	14	.98	" 26,	300	6	.30
	$3012		$31.76		$2140		$13.05
	2140		13.05				
	872	+	18.71	= 890.71.			

Analysis.—First find the due date of each item of the account. Each item will draw interest from its due date until the day of settlement, or Jan. 1, 1882. The total interest on the debit side of the account is $31.76, and on the credit side, $13.05. The balance of interest, $18.71, is therefore in favor of the debit side, or is due Mr. Low.

Since both the balance of the account ($872) and the balance of interest ($18.71) are due the same party, the net amount due Jan. 1, 1882, is $872 plus $18.71, or $890.71.

If the balance of interest had been on the credit side of the account, the net amount due would have been $872 *minus* $18.71, or $853.29.

Notes.—1. It will sometimes happen that certain items will fall due after the day of settlement. The interest on such items should be transferred to the opposite side of the account. (See Ex. 8.)

2. If the account has been averaged, the amount due at a given date may be found by calculating the interest on the balance of the account from the time it is due to the date of settlement. If the date of settlement is earlier than the average date, subtract the interest from the balance of the account; if later than the average date, add the interest. (See Art. **574**, Ex. 7, Note.)

3. The interest method is generally used in business. Since it gives the interest on each item and is readily understood, it is more satisfactory to those to whom accounts current are sent than the product method. When interest tables are used, it is shorter than any other method.

590. The following is a common form of an account current including interest:

Dr. GEO. W. CHILDS in % current with A. A. LOW. *Cr.*

1881.		Days.	Interest.	Amounts.	1881.		Days.	Interest.	Amounts.
Oct. 1	Balance.	92	27.60	1800.00	Oct. 31	Cash.	62	10.33	1000.00
" 16	Mdse. as Nov. 15.	47	2.82	360.00	Nov. 19	Note as Dec. 19.	13	1.30	600.00
Nov. 27	" " Dec. 27.	5	.36	432.00	Dec. 4	Cash.	28	1.12	240.00
Dec. 18	Bill of H. C. & Co.	14	.98	420.00	" 26	"	6	.30	300.00
1882.					1882.				
Jan. 1	Bal. of Interest.			18.71	Jan. 1	*Bal. of Interest.*		18.71	
					" 1	" " *Account.*			890.71
1882.			31.76	3030.71				31.76	3030.71
Jan. 1	Balance.			890.71					

591. RULE FOR THE INTEREST METHOD.—*First find the due date of each item of the account. Then find the interest on each item from the date it becomes due to the day of settlement. The difference between the sums of the debit and the credit interest will be the balance of interest.*

To find the net amount due, when the balance of interest and the balance of items are on the same side, take their sum; when on opposite sides, take their difference.

592. OPERATION.—PRODUCT METHOD.

Dr.				*Cr.*			
Due.	Am't.	Days.	Products.	Due.	Am't.	Days.	Products.
Oct. 1,	$1800 ×	92 =	165600	Oct. 31,	$1000 ×	62 =	62000
Nov. 15,	360 ×	47 =	16920	Dec. 19,	600 ×	13 =	7800
Dec. 27,	432 ×	5 =	2160	" 4,	240 ×	28 =	6720
" 18,	420 ×	14 =	5880	" 26,	300 ×	6 =	1800
	$3012		190560		$2140		78320
	2140		78320	$872 + $18.71 = $890.71.			
	872		6) 112240				
			$18.706				

ANALYSIS.—By multiplying the number of dollars by the number of days, and taking the sum of the products on each side of the account, we find that the total debit interest is equivalent to the interest of $190560 for 1 day, and the total credit interest to the interest of $78320 for 1 day. The balance of interest is therefore equivalent to the interest of $112240 for 1 day. The interest of $1 for 1 day is $\frac{1}{6}$ of a mill (**446**), and of $112240, 18706 ($\frac{1}{6}$ of 112240) mills, or $18.71. Since the balance of items ($872) and the balance of interest ($18.71) are both due the same party, the net amount due is their sum, or $890.71.

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	14	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	1	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]			[illegible]	[illegible]	[illegible]
[illegible]						[illegible]

[illegible]

[illegible]

2. **What is the net amount due on the following account, July 1, 1882, at 6%?**

Dr. C. H. MILLS in % current with G. F. SWORTFIGUER. *Cr.*

1881.				1881.			
July 1	Balance.	1275	46	Nov. 14	Mdse. 4 *mo.*	587	19
Sept. 13	Draft $1012.	871	52	1882.			
1882.				Mar. 13	" 30 *da.*	612	35
Jan. 4	" $1017.	913	27	Apr. 27	" 60 *da.*	846	93
May 17	" $1024.	345	63	June 3	Cash.	500	00

3. What is the balance of the following account, Apr. 1, 1882, at 6%?

Dr. W. J. HILLIS in account with LANGRAVE SHULTS. *Cr.*

1882.				1882.			
Jan. 16	Dft. M. & C.	937	64	Jan. 1	Balance.	3456	75
" 31	" B. & D.	856	75	" 27	Sales as Mar. 15	1225	19
Mar. 3	" W. & Y.	1749	30	Feb. 4	Mdse as Mar. 6	673	75
" 24	" V. & C.	912	38	" 28	Sales as Mar. 19	2428	35

4. Find the amount due Aug. 1, at 6%, on the account represented in Ex. 7, Art. **574**. (See Note, Ex. 7, Art. **574**.)

5. Find the amount due Oct. 1, 1882, at 6%, on the account represented in Ex. 4, Art **582**.

6. Find the balance due Apr. 1, 1882, at 6%, on the following account current.

PHENIX MILLS in % current with JAMES TALCOTT, New York, Apr. 1, 1882.

Date.	Dr.	Amounts.		Date.	Cr.	Amounts.	
1882.				1882.			
Mar. 1	Balance.	45108	34	Mar. 31	Net Proceeds		
" 16	Draft $676.	1000			of Account		
" 18	" $675.	2000			Sales due Apr.		
" 24	" $678.	5000			26, 1882.	12505	70
" 28	Cotton Bill.	3176	42		(See Ex. 4,		
" 30	Transportation.	875	10		Art. **586.**)		

7. **Find the gain or loss on the following consignment account, taking as the day of settlement Jan. 29, 1881, the day the draft for the balance of the account was drawn and sold and reckoning interest at 6% (365 days to the year).**

Cons. F. L. BRUCKMANN, ♯14.

1880.		*Dr.*	Days.	Interest.		Amounts.	
Apr.	25	Mdse. Net Cash.	279	300	17	6544	72
"	25	Clearance.					20
May	10	Insurance.	***	*	**	40	
1881.							
Jan.	29	Balance of Interest to debit.				***	**
"	29	*Gain.*				***	**
				***	**	****	**
1880.		*Cr.*					
May	7	Draft 18000 Reichsmarks	***	***	**	4258	42
Nov.	20	" 2000 "		*	**	468	75
1881.							
Jan.	29	" 9998 "	0			2368	28
"	29	*Balance of Interest to debit.*		***	**		
				***	**	****	**

8. What was the amount due on the following account Feb. 13, 1881, the estimated due date of a sight draft drawn Jan. 29, 1881, for the balance, reckoning interest at 5% (365 days to the year)?

F. L. BRUCKMANN on account of Consignment ♯14.

1880.		*Dr.*		Days.	Interest.		Amounts.	
Oct.	25	Account Sales	due Jan. 9, 1881	35	44	80	9344	83
Dec.	31	" "	" Mar. 7, 1881				22417	54
1881.								
Feb.	13	*Balance of Interest to credit.*			***	**		
					***	**	*****	**
1880.		*Cr.*						
June	30	Freight	due May 14, 1880	***	**	**	1176	32
May	6	Draft 60 days' sight	" July 18, 1880	***	***	**	8000	
"	6	" 60 " "	" " 18, 1880				10000	
Nov.	19	" 60 " "	" Feb. 1, 1881	**	*	**	2000	
1881.								
Feb.	13	Interest Rm. 22417.54	" Mar. 7, 1881	**	**	**		
"	13	Balance of Interest to credit.					***	**
Jan.	29	Draft at sight *to balance* due Feb. 13, 1881					****	**
					***	**	*****	**

NOTES.—1. The interest on all items falling due after the day of settlement should be entered in the interest column on the opposite side of the account.

Some accountants enter these items of interest on the same side of the account in *red* ink so that they will not be added to the other items, and transfer the "red interest" in one amount to the opposite side.

2. The foregoing represents an account in German marks (reichsmarks) kept in an auxiliary book by a consignor of merchandise to a commission merchant at Hamburg, Germany.

The due dates of drafts, accounts sales, and other items are obtained from the letters from the commission merchant and from accounts sales and memoranda rendered by him. The corresponding consignment account as entered in the books of the consignor is represented in Ex. 7.

9. What was the balance due Jan. 1, 1882, at 6%, on the account represented in Ex. 5, Art. **582.**

10. Find the amount due Mar. 1, 1883, at 6%, on the account represented in Ex. 6, Art. **582.**

11. Calculate the interest Jan. 1, 1883, in the following partial account current, and find the total amounts. (Interest 6%, 365 days to the year.) (See Art. **587,** 3.)

G. D. SLOCUM in account with W. B. MCMECHAN.

1882.		*Dr.*	Days.	Interest.		Amounts.	
May	1	Totals from statement of May 1. 1882.		1387	63	28765	79
"	6	Draft H. B. Claflin & Co.	240	50	71	1285	43
"	9	" Austin, Nichols & Co.	***	**	**	674	89
"	13	" W. H. Schieffelin & Co.	***	**	**	346	27
"	25	" Early & Lane.	***	**	**	418	43
"	28	" Mitchell, Vance & Co.	***	**	**	576	80
				****	**	*****	**
1882.		*Cr.*					
May	1	Totals from statement of May 1, 1882.		973	42	22413	71
"	5	Sales as June 28, 1882.	***	***	**	7316	84
"	12	" " Aug. 1, 1882.	***	**	**	2110	92
"	18	" " July 13, 1882.	***	***	**	13446	85
"	25	Cash.	***	**	**	2000	
				****	**	*****	**

12. **Find the balance due on the following account Feb. 13, 1881. (5%, 365 days to the year.)**

Dr. A. Weingreen & Co., on account of Cons. #25. *Cr.*

Date.			Days.	Interest.		Amounts.		Date.			Days.	Interest.		Amounts.	
1880.								1880.							
Dec.	31	Acc. Sales due Feb. 19, 1881.				22587	89	Aug.	2	Freight.	***	**	**	653	1
								Nov.	19	Draft due Feb. 1, 1881.	**	**	**	18000	
								1881.							
								Feb.	13	Interest Rm. 22587.89.	*	**	**		
1881.								Feb.	13	Balance of Interest.				**	**
Feb.	13	*Balance of Interest.*		**	**			Jan.	29	Draft *to balance* due Feb. 13, 1881.				****	**
				**	**	*****	**					**	**	*****	**

13. Find the net gain or loss on the following consignment account, Jan. 29, 1881. (Interest 6%, 365 days to the year.)

Dr. Cons., A. Weingreen & Co., #25. *Cr.*

Date.			Days.	Interest.		Amounts.		Date.			Days.	Interest.		Amounts.	
1880.								1880.							
June	30	Mdse.	***	***	**	4932	86	Nov.	20	Draft Rm. 18000	**	**	**	4218	75
July	3	Clearance.	***		**		20	1881.							
Aug.	1	Insurance.	***		**	25		Jan.	29	" " 3869	0			916	47
1881.								"	29	*Bal. of Interest.*		***	**		
Jan.	29	Bal. of Interest.				***	**								
"	29	*Gain.*				**	**								
				***	**	****	**					***	**	****	**

14. Find the amount due July 1, 1881, on the account represented in Ex. 7, Art. **582.**

15. What was the balance due Jan. 1, 1883, on the account represented in Ex. 8, Art. **582** ?

16. Find the balance of the following account, Mar. 31, 1882, at 6%.

Dr. James A. Douglas in % current with J. H. Hoyt. *Cr.*

1882.					1882.				
Feb.	28	Balance.	18452	50	Mar.	8	100 N. Y.C.	14537	50
Mar.	2	Draft.	700		"	11	50 H. & St. J.	5162	50
"	11	100 N. W.	14062	50	"	17	Cash.	16000	
"	18	200 H. & St. J.	20875		"	24	100 N. W.	14437	50

STOCKS AND BONDS.

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595. "**Stock**" is a term applied to the share capital of a company, and represents an interest in its property over and above its liabilities, and in the profits of its business after the expenses and interest on its bonds have been paid.

1. A *Dividend* is that part of the profits of a company which is divided among the stockholders, and is a certain amount per share or a certain per cent. of the par value of the stock.

2. The Capital Stock of a company is divided into shares usually of $100 each. Shares of $50 and $25 are called half-stock and quarter-stock respectively.

3. A *Stock Certificate* is a written instrument issued by a company, and signed by the proper officers, certifying that the holder is the owner of a certain number of shares of its Capital Stock.

4. The *Par Value* is the sum for which the shares or certificates were issued, or the amount mentioned on their face. The *Market Value* is the amount for which they can be sold.

596. A **Preferred Stock** is one taking preference of the ordinary stock of a corporation in the payment of dividends.

Thus, the holders of preferred stock of a certain railroad are entitled to 6 per cent. on their stock out of any one year's earnings, before the common stock can receive any dividend. After such payment, the balance of earnings, if any remain, may be divided to the common stock.

Preferred stocks are generally the result of a reorganization of a railroad. For instance, the holders of the common stock may save the road from passing out of their hands by the payment of a certain sum of money, for which preferred stock is issued. In other cases, preferred stocks have been issued in payment of floating or unsecured debts.

In some reorganizations, there are two or more classes of preferred stock.

597. A **Bond** is the obligation of a Corporation, City, County, State, or Government to pay a certain sum of money at a certain time, with a fixed rate of interest payable at certain periods.

1. Bonds of business corporations are usually secured by a mortgage on the whole or some specified portion of their property.

2. *Coupon Bonds* are those with small certificates attached representing the different installments of interest payable at the different periods specified, during the time the bond has to run, which are to be cut off and collected from time to time as the interest becomes due.

3. Bonds are also issued without coupons, in what is known as the registered form. In this case the bond is only payable to the registered owner, or his assignee, and the interest is paid by check or in cash, to the owner or his attorney.

4. Bonds are sometimes issued with coupons attached payable to bearer, but the principal of which may or may not be registered at the choice of the owner.

5. Bonds are known as First Mortgage, Second Mortgage, etc., Consols, Sinking Fund, Income or otherwise, according to their priority of lien, the class of property upon which they are secured, or other characteristics. *Income bonds* are generally bonds on which the interest is only payable if earned, and ordinarily are not secured by a mortgage.

Bonds are also named from the rate of interest they bear, or from the dates at which they are payable or redeemable, or from both; as, U. S. 4's 1907, Virginia 6's, Western Union 7's, coupon, 1900, Lake Shore reg. 2d, 1903.

6. In speaking of the income from bonds the term "interest" is used, as it is the consideration received for the use of money loaned, while that derived from an investment in stock is called "dividend," because it is money divided to the stockholders from the profit of carrying on the business, after the fixed charges have all been paid.

7. Bonds are issued in denominations of $50 to $50000.

GOVERNMENT BONDS.

598. 4½'s of 1891. Redeemable at the option of the Government after Sept. 1, 1891. The amount outstanding July 1, 1887, was $250,000,000. Interest is payable Mar. 1, June 1, Sept. 1, and Dec. 1.

599. 4's of 1907. Redeemable at the option of the Government after July 1, 1907. The amount outstanding July 1, 1887, was $737,800,600. Interest is payable Jan. 1, Apr. 1, July 1, and Oct. 1.

600. Refunding Certificates. These certificates are of the denomination of $10, bear interest at 4%, and are convertible at any time, with accrued interest, into 4% bonds. The amount outstanding July 1, 1887, was $175,250.

601. Currency 6's. These bonds were issued to aid in the construction of the Pacific railroads. Principal and interest are payable in lawful money of the United States. Payable 30 years after date, and maturing at different dates from 1895 to 1899. The amount outstanding July 1, 1887, was $64,623,512, all registered.

602. Denominations. The coupon bonds of the various issues are in denominations of $50, $100, $500, and $1000. The registered bonds are in denominations of $50, $100, $500, $1000, $5000, and $10000. Of the 4½'s of 1891, and the 4's of 1907, there are, in addition to the above, registered bonds of the denominations of $20,000 and $50,000.

603. Coupon bonds, being payable to bearer, pass by delivery without assignment, and are therefore more convenient for sale and delivery than registered bonds, which must be assigned by the party in whose name they are registered. The interest coupons being also payable to the bearer will be cashed by any bank or banker in any part of the United States.

1. The interest on registered bonds is paid by checks, made to the order of the registered owner and sent to him by mail. These checks, when properly endorsed, can be collected and cashed through any bank or banker.

2. Coupon bonds may be converted into registered bonds of the same issue, but there is no provision of law for converting registered bonds into coupon bonds.

604. The quotations of government bonds at the New York Stock Exchange were as follows, July 1, 1887:

	Bid.	Asked.		Bid.	Asked.
U. S. 4½'s, '91 reg.	109⅜	109⅝	U. S. cur. 6's, 1896	126¾	——
U. S. 4½'s, '91 c.	109½	109¾	U. S. cur. 6's, 1897	129¾	——
U. S. 4's, 1907 reg.	128¼	128½	U. S. cur. 6's, 1898	132¾	——
U. S. 4's, 1907 c.	128¼	128½	U. S. cur. 6's, 1899	134¾	——
U. S. cur. 6's, 1895	123½	——	Dist. of Col. 3–65's	121½	——

All Government Bonds are dealt in and quoted "flat"—*i. e.*, the quoted market price is for the bond as it stands at the time, including the accrued interest—except that after the closing of the transfer books* the registered bonds are quoted ex-interest—that is to say, the interest then coming due belongs to the holder of the bond at the time of the closing of the books, and does not go with the bond to the purchaser.

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* The transfer books of U. S. registered bonds are closed for the month preceding the day on which the interest is paid.

During the period in which the transfer books remain closed, the quoted price of coupon bonds includes the accrued interest falling due on the first of the ensuing month, while that of registered bonds does not. If, in the month of December, when the books are closed preparatory to the payment of the interest due January 1, the coupon Four per cents are quoted at 118, the equivalent for the registered bonds of the same issue would be 117, the three months' interest being equal to one per cent.

NEW YORK STOCK EXCHANGE.

605. The **New York Stock Exchange** is an unincorporated body of brokers, whose business is to buy and sell stocks, bonds, and other representatives of value.

1. The floor of the Exchange is open for business from 10 A. M. to 3 P. M. There are two regular calls of Stocks daily; three of State and Railroad Bonds; and three of United States Bonds. Transactions are not, however, confined to the regular calls, but are continually taking place on the floor of the Exchange between the hours named above.

2. In Wall Street, there are what are known as strictly commission houses, who take and execute orders for securities, charging the regular commission, and, when customers desire, loaning funds on the securities on a deposit of 10 to 20% of market value being made. This is what is known as buying on a margin (**609**), where the customer intends to sell soon again, and merely buys for speculative purposes. Such houses will usually sell stocks "short" (**610**, 10) for their customers on a similar margin.

There are other houses which make no advances, and require customers to pay outright for securities when bought.

There are also houses which combine a banking and brokerage business, taking deposits and loaning money on any securities marketable at the Exchange, and buying and selling stocks on commission. Some of these extend the privilege of marginal business to their customers, while others do not.

There are other members who operate exclusively for their own account.

606. **Quotations** are made at so much per cent. on the basis of a par value of $100 per share of stock, except in the case of mining securities and Sutro Tunnel stock, which are quoted at so much per share, without reference to their par value.

For example, the par value of Morris and Essex stock is $50, but the quotation, if the stock were worth just par in the market, would be 100%; or, if the quotation is 110, it means $110 for $100 worth of the par value, which, in the case of this stock, would be two shares, while in the case of a stock the par value of which is $100 per share, it would be for one share.

On the other hand, if Sutro Tunnel, the par value of which is $10 per share, is quoted at 2, it means $2 per share; and, in like manner, if Homestake, the par value of which is $100, is quoted at 30, it means $30 per share.

607. Commission.—The regular charge for buying and selling securities dealt in at the Stock Exchange, except mining stocks, is one-eighth of one per cent. ($\frac{1}{8}$%) on par value, or $12.50 on 100 shares of stock of the par value of $100 each.

608. Stocks are usually bought or sold either "cash," "regular way," "seller three," or "buyer three." A stock sold "cash" is deliverable the day sold; a stock sold "regular way" is deliverable the next day, or bought "regular way" is to be paid for the next day. Where nothing else is specified, "regular way" is always understood. When a stock is reported as bought "seller three," it is meant that the seller of the stock can deliver it on either of the three days at his option, but is not required to deliver until the third day. On the other hand, when a transaction is made "buyer three," the buyer can demand delivery of the stock at any time within three days, but must take it and pay for it by the third day.

1. Transactions on any of the above terms carry no interest.
2. If the option is over three days, six per cent. on the selling value of the stock is paid by buyer to seller.
3. One day's notice is required of intention to terminate an option of a longer period than three days.
4. The Stock Exchange does not recognize any contract for over sixty days. Should a stock pay a dividend during the pendency of a contract, the dividend belongs to the purchaser of the stock, unless otherwise previously agreed.

609. A **Margin** is a deposit made with a broker, by a person who wishes to buy or sell stock for speculation to enable the broker to "carry" the stock and protect himself against loss. It is usually 10% of the par value of the stock.

1. A person desiring to speculate in stocks, deposits with his broker $1000 as a margin, and directs him to purchase 100 shares of a certain stock at 90. The broker would pay for the stock $9000, $1000 of which being furnished by the speculator, and the remainder, $8000, by the broker. The broker charges legal interest on the amount furnished by him for "carrying" the stock. (See Ex. 52, Art. **611**.)
2. The margin deposited with the broker is simply to protect the broker against losing any money should the stock move in the wrong direction. In case of its so doing, the margin must be made good by the deposit of an additional amount, otherwise the broker will sell the stock to protect himself from losing any of the money he has advanced.

610. Explanation of Words and Phrases used in Wall Street.

1. ***Bear.*** An operator who is "short" of stock. He wishes to buy at a lower rate, and tries to depress the price of the stock of which he is "short."

2. ***Bull.*** An operator who is holding stock for an advance. He is said to be "long" of the stock. Bulls try to advance the prices of the stocks of which they are "long."

3. *b. 3* (*Buyer 3*), *10, 20, 30, etc.* Meaning at the buyer's option, within three days, ten days, etc. When in a stock transaction, the buyer has the privilege of taking the stock at any time during the number of days mentioned. In buyer's options, when the option is for more than three days, six per cent. interest is charged the buyer, and the seller is entitled to one day's notice.

4. *b. c., "between calls."* The sale not taking place on the call of the stock, but after the first call and before the second.

5. *Collaterals.* Stocks, bonds, notes, or other value given in pledge as security, when money is borrowed.

6. *Cover, to "cover one's shorts."* Where stock has been sold short, and the seller buys it in to realize his profit, or to protect himself from loss, or to make his delivery. This is "covering short sales."

7. *Differences.* When the price at which a stock is bargained for and the rate on day of delivery are not the same, the broker against whom the variation exists, frequently pays the "difference" in money, instead of furnishing or receiving the stock.

8. *Ex.-Div., Ex.-Dividend.* When the price of a stock does not include, and the stock does not carry to the buyer a recently declared dividend.

9. *Seller, 3, 10, 20, 30, etc.* Sold deliverable at seller's option, within the number of days named. When seller's options are for more than three days, the buyer pays six per cent. interest, unless "flat" is specified in the contract, and the seller must give one day's notice of delivery.

10. *Short.* When one has sold stock which he does not own, hoping to realize a profit by buying in at lower prices, he is said to be "short."

11. *Watering a Stock.* The act of increasing the quantity of a stock without a corresponding increase in the value of the property which it represents. This is usually done in the reorganization of a railroad, or in the consolidation of two or more railroads.

EXAMPLES.

611. *1.* A bank with a capital (**595**) of $250,000, declares a semi-annual dividend of 3½%. What is the amount of the dividend, and how much will a stockholder receive who owns 16 shares of $100 each (**595**, 2)?

2. An insurance company divides among its stockholders $18000. What is the rate of the dividend, the capital stock being $225000? How much is paid to Mr. A., who has a certificate (**595**, 3) for 25 shares?

3. A gas company declares a dividend of 5%, and divides among its stockholders $125000. What is its capital stock ?

4. The board of directors of a mining company declared a dividend of $100,000, being five cents per share (par value $10) on the capital stock of the company. What was the capital stock, and in how many shares was it divided ? The dividend was what per cent. of the capital stock ?

5. An installment of 10% was assessed and called on the capital stock of a new railroad company. How much was paid by Mr. B., who had subscribed for 50 shares (par value $100) ?

6. A railway company, whose capital stock is $1,750,000, declares a dividend of 3½ per cent. What was the amount of the dividend ?

7. The Union Pacific Railway paid to its stockholders, in 1879, $2,204,700. What was the par value of its stock, the rate of the dividend being 6% ?

8. A quarterly dividend of 3½% was declared by a manufacturing company. What was the capital stock, the amount of the dividend being $2100 ?

9. What is the market value of 200 shares of stock at 116⅝ ?

10. How many shares of W. U. Tel. can be bought for $43725 at 79½% ?

11. What is the total par value (**595**, 4) and the total market value of 100 shares Lake Shore at 118⅝ (**606**), 300 sh. N. J. Central at 89¾, 500 sh. W. U. Telegraph at 78⅛, 200 sh. U. S. Express at 73¼, 500 sh. N. Y., L. E. & W. com. at 40⅞, and 800 sh. N. Y., L. E. & W. pref. (**596**) at 90⅜ ?

12. What is the cost of 250 shares Tex. & Pac. at 50⅝ and 100 shares Ohio & Miss. pref. at 104, brokerage ⅛% (**607**) ?

13. What are the proceeds of 600 shares Morris and Essex (half stock, **595**, 2) sold through a broker at 121½ ?

14. What are the proceeds of the following stocks sold through a broker ? 200 shares Union Pacific at 117⅜, 2000 shares N. Y., O. & W. at 27¼, 800 shares A. & T. H. pref. at 88, and 600 shares Chi. & Alton at 131¼.

15. Find the cost of 10 shares Manhattan Bank at 135, $5000 Erie 7's (**597**, 5) cons. gold bonds (**597**) at 128, $1000 Toledo and Wabash 2d (**597**, 5) at 108⅞, $5000 C. R. I. & P. 6's, 1907, coupon (**597**, 2) at 129, and $5000 Ohio Southern Income (**597**, 5) *at 45, usual brokerage.*

16. Find the proceeds of $15000 U. S. 4's, registered, 1907 (**599**), b. 3, at 117½, and $10000 U. S. 4½'s coupon (**598**) at 114½, usual brokerage.

17. How much must be invested in U. S. 4½'s, 1891, to produce a quarterly income of $675, bonds selling at 114⅜ ?

18. When Ohio 6's are sold at 109½, what is received for six $500 bonds, brokerage ¼% ?

19. When Pittsburg, Fort Wayne and Chicago 2d 7's, 1912, are worth 135, what will $12000 in bonds cost ?

20. How many $500 bonds shall I receive for $4735 invested in U. S. 4's at 118⅜ ?

21. How much must be sent to a broker that he may purchase $8000 U. S. 4½'s at 102¾, commission ⅛% ?

22. An executor sold Central of New Jersey stock at 52⅝, and purchased with the proceeds $42000 in U. S. 4's, 1907, at 100⅞. What was the par value of the stock sold, usual brokerage ?

23. A broker bought on his own account 200 sh. Nor. Pac. pf. at 69⅞, and sold the same the same day at 73¾. What was his gain ?

24. How many shares of Ill. Cen. bought at 129⅜, and sold at 132⅜, usual brokerage, will produce a gain of $1375 ?

25. What income will be produced by investing $235250 in 4% bonds at 117⅝ ?

26. The common stock of a railroad company is $46,000,000 and the preferred stock (**596**) $8,000,000. The company declares a dividend of 3½% on the preferred stock and 2% on the common stock. What is the surplus, if the net earnings are $1,317,645 ?

27. Bought June 4, 800 sh. Ohio & Miss. pref. at 35½, s. 30. The stock was delivered June 24. What was the amount paid including interest (**610**, 9) ?

28. Bought May 16, 200 sh. Lake Shore at 116¾, b. 60, ar called for the stock July 5. What was the cost including interest

29. Jan. 10, sold 100 sh. Phil. & Read. at 65¼, s. 3. Jan. 1 the stock was quoted at 68½. How much was the differenc (**610**, 7) paid by the seller in settlement ?

30. How much must one invest in 4½% bonds, when they ar selling at 121½, in order to have an income of $1800 per year ?

31. A man invests $28020 in 4% stock at 116⅝, brokerage ⅛% What is his income ?

32. I sell 200 sh. H. & St. J. pf. at 111⅜, and $10000 N. Y. Elevated 1st mortgage bonds at 119. What will be the net proceeds of the sale, allowing usual brokerage?

33. A person sells 26 shares 3½% stock at 86, and loans the proceeds at 5%. What is the increase in his income?

34. July 1, 1887, the interest-bearing debt of the United States was as follows: 4½'s, $250,000,000; 4's, $737,975,850; 3's $14,000,000; Pacific R. R. 6's, $64,623,512. What is the total interest for one year?

35. The capital stock of a railroad company was "watered" (**610**, 11) by declaring a stock dividend of 10%. If the market value of the old stock was 110, what should be the value of the new stock?

36. Jan. 1, 1882, the A. & B. R. R., having a capital stock of $20,000,000, was consolidated with the B. & C. R. R., having a capital stock of $32,000,000. The new company was organized under the name of the A., B., & C. R. R. For every share of the A. & B. R. R. there was issued 1⅕ shares of the new stock, and for every share of the B. & C. R. R. there was issued 1⅛ shares of the new stock. What was the capital stock of the new company, and how much was the stock "watered"?

37. Before the consolidation, the stock of the A. & B. R. R. was worth 1.20 in the market, and the stock of the B. & C. R. R. 90. What should be the quotation of the new stock?

38. During the year 1881, the A. & B. R. R. divided among its stockholders $1,600,000, and the B. & C. R. R., $1,920,000. During the year 1882, the new company divided an amount equal to the total dividends of the two companies in the preceding year. What were the rates of the dividends of the two companies in 1881, and the rate of the dividend of the consolidated company in 1882?

39. Mr. A. had 10 shares of the A. & B. R. R., and 16 shares of the B. & C. R. R. What was the total amount of his dividend in 1881? How many shares of the new stock did he receive, and what was the amount of his dividend in 1882?

40. The gross earnings of the M. C. R. R. for the year ended Dec. 31, 1880, were $9,085,749; operating expenses and taxes, $5,738,751; interest and rentals, $1,586,410. After declaring a dividend, there was a surplus of $261,532. What was the rate of the dividend, if the amount of the stock was $18,738,200?

41. A gentleman bought bank stock, paying regular annual dividends of 6%, at 120. What was the rate per cent. of his income, or what per cent. did he receive on the money invested?

ANALYSIS.—Since dividends are reckoned on the par value of the stock, the dividend on 1 share of $100 would be $6. Since each share costs $120, and pays $6 income, the per cent. would be $6÷$120, or 5%.

NOTE.—The above analysis will not apply to bonds that mature at a certain fixed time, unless the investor expects to sell the bonds before maturity at the cost price. If 6% bonds that mature in 1899 are purchased in 1889 at 120, and are sold at the same rate before maturity, they will pay 5% on the investment, or cost. If the bonds are held until maturity (1899), or for 10 years, the owner would receive from the government the par value only, or $100 for a bond of that amount, and the bonds would yield less than 5%. If 6% bonds, maturing in 10 years, are purchased at $1.07\frac{72}{100}$ and held until maturity, they will pay 5% on the investment. (See Ex. 59.) If 6% bonds, that mature in 2 years, are purchased at more than 112, there would be a loss of interest to the purchaser instead of a gain.

42. Which is the better investment, stock paying a regular annual dividend of 5% and bought at 80, or stock paying 8% dividends, and bought at 120?

43. If insurance stock paying regular dividends of 10% annually is bought at 137¼, brokerage ¼%. what per cent. of income will it produce?

44. Which investment will produce the greater annual income and how much, $20,000 invested in Chemical Bank stock at 2000 which pays dividends of 15% every two months, or the same amount invested in Chatham Bank stock at 125 which pays regular semi-annual dividends of 3%?

45. What rate can you afford to pay for stock paying regular annual dividends of 10%, in order to realize 6% on the investment?

46. At what price must 8% stocks be purchased to afford 5% on the investment? To afford 6%?

47. Stocks bought at 80 pay regular dividends of 5%. What is the rate per cent. on the investment? At what rate should they be purchased to afford 4% on the investment? To afford 8%?

48. Purchased 400 shares Lake Shore at 118½, and 200 shares Chesapeake and Ohio 2d pref., at 24⅜. Sold the Lake Shore at 113⅝, and the Chesapeake and Ohio at 22¾. What was the loss, usual brokerage, no interest?

49. The gross earnings of the Union Pacific Railway Co. for 1879, were $13,201,077.66; the operating expenses were $5,475,503.44. What were the surplus earnings, and what per cent. of the gross earnings were the operating expenses?

50. A synopsis of the report of the N. Y. C. & H. R. R. R. for its fiscal year ended Sept. 30, 1881, is as follows: Gross earnings from passengers, $6,958,038; from freight, $20,736,749; from miscellaneous, $4,653,608; expenses, $19,464,786; interest, rentals, and taxes, $4,990,783. What was the surplus for the year after the declaration of a dividend of 8% on a capital stock of $89,229,300? The expenses were what per cent. of the total earnings?

51. The L. S. & M. S. Railway reported as follows for the year ended Dec. 31, 1880: Gross earnings, $18,749,461; operating expenses and taxes, $10,418,105; interest, rentals, dividend on guaranteed stock, and $250,000 for the sinking fund, $3,000,374. After paying a dividend of 8%, there was a surplus for the year of $1,373,662. What was the amount of the dividend, and the capital stock?

52. July 26, a broker received from a customer a remittance of $1000 as a margin (**609**) and purchased for him 100 shares of St. Paul Common at 59. On Aug. 2, the broker sold the stock at 64½. What was the customer's profit?

OPERATION.

	Dr.					
July 26.	To 100 shares St. Paul Com. 59 .	$5900				
	Commission ⅛% . .	12.50	5912	50		
Aug. 2.	Interest $5912.50, 7 days . . .		*	**	****	**
	Cr.					
July 26.	By margin deposited		1000			
Aug. 2.	" 100 shares St. Paul Com. 64½ .	$6450				
	Commission ⅛% . .	12.50	6437	50		
Aug. 2.	Interest $1000, 7 days		*	**	****	**
	Balance				****	**

The profit is equal to the balance less $1000, the original deposit.

53. Aug. 30, a broker purchased for the account of a customer 300 shares Northwestern Railroad stock at 78. He deposited as a margin $3000. On Sept. 22, the stock was sold at 74¾. What was the loss? (Interest 6%, usual commission.)

54. May 10, a speculator deposited with his broker $5000 as a margin, and directed him to purchase for his account 500 shares N. Y., L. E., & W., pref. at 90⅜. May 20, the stock was sold at 94⅛. What was the gain, interest 6%, usual brokerage?

55. Sept. 10, I deposited with my broker $5000 as a margin, and he purchased for me 200 sh. Cen. Pac. at 90½, 200 sh. Morris & Essex (half stock) at 122¼, 200 sh. Tex. & Pac. at 49¾. The stocks on Sept. 30 were quoted as follows: Cen. Pac. 80¾, Morris & Essex 120⅛, Tex. & Pac. 41⅝. How much should I have deposited with my broker to make my margin of 10% good, and to cover commission for buying and selling, and interest? If I had been unable to have made an additional deposit, and the broker had "sold me out," what would have been my loss?

56. An operator, supposing Erie would decline in value, ordered his broker to sell short 100 shares at 50, and at the same time deposited with him as a margin $1000. The broker on receiving the order sold for his account 100 shares at 50, and borrowed the stock for delivery. When the market price declined to 45, he ordered the broker to "cover his short sale" (buy the stock for delivery), and return the stock to the party from whom it was borrowed. What was the gain, usual brokerage?

OPERATION.

Cr.		
By margin deposited	$****.**	
" 100 shares Erie borrowed and sold at 50. . .	****.**	[illegible]
Dr.		
To 100 shares Erie bought and returned at 45 .	$****.**	
" commission for selling the stock ⅛% . . .	**.**	
" " " buying and returning the stock ⅛%.	**.**	****.**
" amount to credit.		****.**

The net profit equals the balance less the margin deposited.

NOTE.—There is no interest charged on short sales, but it sometimes happens that a small bonus has to be paid for the use of the borrowed stock.

57. A broker sold "short" for me 400 sh. C. B. & Q., at 135¾, and 100 sh. C. R. I. & P., at 132½. My "short" sale on C. B. & Q. was "covered" at 131½, and C. R. I. & P. at 133¾. What was my net profit, usual brokerage? (No interest.)

58. Sold Aug. 11, 500 shares Chicago & Alton, s. 3, at 94½, and covered my short sale Aug. 14, at 91. What was my profit, allowing the usual brokerage?

59. At what price may 6% bonds, maturing in 10 years, be purchased, so that the investment will pay 5%?

NOTE.—Tables have been constructed on various plans, and different methods are used by bankers and financiers, for the solution of problems relating to bond investments; two of which are given below.

ANALYSIS.—1. In the following method, it is presumed that the accruing interest is not reinvested, but that a sufficient part of it is set aside as a sinking fund to make up the amount which was originally paid out as premium.

A $1000 bond in 10 years at 6% would amount to $1600 ($1000 + 10 × $60). $1 in 10 years at 5% would amount to $1.50. To amount to $1600, the principal, or the amount paid for the bond, must be as many times $1 as $1.50 are contained times in $1600, or $1066.66⅔ (106⅔%).

If a $1000 bond is purchased at 106⅔, it will be necessary to set aside as a sinking fund each year $6.66⅔ (⅔%) to make up the premium in 10 years. The annual interest, $60, less $6⅔, the annual sinking fund, is $53⅓, which is 5% of $1066⅔, the cost of the bond or the amount invested.

If the amount set aside as a sinking fund is placed at interest, either simple or compound, 6% bonds, maturing in 10 years and purchased at 106⅔, would pay a little more than 5%.

2. The following method anticipates compound interest throughout; *i. e.*, the interest is immediately reinvested at compound interest.

The holder of a $1000 bond would receive $60 interest annually, and $1000, the face of the bond, in 10 years. If money is worth 5%, the several interests in the 10 years at compound interest would amount to $754.674 ($1 placed at compound interest at the beginning of each year would amount in 9 years to $11.5779 (**484**). $11.5779 plus $1 of the last interest = $12.5779. $60 would amount to 60 times $12.5779, or $754.674). $1000, the principal, plus $754.674, the compound amount of the interest, equals $1754.674, the total value of the bond at maturity, money being worth 5%. The present worth of $1754.64, due in 10 years, at 5% compound interest, is $1754.64 ÷ $1.6289 (**483**), or $1077.19. Hence the bonds must be purchased at $1.07\frac{72}{100}$ to pay 5% on the investment. (See Ex. 41. Note.)

60. What must I pay for 6% bonds, maturing in 15 years, that my investment may yield 3½%? (Both methods.)

61. 6% bonds, maturing in 10 years and bought at 106⅔, pay what per cent. on the investment? (See 1st analysis, Ex. 59.)

ANALYSIS.—A $1000 bond would amount in 10 years at 6% to $1600. If $1066.66⅔ is paid for the bond, the net interest for 10 years is $1600 − $1066.66⅔, or $533.33⅓; and for one year $533.33⅓ ÷ 10, or $53.33⅓. An income of $53.33⅓ on an investment of $1066.66⅔ is equivalent to 5% ($53.33⅓ ÷ $1066.66⅔).

62. What rate of interest do I receive on my investment, if I buy 7% bonds maturing in 20 years at 133⅓?

TAXES.

612. A **Tax** is a sum of money assessed on persons and property to defray the expenses of a State, county, town, corporation, or district.

1. In certain States all citizens above 21 years of age are required by law to pay a certain tax on the person. This tax is called a *Capitation* or *Poll Tax.*

2. The expenses of States, counties, towns, etc., are paid by a direct tax upon the property or polls of the same. The methods of assessing taxes differ in the several States. In some States, a certain percentage of the whole tax is assessed upon the polls, while in others the poll tax is a fixed amount for each citizen. In certain States, the whole tax is paid by the owners of the property of the same. In the States of New York, Pennsylvania, and some other States, there is a direct tax levied upon certain corporations doing business in the State.

3. The expenses of the United States government are paid by duties on imports; the internal revenue (the tax upon distilled spirits, fermented liquors, tobacco, snuff, and cigars); sales of public lands; tax on circulation of national banks; customs fees, fines, penalties, and forfeitures; fees, consular, letters patent, and land; profits on coinage, etc.

613. Real Estate is fixed property; as land, houses, etc.

614. Personal Property is movable property, as money, stocks, bonds, mortgages, furniture, merchandise, etc.

615. An **Assessor** is a person appointed or elected to estimate the valuation of all property liable to taxation.

616. A **Collector** or **Receiver** of taxes is a person appointed or elected to collect or receive the taxes of a city, town, village, or district.

Collectors receive a commission on the amount collected or a fixed salary

EXAMPLES.

617. *1.* For the fiscal year 1879, the N. Y. State tax levy was at the rate $2\frac{863}{1000}$ mills. How much would this rate produce, the valuation of the taxable property being $2,686,140,000?

2. The rate of taxation of a certain county was 3¼ mills, and the amount of the tax $40,653.48. What was the valuation of the property?

3. The following were the rates of taxation of New York for State purposes, 1880: —schools, 1.085 mills; general purposes, 1.475 mills; new capitol, .6 mills; canals, .34 mills. What was the total rate of taxation, and how much was raised by a county whose valuation was fixed by the State Board of Equalization at $11,047,534? How much was raised for school purposes?

4. Taxes were levied in a certain town for the following purposes:—support of poor, $2,000; roads and bridges, $500; accounts audited by town auditors, $2,876.10; accounts audited by supervisors, $19.48; county expenses, $9,774.72 less a surplus of $6,055.90 in the county treasury; state and school tax, $15,079.88; surplus tax, $868.98. What was the rate of taxation, the total valuation of the property, as made by the town assessors, being $4,321,252? What was the tax of Mr. A., whose valuation was $7,300?

Note.—To save labor in the calculation of taxes, a table similar the following is usually prepared by the accountant.

Tax Table.—Rate, 5.8 mills on $1.

	0	1	2	3	4	5	6	7	8	9
1	.0580	.0638	.0696	.0754	.0812	.0870	.0928	.0986	.1044	.1102
2	.1160	.1218	.1276	.1334	.1392	.1450	.1508	.1566	.1624	.1682
3	.1740	.1798	.1856	.1914	.1972	.2030	.2088	.2146	.2204	.2262
4	.2320	.2378	.2436	.2494	.2552	.2610	.2668	.2726	.2784	.2842
5	.2900	.2958	.3016	.3074	.3132	.3190	.3248	.3306	.3364	.3422
6	.3480	.3538	.3596	.3654	.3712	.3770	.3828	.3886	.3944	.4002
7	.4060	.4118	.4176	.4234	.4292	.4350	.4408	.4466	.4524	.4582
8	.4640	.4698	.4756	.4814	.4872	.4930	.4988	.5046	.5104	.5162
9	.5220	.5278	.5336	.5394	.5452	.5510	.5568	.5626	.5684	.5742

5. Find from the above table the tax on $16750.

OPERATION.

Tax on $16000 is $92.80
" 750 " 4.35
" 16750 " 97.15

Analysis.—By looking in the table opposite 1 and under 6, we find that the tax on $16 is $.0928, and by removing the point 3 places to the right, we find the tax on $16000 to be $92.80. In the same manner, the tax on $750 is found to be $4.35. The tax on $16750 is $92.80 plus $4.35, or $97.15.

6. How much was paid by Mr. B. on an assessment of $6400, the collector charging a commission of 1% additional ? (Use table.)

7. Mr. D. being delinquent was charged 5% additional. How much was he obliged to pay on a valuation of $9500 ?

8. What was the total tax, including commission of 1%, of Mr. C., whose real estate was assessed at $24000, and personal property at $15500 ?

Find the tax on
9. $7200 at 8.4 mills on $1.
10. $27500 at $2.174 on $100.
11. $4800 at $21.871 on $1000.
12. $9600 at 6.8 mills on $1.

Find the tax on
13. $248000 at $2.131 on $100.
14. $14100 at $16.476 on $1000.
15. $240500 at $0.889 on $100.
16. $13500 at $29.142 on $1000.

17. A has 6 lots worth $1200 each; B has 8 lots worth $1500 each; and C has 10 lots worth $1000 each. Divide a tax of $1360 for street improvements between them.

18. A rate of $6.34 on $100 produces a tax of $4216.10. What is the property assessed at ?

19. A pays $17.25 more tax than B, their valuations being equal. Living in different towns, the rates of taxation are $3.176 and $3.291 on $100 respectively. What is the valuation of their property ?

20. The cost of a new school house was $3800. What was the rate of the tax on $100, the valuation of the district being $325000 ?

21. If the assessor's valuation of certain property is 60% of its actual value, and the tax is $2\frac{3}{4}$% of the assessment, the tax is what % of the actual value ?

22. In the City of Brooklyn, N. Y., the following is the rule regarding the payment of taxes :

> Rebate at the rate of $7\frac{3}{10}$% per annum allowed on payments made during the month of December for the unexpired portion thereof. On all bills paid after Jan. 1, interest at the rate of 9% per annum will be added, to be computed from Dec. 1 to the date of payment.

According to the above rule, how much tax was paid Dec. 16, by Mr. A., the valuation of whose property was $7500, the rate of tax being $2.376 per $100 ? How much was paid by Mr. B., on a valuation of $12500, Mar. 26 ? (365 days to the year.)

23. What is the total tax on 8375 pounds tobacco at 8c.. 4360 gallons distilled spirits at 70c., 2165 barrels beer at $1 ?

DUTIES.

618. Duties or **Customs** are taxes assessed by the Government upon imported merchandise for the purpose of revenue for the support of the government and for the protection of home industry.

1. The waters and shores of the United States are divided into collection districts; in each of which there is a port of entry and one or more ports of delivery. Thus, the district of Boston and Charlestown comprises all the waters and shores within the counties of Middlesex, Suffolk, and Norfolk. Hingham, Weymouth, Cambridge, Roxbury, and Dorchester, the ports of delivery. All ports of entry are also ports of delivery.

2. The principal officer of every district is the collector, who is assisted by deputy-collectors, surveyors, appraisers, weighers, gaugers, inspectors, etc. The duties of the above vary in the several collection districts and ports. There is also in the leading ports of entry, a "naval officer," whose department is a check upon that of the collector.

619. A Custom-House Broker is a person who makes entries, secures permits, and transacts other business at custom-houses for merchants. He is familiar with the tariff laws and the details and regulations of custom-house business, and usually acts under a power of attorney.

620. Duties are of two kinds, *ad valorem* and *specific.*

621. An Ad valorem Duty is a tax assessed at a certain per cent. on the dutiable value of the merchandise; as silks at 60%, watches at 25%, linens 30, 35 and 40%, china 45 and 50%.

1. The dutiable value of merchandise is its market value at the port of export, but not less than its invoiced cost, commission added, whether paid or not. There is no duty on the freight or transportation from the port of export. The appraised value is sometimes greater than the invoice value (**623**).

2. In reducing foreign money to U. S. money for the purpose of calculating duties, if the cents of the result are less than 50, they are rejected; if more than 50, $1 is added to the dollars.

cific Duty is a tax assessed at a certain sum per yard, gallon, or other weight or measure, with- the value; as leaf tobacco at 35¢ per pound, clay glass per square foot, brandy $2 per proof gallon, board measure, salt (in bulk) 8 cts. per 100 per square yard.

e duties are calculated, allowances are made for tare (the arrel, or cask), leakage (of liquids in barrels), and breakage , usually 5%).

stom House ton contains 2240 lbs. ([illegible] the hundred- the quarter 28 lbs.

ods, there is both a specific and an ad valorem duty (some- ned duty); as statuary marble, $1 per cubic foot and [illegible] s. per pound and 35%.

voice ([illegible]) is a statement made by the seller or andise giving a description of the same, and show- ures, quantity, value, charges, and other details. [illegible]

weights and measures of the

[illegible]

Art. 566.

[illegible]

customs who ex- dutiable value

usually made is called the

package at least one of every store for examination. wharf where unloaded.

625. A Bonded Warehouse is a place for the storage of merchandise on which the duties or taxes have not been paid.

1. If an importer does not desire to place his goods at once in the market, or anticipates exporting the same, by giving a bond for the payment of the duties and making the entry in the proper form, he may have the merchandise stored at his own risk in a bonded warehouse, and thus defer the payment of the duties.

2. Merchandise may be withdrawn from a bonded warehouse for exportation to Canada or other foreign country, without the payment of the duty on the same.

3. Merchandise is frequently sold "in bond" at prices which do not include the duty.

4. Merchandise that may be in warehouse under bond for more than one year, will be liable when withdrawn for 10% additional duty.

5. Any goods remaining in public store or bonded warehouse beyond three years shall be regarded as abandoned to the government, and sold under certain regulations and the proceeds paid into the Treasury.

626. Drawback.—When distilled spirits, fermented liquors and tobacco upon which an internal revenue tax has been paid, and foreign merchandise upon which an import duty has been paid, are exported, the tax or duty upon the same is refunded. Such return of the tax or duty is called a **Drawback.**

627. The **Free List** is a list of articles which are exempt from duty.

In making entries of free goods, the value as given in foreign money must be reduced to U. S. money (See Ex. 23, Art. **628**), permits must be obtained to land the goods, and certain packages are sent to the public store for examination.

EXAMPLES.

628. *1.* A merchant imported from Lyons an invoice of silk, the dutiable value (**621**, 1) of which was 48765 francs. What was the dutiable value of the same in U. S. money, and what was the duty at 60% (**621**)?

NOTES.—1. For foreign moneys of account and their values in United States money, see Art. **566**.

2. 48765 francs at 19.3¢ = $****. (See Art. **621**, 2.) 60% of $**** = $****.**.

2. Find the duty on 1617 pounds of almonds, at 6 cts. per pound.

[illegible]

[illegible] paid at the rate of

According to the weigher's return, the net

pounds. How much additional duty was

having fixed the duty at 3¼ cts. per

[illegible] on an invoice of linens from Ireland,

[illegible] value £534 14s.

[illegible]

15. On a certain invoice of 34216 pounds of pepper, there are discounts for damage as follows: 12% on 6190 pounds, 8% on 6438 pounds, and 5% on 9642 pounds. After deducting the discount, what would be the duty on the remainder at 5 cents per pound?

16. The duty on burlaps is 30% ad valorem. What is the amount chargeable on a bale containing 50 webs, each being 54 *yds.* and 16 *in.* long, and 27 *in.* wide, and valued at 30 cents per *sq. yd.*?

17. What is the amount of duty chargeable on 2465 pounds of wool, valued at £171 8s., when the rate of duty is 10 cts. per pound and 11% ad valorem?

18. The duty on certain glass plates being 35 cents per *sq. ft.*, find the duty on 316 boxes, each containing 20 plates, and each plate being 24 *in.* by 30 *in.*

19. Find the duty at 25%, on one engraving, cost in London £34 5s., case and shipping charges 15s., commission 2½%.

20. What is the duty at $1 per *cu. ft.* and 25%, on a block of marble 2×3×7 *ft.*, imported from Italy, dutiable value 3450 lire?

21. Find the duty on 4175 *lbs.* cloves at 5¢ per *lb.*, 476 *lbs.* cinnamon at 20¢, and 5437 *lbs.* rice at 2½¢.

Make the extensions, find the dutiable value, and calculate the duty on the following invoices and accompanying entries:

22. Entry of merchandise, imported by Tefft, Weller & Co., from Berlin in the Str. "Silesia." Arrived Jan. 14, 1882. New York, Jan. 16, 1882.

Marks.	Nos.	Packages and Contents.	[illegible]
T. W.	351	One case half silk goods, . . .	Rm. 2399.8[illegible]
		Commission 2½%, . . .	**
			****
		Rm. ****.** @ 23.8¢ =	$
		60% of $*** = $***	

he total duty on 3 cases machinery, total value £23
%; 7 *cwt.* 6 *qr.* 14 *lb.* (622, 2) castings at 1¼*c.* per
vt. 0 *qr.* 26 *lb.* chain at 2½*c.* per pound; and 3 *cwt.*
ain at 2*c.* per pound.

voice of woollen cloth weighing 516 pounds, and
27 16*s.*, was imported from England. What was the
s. per pound and 35%?

nporter on making his entry at the custom-house,
y on 38716 pounds (Invoice weight) of tobacco, at
ound. According to the return of the custom-house
net weight was 38472 pounds. How much of the
inded when the entry was liquidated?

uty on 28432 pounds of sugar was paid at the rate of
ound. According to the weigher's return, the net
28218 pounds. How much additional duty was
e appraiser having fixed the duty at 3¼ *cts.* per

is the duty on an invoice of linens from Ireland,
e £424 15*s.* 6*d.*, at 35%? Dutiable value £384 14*s.*

is the duty on an invoice of porcelain vases from
, dutiable value 9843 francs? Dutiable value 7896
%?

he duty on 475 *cu. ft.* of statuary marble imported
dutiable value 16425 lire, at $1 per cubic foot,

is the duty on 37420 pounds of pig iron at $7 per
)?

the duty on an invoice of leather goods from Vienna,
e 6429 florins, at 35%.

is the duty on an importation of toys from Germany
e 8437 marks, at 50%? Dutiable value 7416 marks

e duty at 28 cents per *sq. yd.* and 35%,
ls carpet, 27 *in.* wide, invoiced at 3*s.* 6*d.*
ges (less consul's fee) £2 16*s.* 9*d.*?

NOTE.—The following is an entry of free goods. Free goods are entered and the foreign monetary units reduced to U. S. money for statistical purposes in the same manner as dutiable goods.

23. Entry for consumption of merchandise, imported by W. H. SCHIEFFELIN & Co., in the Str. "Ailsa" from Savanilla, on the 10th day of January, 1882. New York, Jan. 12, 1882.

	Free.
33 bales Medicinal Bark, ...	$310.
Packing, ...	12.
Commission 2½%, ...	**.**
(Pesos of U. S. of Columbia), . .	$***.**
@ 82.3¢, ...	$****.

24. Invoice of one package merchandise, purchased by GLADHILL & Co. for account of D. BUCKLEY & Co., New York, and forwarded for shipment to D. & C. MACIVER, Liverpool.

		£.	s.	d.
D. B.	4 Pieces Drab Cotton Pantaloon 32 *in.* wide, .			
207	#1729 79 *yd*, ...			
	30 80, ...			
	31 77½, ...			
	32 79, 315½ (less 1/37) 307 @ 2s. 2d., . .	**	*	*
	1½% discount, ...		**	
		**	**	*
	Verification and Commissioner's fee, .		14	10
	2½% Commission, ...		16	5
		**	*	*
	Less Consul's Certificate (not dutiable),		14	10
		33	11	7

Entry of merchandise, imported by D. BUCKLEY & Co. in the Str. "Catalonia" from Liverpool. New York, Jan. 12, 1882.

		35%.
D. B.	One case cotton, ...	33–11–7
207	@ 4.8665 =	$***.
	Duty 35% of $*** = $**.**	

25. Invoice of 700 bales leaf tobacco shipped by F. B. DEL RIO & Co., per Str. "Niagara" for New York, and consigned to FREDERICK DE BARY & Co.

F. B.	700 bales 83077 *lbs.* (See page 343, Spain)		$35000	
	CHARGES.			
3328/4027	Baling, ...	$525.		
	Export duties, ...	3407.39		
	Consul fee, ...	2.75		
	Small charges, ...	49	****	**
			*****	**
	Commission 2½%, ...		***	**
	Spanish gold, ...		$*****	**

HAVANA, Dec. 27, 1881.

Custom House, New York, Collector's Office, Jan. 4, 1882.

Bond No. 9817.

Entry of merchandise, imported on the third day of January, 1882, by FREDERICK DE BARY & Co., in the Str. "Niagara" from Havana.

Marks.	Nos.	Packages and Contents.		&c.	
F. B.	3328/4027	700 bales Leaf Tobacco, ...		84240 *lbs.*	$30958.74 @ .93,2 $*****.
		Duty 84240 *lbs.* @ 35¢ =	$*****.		
		†Weighers return 83675 *lbs.* at 35¢ =	*****.**		
		Refund, ...	$***.**		
		† Added by the liquidator.			

26. What is the duty on an invoice of crockery invoiced at £1275 16s. 6d. *f. o. b.* (free on board), at 40%?

27. What is the duty on 28916 pounds of steel rails at 1¼¢ per pound, and 11438 pounds of tin plates at 1$\frac{1}{10}$¢ per pound?

28. The duty on spool thread of cotton, contai
the spool, is 6¢ per dozen spools and in addi
valorem. What is the duty on 11160 spoo

PARTNERSHIP.

?artnership is the association of two or more persons ıeir capital and services for the purpose of conducting ıe gains or losses being shared in such proportion as ›ulated in the agreement.

ness association is called a Firm, House, or Company; and each the association is called a Partner.

\ **Special Partner** is one who takes no active part in ss, and whose liability is limited to the amount of his . In order to thus limit his liability, the amount of ıent must be duly advertised, and he must take no in the business.

rtners who conduct the business are called **General** Their private property is liable for the debts of the ›.

'he **Capital** or **Capital Stock** is the money or other hich is invested in a business.

ners' accounts are used to show the amounts invested. firms, the investments are entered in the partners' "stock ac- l the amounts withdrawn by the partners during the year and are entered in their "private accounts."

\ **Resource** or **Asset** is any kind of property belong- :oncern having a financial value.

\ **Liability** is a debt owing by the concern.

Che **Net Worth** of a concern is the excess of its ver its outside liabilities.

636. Gains or Losses, how shared.—In most partnerships, the gains or the losses are divided according to certain fractions or percentages; the inequalities of the investments are adjusted by allowing interest upon the same; and the partners receive salaries for their services rendered. (See Ex. 36, Art. **639**.) Sometimes the net gain or net loss is shared in proportion to the investments (Ex. 13, Art. **639**), or the average investments. (Ex. 17, Art. **639**.) In joint stock companies the gains (dividends) and the losses (assessments) are shared in proportion to the investments or the amounts of stock held.

637. Gains or Losses, how found.—When the books have been kept by "Single entry," and when no books have been kept, the gain is found by subtracting the net worth (**634**) at commencing, or the investment, from the net worth at closing; and the loss, *vice versa.*

When the books have been kept by "Double entry," the gain may be found as above, or by subtracting the sum of the separate losses from the sum of the separate gains. The results by the two methods should be the same and should prove each other.

EXERCISES.

638. In the following exercises find the gain or the loss:

1. Capital at commencing, $5000; capital at closing, $3000.

2. Capital at commencing, $5000; capital at closing, $8000.

3. Capital at commencing, $5000; insolvency at closing, $1000.

4. Capital at commencing, $5000; insolvency at closing, $7000.

5. Insolvency at commencing, $5000; capital at closing, $2000.

6. Insolvency at commencing, $5000; capital at closing, $6000.

7. Insolvency at commencing, $5000; insolvency at closing, $4000.

8. Insolvency at commencing, $5000; insolvency at closing, $9000.

Find the capital or the insolvency at closing:

9. Capital at commencing, $5000; gain during the year, $3000.

10. Capital at commencing, $5000; gain during the year, $6000.

11. Capital at commencing, $5000; loss during the year, $2000.

12. Capital at commencing, $5000; loss during the year, $8000.

13. Insolvency at commencing, $5000; gain during the year, $1000.

14. Insolvency at commencing, $5000; gain during the year, $7000.

15. Insolvency at commencing, $5000; loss during the year, $4000.

16. Insolvency at commencing, $5000; loss during the year, $9000.

Find the capital or the insolvency at commencing:

17. Capital at closing, $5000; gain during the year, $3000.
18. Capital at closing, $5000; gain during the year, $6000.
19. Capital at closing, $5000; loss during the year, $4000.
20. Capital at closing, $5000; loss during the year, $9000.
21. Capital at closing, $8400; gain during the year, $4100.
22. Capital at closing, $3700; gain during the year, $5200.
23. Insolvency at closing, $5000; gain during the year, $1000.
24. Insolvency at closing, $5000; gain during the year, $8000.
25. Insolvency at closing, $5000; loss during the year, $2000.
26. Insolvency at closing, $5000; loss during the year, $7000.

EXAMPLES.

639. *1.* A and B are partners, A sharing ⅔ of the gain or loss and B ⅓. A invests $5000, and B $2350. At the end of the year their resources and liabilities are as follows: merchandise on hand, per inventory, $2000; real estate, $7000; cash on hand and in bank, $1532; due on personal accounts, $1640.25; notes on hand, $1000; notes outstanding, $800; owing by the concern to sundry persons, $4471.69. What is the amount of net resources belonging to each partner?

FIRST OPERATION.

RESOURCES.		
Merchandise on hand, . .	$2000	
Real estate, ...	7000	
Cash on hand, . . .	1532	
Personal accounts, . .	1640.25	
Bills receivable, . . .	1000	$13172.25
LIABILITIES.		
Bills payable, . . .	$800	
Personal accounts, . .	4471.69	5271.69
Present worth, ...		$7900.56
Investments (subtracted), . . .		7350.
Total net gain, . .		$550.56

$\frac{2}{3}$ of $550.56 = $367.04, A's share of the gain.
$\frac{1}{3}$ of $550.56 = 183.52, B's share of the gain.

A's investment, .	$5000	B's investment, .	$2350.
Plus his gain, .	367.04	Plus his gain, .	183.52
A's present worth,	$5367.04	B's present worth,	$2533.52

$5367.04 + $2533.52 = $7900.56, total present worth, as above.

SECOND OPERATION.

ANALYSIS.—Theoretically, all the resources of a business belong to the creditors and the partners (proprietors), the partners' investments being regarded as liabilities; hence, the resources and liabilities—including the partners' accounts—should be equal. If, in a statement of the condition of a business, the resources and liabilities thus considered should not be equal, it is evident that the partners' accounts do not show their true interests, and the inference is that a gain or loss has occurred which has not been entered to their accounts. The excess of resources over liabilities would in such case show the gain, as would the excess of liabilities over resources show the loss. In order to restore the equilibrium, the gain should be credited or the loss debited to the partners' accounts.

1. STATEMENT BEFORE ADJUSTING PARTNERS' ACCOUNTS.

RESOURCES.		LIABILITIES.	
Merchandise, . . .	2000	Bills payable, . . .	800
Real estate, . . .	7000	Personal accounts, . .	4471.69
Cash, ...	1532	A's investment, . .	5000
Personal accounts, . .	1640.25	B's do. .	2350
Bills receivable, . .	1000		12621.69
	13172.25		
	12621.69		
Excess of resources (net gain).	550.56	A's $\frac{2}{3}$, $367.04 ; B's $\frac{1}{3}$, $183.52.	

2. STATEMENT AFTER ADJUSTING PARTNERS' ACCOUNTS.

RESOURCES.		LIABILITIES.	
Merchandise, . . .	2000	Bills payable, . . .	806
Real estate, . . .	7000	Personal accounts, . .	4471.69
Cash, ...	1532	A's investment and gain, .	5367.04
Personal accounts, . .	1640.25	B's do. do.	2533.52
Bills receivable, . .	1000		
	13172.25		13172.25

2. A and B are partners, A sharing ⅔ of the gain or loss and B ⅓. A invested $5000, and B $2350. During the year the concern gained on merchandise, $955.56 ; on real estate, $315. The expense account showed a loss of $675 ; the interest account, $45. What was the net gain, and the balance of each partner's account.

NOTE.—The above example is the complement of Ex. 1. The books having been kept by double entry, the separate gains and losses are given, and the net gain thus found. The loss and gain account and the partners' accounts are shown in the following operation in "skeleton ledger" form.

OPERATION.

A.

Balance,	5367	04	Investment, Gain, . .	5000 367	 04
	5367	04		5367	04
			Balance, .	5367	04

B.

Balance,	2533	52	Investment, Gain, . .	2350 183	 52
	2533	52		2533	52
			Balance, .	2533	52

LOSS AND GAIN.

Expense, . .	675		Mdse., . .	955	56
Interest, . .	45		Real Estate,	315	
A's, Gain ⅔, .	367	04			
B's " ⅓, .	183	52			
	1270	56		1270	56

3. A and B started in business July 1, 1881. Each put into the concern $2200. The resources on Jan. 1, 1882, were as follows: goods, $4000 ; bills receivable, $1500. The liabilities were $580. A has drawn out cash, $3000 ; and B, $2000. How much is due each partner, the gain or loss being divided equally ?

NOTES.—1. It must be borne in mind that the amounts drawn out by the partners are as fully resources of the business as if charged to an outside party.

2. In examples 3, 4, 5, and 7, open accounts with the partners and make statements of resources and liabilities.

4. A and B are partners. They have cash and notes on hand to the amount of $6475.28. A has drawn from the concern $2478.30, and B has drawn $1016.48. A invested $4287.46, and B, $1037.75. The firm owes sundry persons $5016.82. What is each partner's present interest in the concern, if they share equally in gains and losses?

5. On Jan. 1, my brother and I started a business in which I invested $900, and he $400. We now propose to separate, and the business stands as follows: stock in store $1800; cash on hand and in bank, $1200; outstanding accounts, considered good, $1200. According to the agreement, I am entitled to $\frac{2}{3}$ of the net gain, and my brother $\frac{1}{3}$. During the time of the copartnership, I have drawn $4000 and he, $2800. Of the assets given above, how much are we each entitled to?

6. C, D, and E are partners, each investing $10000, and each to share $\frac{1}{3}$ of the gain or loss. The resources and liabilities at the close of business are found to be as follows, viz.: Merchandise on hand, per inventory, $8159.50; cash on hand, $5012.88; personal accounts due the firm, $4235; notes and accepted drafts (bills receivable) on hand, $5000; real estate, $8000; bonds and stocks, $12000; owing by the firm to sundry persons, $5505; firm's notes outstanding (bills payable), $3000. C has withdrawn during the year $1247.87; D, $1400; and E, $1489. What is each partner's interest in the concern at closing?

7. C, D, and E are partners, sharing the gains and losses equally. C's net investment was $8752.13; D's, $8600; and E's $8511. During the year the firm's gains were as follows: Merchandise, $8529; stocks and bonds, $650; interest, $985.25. The cost of conducting the business was $2125. What was each partner's interest at closing? (See Ex. 2.)

8. M and N are partners, M sharing $\frac{3}{4}$ of the gain or loss and N $\frac{1}{4}$. M invested $15000 and N $5000. At the close of the business year, the resources and liabilities of the concern are as follows: cash on hand, $2128; bills payable, $4000; bills receivable, $3000; the firm owes sundry persons, $8375; due the firm from sundry persons, $16427; rent paid in advance, $375; mortgage held by the concern on the property of A. G. Pope, $5000; accrued interest on the same, $150; store fixtures valued at $835; merchandise on hand, $9416; accrued interest on firm's notes outstanding, $112; accrued interest on notes held by the firm, $75.

M has withdrawn $2465; and N, $2275. According to the agreement, each partner is to receive a salary of $2500. What are the separate interests at the close of the business?

9. A owns a business, the good will of which is estimated at $10000, and the stock on hand at $15000. B and C agree to unite with him on the following conditions: B to invest $25000 cash, and C to devote his entire time to the business, for which he is to receive, in addition to his interest, an annual salary of $1000. The capital is to be kept intact, and no interest to be allowed therefor. The gain or loss to be divided equally between the three partners. At the end of the year the resources, including good will, book accounts, notes, inventories, *etc.*, but not including amounts drawn by the partners, amount to $67000, and the liabilities to outside parties, to $10500. C has drawn out during the year $2500; B, $1575; A, $2000. Of the resources above named there are bad debts not to be counted, amounting to $575. What is the condition of each partner's account?

10. A and B are partners, A investing $\frac{2}{3}$ of the capital, and B $\frac{1}{3}$; the gains or losses to be shared in the same proportion. The following is an exhibit of the business, excepting the partners' accounts, at the close of a certain period: *Resources*, cash, $3775; Stone & Co., $150; A. R. Mead, $1200; bills receivable, $5500; interest on the same, $125; merchandise, $5140. *Liabilities*, L. Blair, $500; W. H. Rice, $723; Martens & Bultman, $517.64; bills payable, $3300; interest on the same, $169. The net gain during the year was $3174. What was each partner's original investment?

11. Upon a close valuation of the personal accounts due the firm in the preceding example, the partners are convinced that Stone & Co.'s is worth no more than 50% of its face; and A. R. Mead's, 25% of its face. Upon this valuation what would be the gain, and what the condition of the partners' accounts at closing?

12. A, B, and C are partners, A investing $25000 capital, B $5000, and C nothing. The proportionate interests are: A 60%, B 25%, C 15%. At the expiration of the term of copartnership, and after the gains and losses have been adjusted, A's credit of capital stands intact, B has a credit of only $1000, while C has overdrawn his account $8534. C being insolvent, how much must *B pay into the* concern to adjust his share of the loss?

13. A and B are partners in business, the gain or loss to be divided in proportion to investment. A invested $8750; B invested $4000. The net gain is $2726.15. What is each partner's share?

First Operation.—Fractional Method.

Analysis.—Since A's investment, $8750, is $\frac{8750}{12750}$ of the total investment, he is entitled to $\frac{8750}{12750}$ of the gain; and for a similar reason, B is entitled to $\frac{4000}{12750}$ of the gain.

$\frac{8750}{12750} = \frac{35}{51}$; $\frac{35}{51}$ of $2726.15 = $1870.89, A's gain.
$\frac{4000}{12750} = \frac{16}{51}$; $\frac{16}{51}$ of $2726.15 = $855.26, B's gain.

Second Operation.—By Proportion.

Analysis.—The total investment is to each partner's investment as the total gain is to each partner's gain.

$12750 : $8750 : : $2726.15 : $1870.89, A's gain.
$12750 : $4000 : : $2726.15 : $855.26, B's gain.

Note.—Cancel any factor common to the given extreme and either of the means.

Third Operation.—By Percentage.

Analysis.—$2226.15, the gain, is 21.3816% of $12750, the total investment. The partners' gains are therefore 21.3816% of their respective investments.

21.3816% of $8750 = $1870.89, A's gain.
21.3816% of $4000 = $855.26, B's gain.

Note.—In order to produce exact results by this method, it is necessary to extend the number expressing the rate per cent. of the gain or loss to several decimal places.

14. E, F, G, and H enter into a joint speculation. E advances $5000, F $700C, G $8000, and H $10000, the gain or loss to be divided according to investment. They gain $14285. What is the share of each?

15. Four merchants ship goods on joint account. A puts in $6000, B $5500, C $4200, and D $4800. What will be each man's share, if the gain is $9200?

16. Five persons having claims against the government, placed their claims in the hands of an agent for collection. A's claim amounted to $500, B's to $425, C's to $300, D's to $250, and E's to $175; but, after the agent had deducted his fees, there remained only $1237.50. How much did each claimant receive?

17. A and B are partners, gain or loss to be divided in proportion to average investment. A invests, Jan. 1, $4000; Mar. 1, $2000; Oct. 1, $3000; and withdraws July 1, $1500; Dec. 1, $1000. B invests, Jan. 1, $6000; Sept. 1, $3000. They close their books Jan. 1 of the following year and find they have gained $3456. What is each partner's share?

NOTE.—An *Average Investment* is an investment for a certain period of time equivalent to several investments for different periods of time.

OPERATION.

A invested	Jan. 1,	$4000 × 12 =	$48000	
"	Mar. 1,	2000 × 10 =	20000	
"	Oct. 1,	3000 × 3 =	9000	77000
A withdrew	July 1,	1500 × 6 =	9000	
"	Dec. 1,	1000 × 1 =	1000	10000
A's average investment for 1 month,				67000

OR,

A invested	Jan. 1,	$4000	× 2 =	$8000
"	Mar. 1,	2000		
		6000	× 4 =	24000
withdrew	July 1	1500		
		4500	× 3 =	13500
invested	Oct. 1,	3000		
		7500	× 2 =	15000
withdrew	Dec. 1,	1000		
		6500	× 1 =	6500
A's average investment for 1 month,				$67000

ANALYSIS.—By the first operation, we suppose each investment to be made for the remainder of the time. To find the average investment, multiply each investment and withdrawal by the interval between its date and time of settlement. Subtract the products obtained from the withdrawals from the products obtained from the investments. The remainder will be the average investment for 1 month, if the time is found in months. A's investment of Jan. 1 is in the business 12 months (Jan. 1 to Jan. 1); the use of $4000 for 12 months is equivalent to the use of $48000 for 1 month. Treating the other investments in like manner, we find A's total investments are equivalent to $77000 for 1 month. A's withdrawals are equivalent to $10000 for 1 month. A's net average investment is therefore equivalent to $67000 for 1 month.

By the second operation, we find the actual amount in the business for each month of the year. Jan. 1, A invested $4000, which was in the business *until Mar. 1*, or for 2 months. Mar. 1, he added $2000, making his total investment $6000, which was in the business until July 1, or for 4 months. July 1, he withdrew $1500, leaving in the business $4500 until Oct. 1, or 3 months, etc. The several net investments as found in this manner are equivalent to $67000 for 1 month.

B's average investment, as found by either of the above methods, is $84000 for 1 month.

A's average investment for the year is $5583.33⅓ ; and B's $7000. To avoid fractions, divide the gain in proportion to the average investments for 1 month. After the average investments are found for a common time, the gain may be divided according to either of the methods under Ex. 15. By the fractional method, A would be entitled to $\frac{67}{151}$ of the gain, and B to $\frac{84}{151}$.

18. C and D are partners, gain or loss to be divided in proportion to average investment. C puts in $6000 for one year, and $7000 for one and a half years ; D puts in $6000 for two and a half years. The net loss is $1565.40. What is each one's share ?

19. A, B, and C are partners. A puts into the concern $3000, but withdraws half of it at the end of 6 months ; B puts in $2000, and adds $500 to it at the end of 4 months ; C puts in $2500 for the whole year. The gain during the year is $1700. What is each one's share ?

20. Three contractors agree to build a road for $10000. A has 25 men at work for 16 days and 30 men for 34 days. B has 40 men for 10 days and 45 men for 40 days. C has 48 men for 50 days. C receives $200 extra for superintending the work. How much is each contractor entitled to ?

21. J, K, and L are partners, gain or loss to be divided according to average investment. J invests as follows : Jan. 1, $6000 ; Apr. 1, $4000 ; K invests, Jan. 1, $8000 ; L invests, Jan. 1, $7000 ; Apr. 16, $2500 ; and draws out June 16, $3500. At the end of the year the net gain is found to be $4135.60. What is each partner's share ? (Time by Compound Subtraction.)

22. A, B, C, and D were partners for two years. When the firm commenced business, A's investment was $6000, B's $3500, C's $2800, and D's $1700. At the end of 8 months, A withdrew $3000. At the end of 10 months, D added $1300 to his former investment. At the end of one year, B withdrew $800. At the close of the two years, they had gained $4727. What was each partner's share of the gain ?

23. A and B are partners for one year, the gain or loss being divided in proportion to their average investments. A invested, Jan. 1, $8000 ; June 16, $1500 ; Aug. 1, $2500 ; and drew out

May 1, $1500. B invested Jan. 1, $10000; Apr. 1, $500; and withdrew Aug. 16, $2500. How much should A invest Sept. 1 to entitle him to one-half the gain?

24. C and D are partners. According to agreement C is to share ¾ of the gain or loss, and D ¼. At the end of the year, D desires to increase his investment so that he will be entitled to a ⅓ interest. How much must D invest, the partners' accounts after the books are closed being as follows: C's debit, $6712.38; C's credit, $27000; D's credit, $9000?

25. R, S, T, and U enter into copartnership with equal capital, upon the following conditions: R to receive as a salary $2000; S, $1500; T, $1200; and U, $1000; the gain or loss to be divided equally. At the close of the year, the net gain, exclusive of salaries, proves to be $5400. To how much of this amount is each entitled?

NOTE.—The excess of the salaries (losses) over the gain is the net loss to the business, and should be charged equally to the partners.

26. X, Y, and Z commence business without capital. According to the partnership contract, X is to receive a salary of $3000; Y, $2500; and Z, $2000; the gain or loss to be divided equally. During the year, X withdraws $3000; Y, $2800; and Z, $1800. What is the balance due each partner, if the gain, without taking into account the partners' salaries, is $9000?

27. A merchant's assets are $12000, and he owes A $1900, B $5000, C $3000, and D $6100. As he is unable to pay his debts in full, he is compelled to make an assignment for the benefit of his creditors. If the expense of making the settlement is $1600, what % of his indebtedness can he pay? What amount will each creditor receive?

28. A and B failed in business. Their liabilities were $64000. The firm's assets amounted to $37500. If the assignee's fees and other expenses were $2300, what % of their indebtedness can they pay? What will C receive, whose claim is $16400?

29. D and E have made an assignment. They owe F $4200, G $16000, H $2500, and K $11800. Their assets amount to $25400, and the expense of making the settlement was $1630. G being a preferred creditor, what % will be paid to the other creditors, and what amount will each receive?

NOTE.—A preferred creditor is one who is paid in full, before any dividend is paid to the other creditors.

30. A lot, whose front is 240 feet, and whose depth is 100 feet, is bought by A, B and C, who pay respectively $3000, $4000, and $5000. How many feet front is each entitled to, if it is divided in proportion to their investments ?

31. M, the owner of a mill, employs S, a miller, under the following conditions : M is to furnish the requisite capital, and S to receive, in lieu of salary, ⅓ of the profits. M has a store connected with the mill, on the books of which are entered all time sales of mill products. The grain, etc., for the mill is furnished by M. At the beginning of the year the value of the grain, flour, feed, etc. is $1727. During the year M's purchases for the mill amount to $19275. S has received for cash sales $16337, of which he has paid over to M $15550. The sales on account, as shown on M's books, amount to $8375; and the value of the products on hand is $2828. During the year S has purchased goods at M's store to the amount of $837.65. How much is owing to S at the expiration of the year ?

32. P and Q are partners, each to receive interest on his net investment at the rate of 6% per annum, and the net gain or loss to be divided equally. P invests, Jan. 1, $5000 ; Mar. 1, $4000 ; June 16, $1500; and draws out Apr. 16, $2500. Q invests, Jan. 1, $8000 ; Sept. 16, 2000 ; and draws out June 1, $1500 ; Nov. 11, $500. At the close of the year the net gain is found to be $4475.25, without taking into account the interest on the partners' accounts. What is the amount due each partner after the gain is adjusted ? (Time by Compound Subtraction.)

NOTE.—Open accounts with P, Q, and Loss and Gain.

33. A and B have been doing business as partners, A sharing ⅗ and B ⅖ of the gains and losses. A invested $4500, average date Mar. 25, 1882 ; and drew out $2700, average date Sept. 12, 1882. B invested $7200, average date June 17, 1882 ; and drew out $3750, average date Oct. 25, 1882. At the time of their dissolution, Jan. 1, 1883, the debts of the firm were all paid and they had on hand belonging to the firm $8750 in cash. How shall the money be divided, each being allowed interest at 6% on his investment and charged with interest at the same rate on the amounts drawn ? (Exact days. Interest 360 days to the year.)

NOTE.—In Examples 33 and 34, open accounts with A and B, and make statements of Resources and Liabilities.

34. A and B are partners, A having $\frac{3}{5}$ and B $\frac{2}{5}$ interest. A advanced in business $12000, average date Jan. 12, 1883; and drew out $1265, average date Oct. 20, 1883. B advanced $7500, average date Apr. 5, 1883; and drew out $2560, average date Nov. 25, 1883. Jan. 1, 1884, the assets are as follows: Cash, $5800; merchandise, $6250; notes on hand, $7300; accrued interest on the same, $387.14; personal accounts, $5700. The liabilities are as follows: Notes outstanding, $4200; accrued interest on the same, $227.65; personal accounts, $2500. Find the balance of each partner's account, 5% of the personal accounts being considered uncollectible, and interest being reckoned on the partners' accounts at 6% per annum (365 days to the year).

35. A, B, and C form a copartnership under the following conditions: A is to manage the business, and to receive therefor $2400 per annum, which amount is to be credited as July 1. He is to receive interest on his salary and to pay interest on sums withdrawn at the rate of 6% per annum. B and C are to furnish the capital, and to receive interest therefor at the rate of 6% per annum. The net gain or loss to be divided equally. B invests, Jan. 1, $10000; Apr. 1, $5000. C invests, Jan. 1, $10000; July 1, $5000; and draws out Sept. 16, $500. A draws out, Feb. 1, $200; Mar. 1, $400; July 11, $500; Oct. 1, $200; Nov. 21, $100. At the end of the year, the gain—without taking into account either the salary to be paid to A or the interest on the partners' accounts—is $8437.16. What will be the balance of each partner's account, when all the items have been properly entered?

NOTE.—In Examples 35 and 36 open accounts with the partners and with Loss and Gain.

36. A, B, and C are partners, A sharing $\frac{3}{6}$ of the gain or loss, B $\frac{2}{6}$, and C $\frac{1}{6}$. Interest is to be reckoned at the rate of 6% per annum (365 days to the year) on the partners' accounts, and each partner is to receive a salary of $1800, to be credited as July 1. A invested, Jan. 1, $16000; and withdrew during the year $4875, average date, Aug. 21. B invested, Jan. 1, $20000; and withdrew $6224, average date, June 18. C invested, Jan. 1, $5000; and withdrew $2625, average date, July 31. Jan. 1, of the following year, the merchandise account shows a gain of $18437.16; the interest account (not including the interest on the partners' *accounts*) a gain of $586.38; sundry consignment accounts show a net gain of $1287.14. The expense account (not including the partners' salaries) shows a loss of $3424.75. What is each partner's interest in the business at closing? How will A be affected if each partner's salary is increased to $2500?

37. A and B unite in conducting a summer hotel, on the following basis: 1. Each is to receive interest at the rate of 6% *per annum* on his investment; 2. A is to receive a salary of $1000 and B of $800 for the season; 3. The profit or loss of the general business is to be divided in the proportion of A $\frac{2}{3}$, B $\frac{1}{3}$; the profit or loss of the livery business attached thereto in the proportion of A $\frac{1}{3}$, B $\frac{2}{3}$; the profit or loss of the bathing business in the proportion of A $\frac{1}{2}$, B $\frac{1}{2}$. A invests an average of $10150 for four months, and B an average of $6750 for the same time. At the close of the business the accounts showing loss and gain stand as follows:

Outgo.	Hotel.	Income.
15150.75		25175.19

Outgo.	Livery.	Income.
1592.75		3279.50

Outgo.	Bathing.	Income.
759.12		1275.30

There is besides an item of service amounting to $375, which at the time could not be easily apportioned in the charges, and which does not appear in the above outgoes. It is agreed that this item, as also the sums severally due the partners for interest and salaries, shall be charged to the several departments of the business in proportion to the net gains. There is, also, an inventory in the livery business amounting to $429.33. How much gain from all sources will each partner get out of the business?

38. A, B, and C are equal partners in a mill, each to receive 6% *per annum* interest on his average investment. C is to superintend the business and receive therefor a yearly salary of $3000; B keeps a store at which the operatives trade, and is to pay to A and C 5% on sales to operatives. A negotiates the products of the mill, for which he is allowed 10% on the net profits as existing before his percentage is taken. A's average investment for the year is $9750; B's $5750; C's $5000. Leaving out the interest, salary and percentages, the net gain for the year is $15000. B's sales to operatives amount to $1575. What share of the $15000 is each partner entitled to?

NATIONAL BANKS.

640. A National Bank is a bank organized under the laws of, and chartered by, the United States.

1. National banks are authorized to discount and negotiate notes, drafts, etc.; to receive deposits; to buy and sell exchange, coin and bullion; to loan money on personal security; and to issue circulating notes.

2. National banks are prohibited from making loans on real estate or on security of their own shares of capital except to secure debts previously contracted.

Real Estate purchased or mortgaged to secure a pre-existing debt shall not be held for a longer period than five years.

They are also prohibited from making loans to one person or association, excepting on business paper representing actually existing value as security, in excess of one-tenth of the capital of the bank.

3. The stockholders of a national bank are individually liable (equally and ratably, and not one for another) for an amount equal to the par value of the capital stock held by them.

641. Circulation. Upon a deposit of registered bonds, the association making the same shall be entitled to receive from the Comptroller of the Currency circulating notes of different denominations (**120**) equal in amount to 90% of the current market value, not exceeding par, of the United States bonds so transferred and delivered, and at no time shall the total amount of such notes issued to any association exceed 90% of the amount at such time actually paid in of its capital stock.

1. Any national bank desiring to decrease its circulation, in whole or in part, may deposit lawful money (specie or legal tenders) with the Treasurer of the United States in sums of not less than $9,000, and withdraw a proportionate amount of bonds held as security for such notes.

No national bank which makes any deposit of lawful money in order to withdraw its circulating notes, shall be entitled to receive any increase of its circulation for the period of six months from the time it made such deposit. Not more than $3,000,000 shall be deposited during any calendar month for this purpose.

2. The State bank circulation wholly ceased after Congress had imposed a penalty of 10% in the form of a tax every time it should be issued. This act took effect Aug. 1, 1866.

642. Redemption.—The circulating notes of national banks are redeemed in lawful money by the banks which issued them and by the Treasurer of the United States at Washington, D. C.

1. Every national bank shall, at all times, keep and have on deposit in the Treasury of the United States in lawful money of the United States, a sum equal to 5% of its circulation, to be held and used for the redemption of such irculation.

2. All national banks which go into voluntary liquidation shall, within six months thereafter, deposit in the Treasury an amount of lawful money equal to the amount of their circulating notes outstanding. The law also requires that full provision shall be made for the redemption of the circulating notes of any insolvent bank before a dividend is made to its creditors. Thus it will be seen that no association can close up its business without first providing for the payment of all its circulating notes, and that the amount deposited for their redemption must remain in the Treasury until the last outstanding note shall have been presented. It is therefore plain that the government, and not the bank, receives all the benefit arising from lost or unredeemed circulating notes.

643. Reserve.—The national banks in the reserve cities* are required by law to hold a lawful money reserve of 25% of their deposits; all other national banks 15%. The excess above legal requirements is called "surplus reserve."

644. Surplus Fund.—The law provides that a surplus fund shall be accumulated, by setting aside, before the usual semi-annual dividend is declared, one-tenth part of the net profits of the bank for the preceding half-year, until the surplus fund shall amount to 20% of its capital stock.

EXAMPLES.

645. *1.* The impairment of the capital stock ($300000) of an insolvent national bank was $216000. What was the rate per cent. of the assessment made upon the stockholders for the purpose of making good the deficiency (**640**, 3)? How much was Mr. A. obliged to pay, who owned 80 shares?

2. What amount of bank notes is issued to a national bank that deposits $780000 in U. S. bonds to secure circulation (**641**)? How much is its redemption fund (**642**, 1)?

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* The reserve cities are New York, Boston, Philadelphia, Baltimore, Albany, Pittsburgh, Washington, New Orleans, Louisville, Cincinnati, Cleveland, Chicago, Detroit, Milwaukee, Saint Louis, and San Francisco.

3. A national bank, desiring to reduce its circulation, deposits with the Treasurer of the United States $27000 in legal-tenders, and sells the bonds withdrawn (**641**) in the market at 118¾. What were the proceeds?

4. The circulation of a national bank having a capital of $150000 is $57600; what is the remaining amount of circulation which it may call for by depositing the necessary amount of bonds (**641**)? What is the par value of the bonds now on deposit? What additional amount of bonds must the bank deposit if the circulation is increased to the maximum?

5. How much is the redemption fund of a bank whose circulation is $427500? What is the amount of bonds on deposit to secure its circulation?

6. The New York associated banks, Mar. 25, 1882, held $58,602,100 in specie and $16,150,900 in legal-tenders. Their deposits on the same date were $285,659,600. What was the excess of reserve (**643**) above legal requirements?

7. Oct. 1, 1881, the national banks of Boston had $8,286,182 in specie, $3,457,379 in legal-tenders, $75,000 in U. S. certificates of deposit, $11,735,499 due from reserve agents, and a redemption fund with U. S. Treasurer of $1,603,628. What was the ratio of the reserve to the deposits, which were $95,776,386? What amount of reserve was required? What was the surplus reserve?

8. What amount of reserve was required by the national banks of the State of Maine, their deposits being $9,558,878?

9. The net earnings of a bank, whose surplus (**644**) is less than 20% of its capital ($300000), are $10475.38. What amount must be carried to the surplus account, and what are the undivided profits after declaring a dividend of 3%?

10. What is the semi-annual tax* at ½% upon a national bank whose average circulation is $462,730 (**641**)?

11. A bank having a capital of $250,000, and a surplus of $50,000, for a period of six months, earned $58693, and declared a dividend of $30000. What was the rate of the dividend? The dividend is what % of the capital and surplus? The net earnings are what % of the capital and surplus?

12. The average daily exchanges at the New York Clearing House for 1886 were $109,000,000, and the average daily balances $5,000,000. The balances were what % of the exchanges?

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* The tax upon capital and deposits was repealed by Act of March 3, 1883.

SAVINGS BANKS.

646. Savings Banks are institutions for the deposit and safe keeping of small sums of money. They are designed to encourage thrift and economy among the working classes.

647. Interest is usually declared Jan. 1st and July 1st of each year, and when declared is carried at once to the credit of each depositor on the books of the bank, when it stands as a deposit, and is entitled to interest the same as any other deposit. Savings banks, therefore, pay compound interest.

No interest is allowed on the fractional parts of a dollar, nor is any interest allowed on any sum withdrawn previous to the first day of January or July, for the period which may have elapsed since the last dividend.

648. Deposits are practically payable on demand, though the right to require a notice of 60 or 90 days is reserved.

In some savings banks, deposits commence to draw interest Jan. 1st, April 1st, July 1st, Oct. 1st; in others, deposits made on or before the first of any month draw interest from the first days of those months respectively.

NOTE.—In certain savings banks, money must be on deposit at least three months before it will be entitled to any interest. In a bank that has the above by-law and that pays interest from the first of each month, money deposited on or before May 1 would draw interest for 8 months on the following Jan. 1, and deposits made on or before June 1 would draw interest for 7 months, Jan. 1. Deposits made in October and November would be treated in a similar manner on the following July 1.

649. According to the laws of the State of New York,

No person shall have a deposit larger than the sum of three thousand dollars, exclusive of accrued interest, unless such deposit was made prior to the passage of the act (May 17, 1875), or pursuant to the order of a court of record, or of a surrogate.

Savings banks are restricted to 5% per annum regular interest or dividend. They must, however, declare an extra dividend at least once in three years, when their surplus earnings amount to 15% of their deposits.

Savings banks are allowed to pay interest on all sums deposited during the first ten days of January and July, and the first three days of April and October from the first of those months respectively.

EXAMPLES.

650. Perform the following examples according to both methods mentioned in Art. **648.** Where no rate is mentioned, 4% is understood.

1. A person deposited Dec. 30, 1881, $150; Feb. 20, 1882, $40; April 1, 1882, $120; May 30, 1882, $60. What amount was due July 1, 1882, nothing having been withdrawn?

Analysis.—If interest begin on the first of each quarter, the first deposit, $150, will draw interest from Jan. 1, or for 6 *mo.*; the second and third deposits, $160 ($40 + $120), will draw interest from April 1, or for 3 *mo.*; the last deposit, made May 30, will draw no interest July 1.

4% *per annum* is 2% for 6 months and 1% for 3 months.

If interest begin on the first of each month, the first deposit, $150, will draw interest from Jan. 1, or for 6 *mo.*; the second deposit, $40, made Feb. 20, will draw interest from March 1, or for 4 *mo.*; the third deposit, $120, made April 1, will draw interest from April 1, or 3 *mo.*; the fourth deposit, $160, made May 30, will draw interest from June 1, or for 1 *mo.*

2. The following deposits were made in a savings bank: July 1, 1881, $100; July 16, $40; Aug. 1, $75; Aug. 29, $45; Sept. 30, $75; Oct. 28, $200; Nov. 25, $30; Dec. 31, $100. What was due Jan. 1, 1882?

Note.—Balance the following accounts Jan. 1 and July 1 of each year.

3. How much interest was due on the following account July 1, 1883? Deposits, Oct. 1, 1881, $200; Dec. 31, 1881, $160; Mar. 24, 1883, $100.

4. Mr. A. made the following deposits in a savings bank: Jan. 1, 1879, $100; May 1, 1879, $140; June 30, 1879, $40; Oct. 1, 1879, $60; Feb. 28, 1880, $120; June 30, 1880, $45; Aug. 29, 1881, $200. What was the balance due Jan. 1, 1882?

5. What is the balance of the following account July 1, 1879, interest being reckoned at 6% until July 1, 1877, and at 5% thereafter: Deposits, Oct. 14, 1876, $200; Mar. 30, 1878, $135; April 1, 1879, $90?

6. Balance the following account July 1. Balance due Jan. 1, $103. Deposits, Jan. 28, $40; Mar. 30, $125; May 26, $80. Drafts, Feb. 20, $20; April 18, $15; May 3, $25; June 16, $100.

Date.		Deposits.	Drafts.
Jan.	1,	103	
"	28,	~~40~~	
Feb.	20,	20	~~20~~
Mar.	30,	~~125~~	
Apr,	18,	85	~~15~~
May	3,	65	~~25~~
"	26,	~~80~~	
June	16,		~~100~~

ANALYSIS.—In order to determine the amounts that are entitled to interest, arrange the account in the following form, and deduct the drafts from the *last* deposits made, by the system of cancellation indicated below. The draft of $20 made Feb. 20 is deducted from the deposit, $40, of Jan. 28, leaving $20. The drafts made April 18, $15, and May 3, $25, are deducted from the deposit of March 30, leaving $85. The draft of June 16, $100, cancels all of the deposit of May 26, and $20 of the deposit of March 30, leaving $65 ($85—$20). The net deposits are as follows: Jan. 1, $103; Jan. 28, $20; March 30, $65.

If interest commence the first of each quarter, the several amounts will draw interest as follows: $103 from Jan. 1, or 6 *mo.*; $20, deposited Jan. 28, and $65 deposited Mar. 30, making $85 from April 1, or 3 *mo.*

If interest commence the first of each month, the several amounts will draw interest as follows: $103 from Jan 1, or 6 *mo.*; $20, deposited Jan. 28, from Feb. 1, or 5 *mo.*; $65, deposited Mar. 30, from April 1, or 3 *mo.*

7. What is the balance of the following account July 1? Balance due Jan. 1, $30; deposits, Feb. 16, $50; Apr. 1, $185. Drafts, Mar. 12, $60; May 10, $50; June 20, $60.

8. Balance the following Jan. 1, 1881. Deposits, July 1, 1880, $300; Aug. 1, $150; Sept. 27, $60; Oct. 12, $325. Drafts, July 16, $150; Sept. 1, $150; Nov. 17, $70; Dec. 18, $140.

9. Balance the following account July 1. Balance due Jan. 1, $364.48. Deposits, Jan. 24, $50; Feb. 16, $80; Apr. 30, $40; June 28, $100. Drafts, Mar. 30, $75; May 19, $10.

10. What was due July 1, 1882, on the following pass-book?

Dr. FRANKLIN SAVINGS BANK in account with A. C. LOBECK. *Cr.*

1881.				1881.		
Jan 1	Four Hundred Dollars.	400		Aug. 1	Two Hundred Dollars.	200
Mar. 15	Ninety Dollars.	90		1882.		
1881.	Interest to July.	*	**	Jan. 16	One Hundred and Sixty dollars.	160
Sept. 16	Two Hundred Dollars.	200				
1882.	Interest to January	*	**	June 1	Eighty Dollars.	80
Feb. 27	Two Hundred and Sixty Dollars.	260				
Mar. [illegible]	One Hundred Dollars.	100				

LIFE INSURANCE.

651. Life Insurance is a contract by which a company (the insurer), in consideration of certain payments, agrees to pay to the heirs of a person, when he dies, or to himself, if living at a specified age, a certain sum of money.

Life Insurance Companies may be classified according to principles of organization the same as Fire Insurance Companies (529).

652. The principal kinds of policies issued by Life Insurance Companies are the following: **Ordinary Life, Limited Payment Life, Endowment, and Annuity.**

Tontine Investment, Reserve Endowment, Semi-Tontine, Semi-Endowment, Yearly Renewable, and other special policies are issued by some companies.

653. Ordinary Life Policies.—On this kind of policy, a certain premium is to be paid every year until the death of the insured, when the policy becomes payable to the persons named in the policy as the beneficiaries.

654. Limited Payment Life Policies.—On a policy of this kind, premiums are paid annually for a certain number of years fixed upon at the time of insuring—or, until the death of the insured, should that occur prior to the end of the selected period. The policy is payable on the death of the insured.

These policies are issued with single payments, or with 5, 10, 15, 20, or 25 annual payments.

655. Endowment Policies.—An Endowment Policy provides (1) **insurance** during a stipulated period, payable at the death of the insured should he die within the period; and (2) an **endowment**, of the same amount as the policy, payable at the end of the period if the insured survive until that time.

These policies are issued for endowment periods of 10, 15, 20, 25, 30, or 35 years, and may be paid up by a single payment, by annual premiums during the endowment period, or by 5 or 10 annual payments.

656. Annuity Policies.—An Annuity Policy secures to the holder the payment of a certain sum of money every year during his life-time. It is secured by a single cash payment.

657. The **Reserve** of life insurance policies is the present value of the amount to be paid at death less the present value of all the net premiums to be paid in the future.

658. The **Reserve Fund** of a Life Insurance Company is that sum in hand which, invested at a given rate of interest together with future premiums on existing policies, should be sufficient to meet all obligations as they become due. It is the sum of the separate reserves of the several policies outstanding.

The legal rate for the reserve fund according to the laws of the State of New York, is 4½%; of Massachusetts, 4%.

659. A **Non-Forfeiting Policy** is one which does not become void on account of non-payment of premiums.

1. According to the laws of the State of New York, after three full annual premiums have been paid, the legal reserve of the policy, calculated at the date of the failure to make the payments, shall, on surrender of the policy within six months after such lapse, be applied as a single payment at the published rates of the company in either of two ways, at the option of the assured. (1) To the continuance of the full amount of the insurance so long as such single premium will purchase term insurance for that amount, or (2) to the purchase of a non-participating paid-up policy.
2. According to the Massachusetts limited forfeiture law of 1880, after two full annual premiums have been paid, and without any action on the part of the assured, the net value (Massachusetts standard) of the policy less a surrender charge of 8% of the present value of the future premiums which the policy is exposed to pay in case of its continuance, shall be applied as a single payment to the purchase of paid-up insurance.
3. Certain companies voluntarily apply all credited dividends to the continuance of the insurance; others voluntarily apply the legal reserve to the purchase of term insurance at the regular rates.
4. In some companies, all limited payment life policies and all endowment policies, after premiums for three (or two) years have been paid and the original policy is surrendered within a certain time, provide for paid-up assurance for as many parts (tenths, fifteenths, twentieths, etc., as the case may be), of the original amount assured, as there shall have been complete annual premiums received in cash by the Company.

660. The **Surrender Value** of a policy is the amount of cash which the company will pay the holder on the surrender of the policy. It is the legal reserve less a certain per cent. for expenses.

TABLE OF RATES.

661. Annual premium for an Insurance of $1,000, with profits.

LIFE POLICIES. Payable at Death, only.					ENDOWMENT POLICIES. Payable as Indicated, or at Death, if Prior.				
AGE.	ANNUAL PAYMENTS.				AGE.	In 10 Years.	In 15 Years.	In 20 Years.	AGE.
	For Life.	10 Years.	15 Years.	20 Years.					
25	$19 89	$42 56	$32 34	$27 39	25	$103 91	$66 02	$47 68	25
26	20 40	43 37	32 97	27 93	26	104 03	66 15	47 82	26
27	20 93	44 22	33 62	28 50	27	104 16	66 29	47 98	27
28	21 48	45 10	34 31	29 09	28	104 29	66 44	48 15	28
29	22 07	46 02	35 02	29 71	29	104 43	66 60	48 33	29
30	22 70	46 97	35 76	30 33	30	104 58	66 77	48 53	30
31	23 35	47 98	36 54	31 03	31	104 75	66 96	48 74	31
32	24 05	49 02	37 35	31 74	32	104 92	67 16	48 97	32
33	24 78	50 10	38 20	32 48	33	105 11	67 36	49 22	33
34	25 56	51 22	39 09	33 26	34	105 31	67 60	49 49	34
35	26 38	52 40	40 01	34 08	35	105 53	67 85	49 79	35
36	27 25	53 63	40 98	34 93	36	105 75	68 12	50 11	36
37	28 17	54 91	42 00	35 83	37	106 00	68 41	50 47	37
38	29 15	56 24	43 06	36 78	38	106 28	68 73	50 86	38
39	30 19	57 63	44 17	37 78	39	106 58	69 09	51 30	39
40	31 30	59 09	45 33	38 83	40	106 90	69 49	51 78	40
41	32 47	60 60	46 56	39 93	41	107 23	69 92	52 31	41
42	33 72	62 19	47 84	41 10	42	107 65	70 40	52 89	42
43	35 05	63 84	49 19	42 34	43	108 08	70 92	53 54	43
44	36 43	65 57	50 61	43 64	44	108 55	71 50	54 25	44
45	37 97	67 37	52 11	45 03	45	109 07	72 14	55 04	45
46	39 58	69 26	53 68	46 50	46	109 65	72 86	55 91	46
47	41 30	71 25	55 35	48 07	47	110 30	73 66	56 89	47
48	43 13	73 32	57 10	49 73	48	111 01	74 54	57 96	48
49	45 09	75 49	58 95	51 50	49	111 81	75 51	59 15	49
50	47 18	77 77	60 91	53 38	50	112 68	76 59	60 45	50

1. The above table represents the maximum rates of the leading New York companies. Surplus premiums or dividends are returned annually commencing at the payment of the second premium.

2. Policies which do not share in the dividends of the company, are issued at fixed rates 15 to 20% less than the above.

3. The above rates are for annual payments only. To obtain semi-annual payments, add 4% and divide by 2. To obtain quarterly payments, add 6% and divide by 4.

EXAMPLES.

662. *1.* Find the amount of premium for an ordinary life policy (**653, 661**) of $5000, issued to a person 35 years of age.

2. What is the first annual premium of a life policy of $6000, issued to a person 30 years old, $1.00 being charged for the policy?

NOTE.—The policy fee is added to the first premium only.

3. Find the annual premium for a 20-payment life policy (**654, 661**) of $4000, issued to a person 28 years old.

4. What annual premium must be paid for a 20-year endowment policy (**655**) of $8000, age of the insured at nearest birthday, 40 years? If the insured dies during the tenth year, how much more would have been paid than if he had been insured on the ordinary life plan?

5. What is the average daily cost of a life policy for $1000, no allowance being made for probable dividends, insurance commencing at age 25? At 35? At 45?

6. How much must a person, aged 35, lay aside weekly to secure a life policy of $1000, payable in 20 annual payments?

7. When 40 years old, a person took out a 20-year endowment policy of $10000. He survived the endowment period. How much less did he receive than he paid as premiums, not reckoning interest?

8. Mr. A. when 26 years old took out an ordinary life policy of $20000. He died aged 41 years 2 months. How much more did his heirs receive than had been paid premiums, no allowance being made for interest?

9. In the above example, supposing money to be worth 6% (simple interest), what was the net gain of the above insurance?

10. The annual premium, without profits, on a life policy of $10000 at age 35 is $222. How much would it be necessary to invest at 6% interest to secure the payment of the annual premium? How much would the insured leave his family at his death?

11. A gentleman, age 30, insures his life for $20000, ordinary life plan. How much must he place in trust so that the interest at 5% will be sufficient to pay the premiums on the policy? At his death, how much does he leave his family?

12. If a man 32 years old takes out a life policy for $5000 and dies just before reaching the age of 40 years, how much less will his total payments be than if he had taken a 20-year endowment policy for the same amount ?

13. Mr. C. when 25 years of age secured a 20-year endowment policy of $6000 ; when he was 30 years of age, he obtained an ordinary life policy of $4000 ; when 35 years of age, he took out a 20-payment life policy of $10000. What was the total annual premium after taking the last policy ?

14. Suppose Mr. C. had died at the age of 40½ years, how much more would his heirs receive than had been paid as premiums ?

15. A single premium for an assurance of $1000, without profits, for a person 32 years old, is $300. What would be the excess of the assurance over the amount produced by placing the money at compound interest (**483**) at 4%, supposing the insured to live 20 years ? 30 years ? What would be the excess of the sum produced by the money at interest at 5%, over the assurance in 30 years ?

16. Mr. B., age 40, has $10000 at interest at 6%, which he intends to leave his family. What will this amount to at compound interest (**483**) in 25 years at 6% ? How much will he leave his family if he takes out a life policy and pays the premium with the interest on his investment of $10000 ?

17. Mr. A., aged 30, secures an ordinary life policy, annual premium $100. How much more would his heirs receive from the insurance company than from the money at compound interest (**484**) at 5%, should he die at the age of 32 ? Of 40 ? Of 50 ? At about what age would the amount received from the money at interest exceed the assurance ?

18. What is the semi-annual premium (**661**, 3) on a 20-year endowment policy for $6000, age 32 ? The quarterly premium ?

19. Mr. A., who will be 35 years of age July 1, takes out Apr. 1 a 20-payment life policy for $10000, premium payable semi-annually. Mr. B., of the same age, takes out Apr. 1 the same kind of policy for $5000, and Oct. 1, another policy of the same kind for $5000, premium payable annually. How much less does Mr. B. pay as premium each year than Mr. A ? (**661**, 3.)

20. An ordinary life policy issued at age 35 for $10000 has, at age 45, a 4% reserve of $1262.60. How much non-participating paid-up insurance will this amount purchase, the single premium rate per $1000 at age 45 being $475.44 ?

REVIEW EXAMPLES.

663. *1.* Add $17\frac{1}{3}$, $28\frac{3}{4}$, $36\frac{1}{2}$, $44\frac{5}{8}$, $89\frac{7}{12}$, and $76\frac{5}{6}$; multiply the sum by 87; subtract $1022\frac{17}{24}$ from the product; and divide the remainder by $234\frac{2}{3}$.

2. Divide eighty-three, and seventy-five hundredths by one hundred twenty-five ten-thousandths; add to the quotient sixty-eight, and six hundred twenty-five thousandths; and multiply the sum by three, and two-tenths.

3. How many minutes in the month of February, 1900?

4. Find the cost of 7312 pounds of meal at $2.25 per *cwt.*

5. The difference in the local time of two places is 1 *hr.* 7 *min.* 13 *sec.;* what is the difference in longitude?

6. Find the number of square yards of paving in a street, 3000 *ft.* long and 50 *ft.* wide.

7. What is the charge for packing, marking, and shipping 251 bales merchandise at 5*s.* 6*d.* per bale?

8. If 46 *T.* 12 *cwt.* of coal are worth $174.75, what is the value of 37 *T.* 8 *cwt.*?

9. How many square yards of linoleum would cover a floor 22 *ft.* 6 *in.* by 15 *ft.* 4 *in.*? Find its value at 63¢ per *sq. yd.*

10. What is the freight of 5 *T.* 9 *cwt.* 2 *qr.* 8 *lb.*, at 70 shillings per ton (2240 *lb.*)?

11. Bought 280 cords of hard wood, at $6.75, and 790 cords of soft wood, at $$3.62\frac{1}{2}$ per cord. Also, 750 bushels of corn, at $62\frac{1}{2}$ cents, and 925 bushels of oats, at $37\frac{1}{2}$ cents per bushel. What was paid for the whole, and what was the average price of wood per cord, and of grain per bushel?

12. Bought on contract 350 reams of foolscap paper, at $$3.83\frac{1}{2}$ per ream, $45\frac{1}{2}$ reams of which were returned as unsuitable, and 275 reams of letter, at $$2.67\frac{1}{2}$ per ream, $37\frac{1}{2}$ reams of which were rejected. How much was paid for the remainder?

13. Feb. 26, 1879, the Nevada Bank of San Francisco sold 100,000 ounces of pure silver to the United States, at $$1.08\frac{1}{2}$ per ounce. At this rate, what is the intrinsic gold value of the standard silver dollar?

14. What is the value of 45000 tons of steel rails at 97*s.* 6*d.* per ton? What is the value per ton in U. S. money? Of total in U. S. money?

15. What will be the cost of painting the walls and ceiling of a room, whose height, length, and breadth are 12 *ft.* 6 *in.*, 27 *ft.*, and 20 *ft.*, respectively, at 24 cents per square yard?

16. What is the total cost of 561^{23} bushels oats at 43 cents per bushel, and 411^{14} bushels corn at 46 cents per bushel?

17. Find the total freight on 68 *cu. ft.* mdse. at 35 shillings per ton (40 *cu. ft.*), and 123 *cu. ft.* at 40 shillings per ton, plus 10% primage on each item.

18. What is the cost of 250 *ft.* 3-ply hose, at 60 *cts.* per foot, less 30 and 10%, and 5 sets couplings at $1.50 each?

19. May 10, A buys a bill of goods amounting to $5000 on the following terms: 60 days, or 1% discount in 30 days, or 2% discount in 10 days. May 20, he makes a payment of $2000, and June 9, of $2500. How much would be due July 9, the end of the 60 days' credit?

20. Oct. 16, B bought a bill of merchandise amounting to $2000 on the following terms: 4 months, or 5% discount in 30 days, or 6% discount in 10 days. Oct. 26 he made a payment of $1000. How much would settle the bill Nov. 15?

21. B bought a bill of merchandise May 16 amounting to $3416.72 on the following terms: 4 *mos.*, or less 5% 30 days. He paid on account June 21 (6 days after the expiration of the 30 days) $3000, with the understanding that he should have the benefit of the discount by paying interest for the time elapsed, at 6% per annum. How much was due Sept. 16, no compound interest being reckoned?

22. A commission merchant in Chicago sells for me 12 bales brown sheeting, each bale containing 800 yards at 7 *cts.* per yard; pays transportation and other charges amounting to $72; and invests the proceeds in flour at $4.80 per barrel. If he charges 2½% for selling and 1¼% for purchasing, how many barrels of flour does he send me?

23. A of Chicago, sends to B of New Orleans, 8000 *bu.* of wheat and 500 *bbls.* of flour with instructions to sell it and invest the proceeds in sugar. B pays freight and cartage $3420; sells the wheat at $1.60 per bushel and the flour at $5.25 per barrel; charges 2½% commission on the flour and 1¢ per bushel on the wheat. How many pounds of sugar are purchased at 8¼ cents per pound, the commission for purchasing being 3%?

24. Mr. B. purchased 36150 pounds of hay at $16.50 per ton, and 16438 pounds of oats at 70 cents per bushel. He sold the hay at a gain of 16%, and the oats at a loss of 8%. What were the proceeds?

25. If I purchase two building lots for $3750 each, and sell one for $\frac{1}{3}$ more than it cost, and the other for $33\frac{1}{3}\%$ less, what is the gain or loss on the two lots?

26. A speculator sells two farms for $6000 each; how much does he gain or lose, if he sells one for 20% more than it cost, and the other for $\frac{1}{5}$ less than it cost?

27. Bought coal by the long ton at $3.64, and sold by the short ton at $4.25. What was the gain per cent?

28. Mr. A offered to sell his horse for 12% more than it cost him, but afterward sold it for $504, which was 10% less than his first asking price. How much did his horse cost him?

29. Find the interest of $375.60 for 1 *yr.* 10 *mo.* 22 *da.*, at 5%?

30. Find the interest of $4128 for 8 *mo.* 26 *da.*, at $4\frac{1}{2}\%$.

31. What is $2\frac{1}{2}\%$ of £159 13*s.* 10*d.*?

32. Find the date of maturity and the net proceeds of a note for $5000, dated May 19, payable 4 months after date, and discounted July 21 at 6%.

33. When the above note became due, its maker had discounted at 6% a new note, payable 90 days after date, whose proceeds were sufficient to pay the first note. What was the face of the new note?

34. Apr. 1, a merchant buys a quantity of coffee on 90 days' credit, with privilege of discounting within 30 days from date of purchase at the rate of 6% per annum for the unexpired time. Apr. 16, he makes a payment of $28000 on account, no actual invoice having been rendered. May 1, he receives the invoice, amounting to $29215, and on the same date full settlement is made What amount is required to cancel the bill? (Exact days, 360 days to the year.)

35. Divide $2000 in such a manner between two brothers, aged 16 and 19 years respectively, so that when they arrive at 21 years of age they will have equal amounts, money being worth 6% simple interest.

36. What would be the share of each if money is worth 6% compound interest?

nd the interest on $1000 from May 18 to Sept. 28, at 4[illegible]
interest and compound subtraction; 2. Ordinary in-
year days; 3. Accurate interest.
nd the amount due on the following note Jan. 1, 188[illegible]
[illegible] Rule and the Mercantile Rule:

Davenport, Iowa, May 1, 1878.
[illegible] I promise to pay Edwin D. Mathews, or order,
[illegible] dollars, with interest at six per cent., for value
E. H. Gordon.

[illegible] the following payments were indorsed:
[illegible] 1878, $[illegible]. Received Dec. [illegible], 188[illegible], $[illegible]
[illegible] 1879, $[illegible]. Received Aug. [illegible], 188[illegible], $[illegible]
[illegible] 1880, $[illegible]. Received Apr. [illegible], 188[illegible], $[illegible]
[illegible] would have been due on the above note [illegible]
[illegible]

[illegible]

48. Average the following account, and find the amount due Sept. 28, 1882, at 6%.

Mar. 16, 1882,	$874.32	on	30 days	credit.
" 31, "	518.65	"	60 "	"
May 5, "	373.78	"	4 months	"
" 21, "	429.31	"	60 days	"
June 18, "	657.70	"	30 "	"
July 3, "	242.28	"	60 "	"
" 24, "	983.75	"	4 months	"
Aug. 19, "	716.30	"	4 "	"
Sept. 13, "	536.60	"	60 days	"

49. Average the following account. What will be the amount due Jan. 1, 1883 ?

Dr. DANIEL S. LAMONT, *Albany, N. Y.* *Cr.*

1882.				1882.			
July 16	Mdse., 4 *mo.*	$876	14	Sept. 10	Cash, . . .	$900	00
Aug. 4	" 60 *da.*	415	65	" 21	" . . .	700	00
Sept. 10	" 30 *da.*	797	38	Oct. 13	" . . .	500	00
" 21	" 30 *da.*	686	96	" 31	Mdse., 30 *da.*	322	16
Oct. 13	" 4 *mo.*	524	27	Nov. 2	Cash, . . .	400	00
" 31	" 30 *da.*	859	75	Dec. 1	Note, 120 *da.*	800	00
Nov. 28	" 60 *da.*	263	31	" 27	Cash, . . .	500	00
Dec. 1	" 60 *da.*	172	64				
" 30	" 30 *da.*	938	52				

50. Prepare an account current, including interest at 6% to Jan. 1, 1883, from the above ledger account, according to the form and method of Art. **590.**

51. Sold five $1000 bonds at 116⅜, and invested the proceeds in railroad stock at 92⅞, which I sold at 98¼. What was the gain on the stock, allowing usual brokerage ?

52. Sold Aug. 11, 1879, 500 shares Chicago & Alton, at 94½, and covered my short sale Aug. 16, 1879, at 91. What was my profit, allowing the usual brokerage ?

53. What annual income will be obtained by investing $9923.75 in bonds, bearing 5% interest, and purchased at 116¾ ?

54. Bought stock at 116⅜ and sold at 112⅞. Loss, $1295. What was the par value of the stock ?

55. The tax levied in a town, having a valuation of $1800000, is $22500. What is the tax on $1, and what is the tax of A, whose real estate is assessed $5000 and personal property $1500 ?

56. What is the duty at 60% on an invoice of silk amounting to 36475 francs ?

57. What is the duty on 50 *cwt.* 3 *qr.* **14 *lb.*** (Long ton table) of steel at 2½*c.* per pound ?

58. Find the duty at 25% on an invoice of mdse. valued at £243 2*s.* 3*d.*

59. What is the duty on a block of marble 2 × 3 × 7 *ft.*, imported from Italy, dutiable value 3450 lire, and duty $1 per cubic foot and 25% ?

60. A, B, and C are partners in business, investing as follows : A, $4000 ; B, $6000 ; C, $8000. The partners are to share the profits and losses in proportion to their investments. Each is entitled to compensation for services at the rate of $150 per month, to be credited the first day of the following month. Interest is to be reckoned on the salaries and on the amounts drawn out at the rate of 6% per annum. At the end of the year B and C purchase the interest of A, and in the payment therefor, it is desired that the remaining members shall so invest that their interests shall be equal. It is mutually agreed that the "good will" of the business shall be valued at $3000 in the final settlement. It is also agreed that a discount of 5% shall be allowed upon all uncollected accounts as a fund to meet bad debts and costs for collecting. A statement of the business previous to closing shows the following results : merchandise, horses, wagon, office fixtures, and cash on hand, $12410; sundry debtors, $17030; sundry creditors, $4050; expense account (not including partners' salaries), $2400 ; profit on merchandise sold, $15290. A withdrew on account of salary Apr. 1, $450 ; July 1, $300 ; Oct. 1, $400. B withdrew Mar. 1, $400 ; Apr. 1, $150 ; June 1, $400 ; Oct. 1, $800 ; Dec. 1, $500. C withdrew Apr. 1, $600 ; July 1, $700 ; Oct. 1, $600 ; Nov. 1, $200. How much must B and C each invest or pay A, and how should the books of the new firm be opened ?

NOTES.—1. B and C, not desiring to have the new books encumbered with the contingent accounts of "good will" and "reserve fund," closed these accounts after a settlement was made with A.

2. The loss or gain may be found from a statement of resources and *liabilities*, or from the Loss and Gain account.

APPENDIX.

GREATEST COMMON DIVISOR.

664. The **Greatest Common Divisor** of two or more numbers is the *greatest* number that will divide each without a remainder; hence it is their greatest common factor.

Thus, 2, 3, 4, and 12 are common divisors of 36, 48, and 60; 12 is their greatest common divisor.

665. To find the greatest common divisor of two or more numbers.

Ex. What is the greatest common divisor of 168, 252, and 420?

OPERATION.

4) 168, 252, 420

7) 42, 63, 105

3) 6, 9, 15

2, 3, 5

$4 \times 7 \times 3 = 84$.

ANALYSIS.—Divide the given numbers by any number that will divide them all without a remainder, and divide the quotients in the same manner until the last quotients have no common divisor. Since 4 will divide all the given numbers, and 3 and 7 will divide successively the resulting quotients, their product, 84, is a common divisor of the given numbers. Since the last quotients have no common divisor or factor, 84 is the *greatest* common divisor.

666. RULE.—*Divide the given numbers by any factor that will divide all of them without a remainder. In like manner divide the resulting quotients, and continue the division until the quotients have no common factor. The product of the several divisors will be the greatest common divisor.*

EXAMPLES.

667. Find the greatest common divisor of the following numbers:

1. 108, 144, and 360.
2. 144, 336, and 240.
3. 165, 550, and 220.
4. 792, 144, and 216.
5. 405, 243, and 324.
6. 378, 126, and 252.
7. 375, 625, and 250.
8. 288, 720, and 864

668. To find the greatest common divisor of two numbers when they are not readily factored.

669. Principles.—*1. A common divisor of two numbers is a divisor of their sum, and also of their difference.*

2. A divisor of a number is a divisor of any multiple of that number.

670. Rule.—*Divide the greater number by the smaller, and divide the last divisor by the remainder; and so continue until there is no remainder. The last divisor will be the greatest common divisor.*

Notes.—1. When the greatest common divisor of more than two numbers is required, find the greatest common divisor of the smallest two first, and of this greatest common divisor and the next greater, and so on, until all the numbers are used. The last divisor will be the greatest common divisor of all the given numbers.

3. If the remainder at any time is a prime number, and it is not contained in the last divisor, there is no common divisor greater than 1; it will therefore be useless to further continue the division.

Ex. Find the greatest common divisor of 391 and 437.

OPERATION.

```
391 ) 437 ( 1
      391
      ---
       46 ) 391 ( 8
            368
            ---
             23 ) 46 ( 2
                  46
                  --
                   0
```

Demonstration.—Since 23 is a divisor of 46, it is a divisor of 368, a multiple of 46 (*Prin.* 2). Since 23 is a divisor of itself and 368, it is a divisor of their sum, 391 (*Prin.* 1). Since 23 is a divisor of 46 and 391, it is a divisor of their sum, 437. 23 is therefore a *common divisor* of 391 and 437, the given numbers.

The greatest common divisor of 391 and 437, whatever it may be, is a divisor of their difference, 46 (*Prin.* 1); also of 368, a multiple of 46 (*Prin.* 2); also of 23, 391 — 368 (*Prin.* 1). Since the divisor of a number cannot be greater than itself, the greatest common divisor of the given numbers cannot be greater than 23. 23 is therefore the *greatest common divisor*.

EXAMPLES.

671. Find the greatest common divisor of the following numbers:

1. 319 and 377.
2. 259 and 629.
3. 589 and 713.
4. 903 and 989.
5. 611, 799, and 987.
6. 744, 984, and 522.
7. 391, 667, and 920.
8. 451, 481, and 737.

ANNUAL INTEREST.

672. When a note contains the words "with interest annually," the laws of New Hampshire and Vermont, if the interest is not paid when due, allow simple interest on the annual interests from the time they become due to the time of payment.

ILLUSTRATION.—A agrees to pay B $6000 in three years from Jan. 1, 1880, with interest annually at 6%. By this contract, $360 becomes due Jan. 1, 1881, and on the first day of January in each year thereafter, until paid; this is the "annual interest." Suppose A does not pay any portion of this interest until Jan. 1, 1883, when the principal becomes due; then A, having had the use of money that his contract required him to pay to B, and B having been deprived of its use, B is entitled to have simple interest added to the annual interest, from the time when the same became due to Jan. 1, 1883; so that on Jan. 1, 1883, B would be entitled to the following sums as interest:

First year's int.	$360	+ 2 *yrs.* simple int. thereon,	$43.20 =	$403.20
Second " "	360	+ 1 " " " "	21.60 =	381.60
Third " "	360	+ 0 (paid when due)	00 =	360
	$1080		$64.80 =	$1144.80

Amount of annual interest	$1080.00
Amount of simple interest accrued upon annual interest .	64.80
Total amount of interest due	$1144.80

In calculating the simple int. upon the annual int., shorten the operation by finding the int. upon the annual int. for the sum of the several periods.

Ex. What is the amount due on the following note July 1, 1885?

$10000. CONCORD, N. H., *January 1, 1882.*

Three years after date, for value received, I promise to pay A. B. THOMPSON, or order, Ten Thousand Dollars, with interest payable annually.

C. A. DOWNS.

OPERATION.

Face of note, on interest from Jan. 1, 1882		$10000.00
Interest from Jan. 1, 1882, to July 1, 1885, 3 *yr.* 6 *mo.*		2100.00
3 items of annual interest ($600 each) are unpaid:		
1st from Jan. 1, 1883, to July 1, 1885,	2 *yr.* 6 *mo.*	
2nd from Jan. 1, 1884, to July 1, 1885,	1 *yr.* 6 *mo.*	
3rd from Jan. 1, 1885, to July 1, 1885,	6 *mo.*	
Int. on the annual int. = int. on $600 for 4 *yr.* 6 *mo.*		162.00
Total amount due July 1. 1885		$12262.00

NEW HAMPSHIRE RULE.*

675. According to the laws of New Hampshire, when payments are made upon a note, or other contract, by virtue of which interest is payable *annually* (**672**), they should be applied in the following order to the payment of—

1. Any simple interest that may have accrued upon the annual interest.

2. The annual interest. 3. The principal.

676. Rule.—*Find the interest due upon the principal and the annual interest at the annual rest (the time when the annual interest becomes due from year to year) next after the first payment. To the payment or payments made before this rest, add interest from the dates when they were made to the date of the rest, unless there is no interest due upon the principal, excepting that which is accruing during the year in which the payment or payments were made, and the payments together are less than the interest thus accruing, in which last case no interest is to be added to the payments. Deduct the payment or payments, with or without interest, as aforesaid, from the amount of principal, annual interest, and simple interest upon the annual interest due at the time of said rest, if such payment or payments equal or exceed the annual and simple interest then due; if less than such annual and simple interest, but greater than the simple interest due upon the annual interest, deduct the same from the sum of the annual and simple interest, and upon the balance of such annual interest find simple interest to the time when the next payment or payments are applied; if less than the simple interest due upon the annual interest, deduct the same from such simple interest and add the balance without interest to the other interest due at the time when the next payment or payments are applied.*

Proceed in like manner to the time of the first annual rest following the next payment, and to the end of the time required.

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* From Report of State Superintendent of Public Instruction (1877).

EXAMPLES.

677. *1.* According to the law of New Hampshire, how much is due Jan. 1, 1886, on a note dated Jan. 1, 1880, for $2000, with interest annually at 6%, the following payments having been made: July 1, 1882, $500; Oct. 1, 1883, $50.

OPERATION.

First annual interest due Jan. 1, 1831, $120 + 2 *yr.* simple interest thereon, $14.40		$134.40
Second annual interest due Jan. 1, 1882, $120 + 1 *yr.* simple interest thereon, $7.20		127.20
Third annual interest due Jan. 1, 1883,		120.00
Principal		2000.00
		$2381.60
First payment, July 1, 1882,	$500	
Interest thereon from July 1, 1882, to Jan. 1, 1883,	15	515.00
Balance of principal due Jan. 1, 1883,		1866.60
Fourth annual interest of $1866.60, due Jan. 1, 1884,		112.00
Second payment, Oct. 1, 1883 (being less than the interest accruing during the year, it does not draw interest)		50.00
Balance of fourth annual interest unpaid		62.00
Fifth annual interest of $1866.60, due Jan. 1, 1885,		112
Sixth annual interest of $1866.60, due Jan. 1, 1886,		112
Simple interest on unpaid balance of fourth annual int. for 2 *yr.*		7.44
Simple interest on fifth annual interest for 1 year		6.72
Balance of principal		1866.60
Amount due Jan. 1, 1886		2166.76

Solve Examples 2, 4, 8, and 9, Art. **505**, according to the New Hampshire Rule, at the legal rate (**436**), supposing each note to contain the words "with interest annually."

VERMONT RULE.

678. The Vermont Rule for notes with interest is essentially the same as the United States Rule (**504**); and for notes "with interest annually," it is the same as the New Hampshire Rule, except that when payments are made on account of interest accruing but not yet due, they draw interest from the date they were made to the annual rest, whether they are greater or not than the interest accruing during the year.

Thus, by the Vermont Rule, the payment of $50, in the above example, would draw interest from Oct. 1, 1883 to Jan. 1, 1884, or 3 months. The unpaid balance of fourth annual interest would be $61.25 ($112 − $50.75).

STORAGE.

679. **Storage** is keeping or storing of goods in a warehouse until they are required for use, sale, or transportation.

Storage is also the name applied to the price or compensation for storing goods in a warehouse.

680. Storage is usually calculated at a certain rate per barrel, bale, bushel, box, or other unit for a certain time.

1. The storage term is one week, 10 days, 20 days, or one month.
2. In some warehouses, storage for a part of a term is charged at the same rate as for a full term.

CASH STORAGE.

681. When the storage is paid or estimated when the goods are taken out of store or the receipt is surrendered, it is sometimes called *cash storage.*

EXAMPLES.

682. *1.* What was paid for the following storage at 6 cents per barrel per month or part of a month, the calculation being made at each delivery? Received Oct. 1, 1800 *bbl.*; Nov. 15, 360 *bbl.*; Dec. 18, 420 *bbl.*; Dec. 27, 432 *bbl.* Delivered Oct. 31, 1000 *bbl.*; Dec. 4, 240 *bbl.*; Dec. 19, 600 *bbl.*; Dec. 26, 300 *bbl.*

OPERATION.

Date.	Received.	Delivered.
Oct. 1	$\cancel{1}\cancel{8}\cancel{0}\cancel{0}$	
" 31	$\cancel{8}\cancel{0}\cancel{0}$ $\cancel{5}\cancel{6}\cancel{0}$	$\cancel{1}\cancel{0}\cancel{0}\cancel{0}$
Nov. 15	$\cancel{3}\cancel{6}\cancel{0}$	$\cancel{2}\cancel{4}\cancel{0}$
Dec. 4	$\cancel{3}\cancel{2}\cancel{0}$ 20	
" 18	420	$\cancel{6}\cancel{0}\cancel{0}$
" 19		$\cancel{3}\cancel{0}\cancel{0}$
" 26		
" 27	432	
On hand	872	

1000 @ .06 = 60.00
240 @ .18 = 43.20
560 @ .18 = 100.80
40 @ .12 = 4.80
300 @ .12 = 36.00
$244.80

ANALYSIS.—All goods delivered are deducted from the oldest receipt on hand. By the system of cancellation indicated in the operation, it can be easily determined when the storage commences. The 1000 bbl. taken out

Oct. 31 were placed in store Oct. 1, and pay 1 month's storage. The 240 *bbl.* taken out Dec. 4 were placed in store Oct. 1, and pay 3 months' storage. Of the delivery of 600 *bbl.*, 560 were placed in store Oct. 1, and pay 3 months' storage, and the remainder, 40 *bbl.*, were placed in store Nov. 15, and pay 2 months' storage, Dec. 19. The lot of 300 *bbl.* withdrawn Dec. 26, were placed in store Nov. 15, and pay 2 months' storage. The separate calculations are placed at the right in the above operation.

Notes.—1. Certain warehouses render bills at the end of each month for all goods taken out during the month. Others render bills monthly for all storage dues, whether the goods have been withdrawn or not.

2. Storage on goods for which negotiable receipts have been issued, and in many other cases, is collected when the receipt is surrendered or the goods delivered.

2. What will be the storage at 5 cents per barrel per month on the following? Received Aug. 1, 800 *bbl.*; Aug. 15, 700 *bbl.*; Aug. 26, 900 *bbl.* Delivered Aug. 12, 400 *bbl.*; Aug. 20, 800 *bbl.*; Sept. 1, 400 *bbl.*; Sept. 8, 800 *bbl.*

3. Find the storage due on the following account June 24, at 3 cents a bale per month or part of a month. Received Apr. 13, 400 bales; Apr. 30, 800 bales; May 16, 200 bales; May 25, 400 bales; June 19, 600 bales. Delivered May 10, 600 bales; May 20, 100 bales; May 28, 700 bales; May 31, 400 bales; June 24, 600 bales.

4. Complete the following storage bill, the rate being 10*c.* per bale per month or part of a month.

Baltimore, Md., *Aug. 31, 1887.*

Messrs. Armstrong, Cator & Co.,

To Merchants' Storage Co., *Dr.*

	Marks and Numbers.	When received.	When delivered.	Rate.	Amount.	
4	Bales A C #174–177	June 4	Aug. 2	20*c.*		80
1	" A C #298	May 21	" 4	30*c.*		30
5	" A. C. & Co. #21–25	July 26	" 7	10*c.*		50
3	" A C #170–172	June 4	" 16	**		**
2	" C # 20–28	Aug. 17	" 20	**		**
6	" A. C. & Co. #15–20	July 26	" 22	**	*	**
1	" A C #173	June 4	" 24	**		**
4	" A C #299–302	May 21	" 27	**	*	**
5	" A. C. & Co. #26–30	July 26	" 31	**	*	
					*	**

Note.—In many cases (see above example), storage is charged for the time the particular packages withdrawn have been in store.

5. The following quantities of wheat were stored at 1*c.* a *bushel* per month or part of a month. What was the amount of storage due June 1, a full settlement being made on that date and a new receipt being given? Received Apr. 7, 600 bushels; Apr. 25, 400 bushels; May 9, 400 bushels; May 27, 300 bushels.

AVERAGE STORAGE.

683. At some warehouses, in computing storage on grain, flour, etc., when there are frequent receipts and deliveries, it is customary to average the time and charge a certain rate per month of 30 days. The process is called *average storage*, or *storage on account.*

684. Ex. Merchandise was received and delivered at a warehouse as follows: Received Oct. 1, 1800 *bbl.* flour; Nov. 15, 360 *bbl.*; Dec. 18, 420 *bbl.*; Dec. 27, 432 *bbl.* Delivered Oct. 31, 1000 *bbl.*; Dec. 4, 240 *bbl.*; Dec. 19, 600 *bbl.*; Dec. 26, 300 *bbl.* Find the average storage due on the above Jan. 1, at 6 cents per barrel per month of 30 days.

685. Operation.—Product Method.

Date.	Received.	Days.	Products.	Date.	Delivered.	Da	Products.
Oct. 1	1800	92	165600	Oct. 31	1000	62	62000
Nov. 15	360	47	16920	Dec. 4	240	28	6720
Dec. 18	420	14	5880	" 19	600	13	7800
" 27	432	5	2160	" 26	300	6	1800
	3012		190560		2140		78320
	2140		78320				
On hand	872	30)	112240	3741⅓ × .06 = 224.48			
			3741⅓				

Analysis.—Assuming that there was nothing withdrawn, the 1800 *bbl.* would be in store from Oct. 1 to Jan. 1, or 92 days. The storage of 1800 *bbl.* for 92 days is equivalent to the storage of 1 *bbl.* for 165600 days. The storage of 360 *bbl.* for 47 days is equivalent to the storage of 1 *bbl.* for 16920 days. In the same manner, we find the total storage, if nothing had been withdrawn, to be equivalent to the storage of 1 *bbl.* for 190560 days. The storage on the goods withdrawn is equivalent to the storage of 1 *bbl.* for 78320 days, thus making the net storage 1 *bbl.* for 112240 days, or 3741⅓ months. 6 cents multiplied by 3741⅓ equals $224.48, the total storage.

686. OPERATION.—BY DAILY BALANCES.

Date.	Received.	Delivered.	Balances.	Days.	Products.
Oct. 1	1800		1800	30	54000
" 31		1000	800	15	12000
Nov. 15	360		1160	19	22040
Dec. 4		240	920	14	12880
" 18	420		1340	1	1340
" 19		600	740	7	5180
" 26		300	440	1	440
" 27	432		872	5	4360
	3012	2140		92	30) 112240
Jan. 1	Bal. on hand	872			$3741\frac{1}{3}$
	3012	3012			

$3741\frac{1}{3} \times .06 = 224.48$

ANALYSIS.—Arrange the receipts and deliveries in the order of their dates as in the operation. Find the number of barrels on hand at each of the dates. The 1800 *bbl.* are in store from Oct. 1 to Oct. 31, or 30 days. The storage of 1800 *bbl.* for 30 days is equivalent to 1 *bbl.* for 54000 days. The total storage is equivalent to the storage of 1 *bbl.* for 112240 days, or $3741\frac{1}{3}$ months. 6 cents multiplied by $3741\frac{1}{3}$ equals $224.48, the total storage.

EXAMPLES.

687. *1.* Find by either of the above methods the average storage at 5 cents per month of 30 days, of the account given in Ex. 2, Art. **682**.

2. Find the average storage, at 3 cents per month of 30 days, of the account given in Ex. 3, Art. **682**.

3. Find the total charge for pasturing cattle per the following statement, at 30 cents a head per week: Received July 5, 18 head; July 12, 10 head; July 20, 30 head; Aug. 2, 40 head; Aug. 10, 20 head; Sept. 1, 10 head; Sept. 4, 24 head; Sept. 17, 26 head; Oct. 2, 20 head; Oct. 27, 18 head; Nov. 1, 6 head; Nov. 2, 16 head. Withdrawn July 7, 4 head; July 9, 8 head; July 14, 10 head; July 17, 6 head; July 23, 20 head; Aug. 4, 20 head; Aug 13, 20 head; Aug. 21, 12 head; Aug. 29, 4 head; Sept. 8, 10 head; Sept. 14, 30 head; Sept. 21, 18 head; Oct. 30, 20 head; Nov. 5, 10 head; Nov. 9, 20 head; Nov. 16, 26 head.

NOTE.—When the account is long, the first method is preferable.

ALLIGATION.

688. Alligation treats of mixing ingredients of different values to find the value of the mixture, or to produce a mixture of a given value.

NOTE.—Alligation is sometimes and more properly called *Average*.

ALLIGATION MEDIAL.

689. Alligation Medial is the process of finding the average value of a mixture, the rates and quantities of the ingredients being given.

EXAMPLES.

690. *1.* A grocer mixes together 7 pounds of coffee at 26 cents per pound, 4 pounds at 27 cents per pound, and 10 pounds at 34 cents per pound. What is the value of a pound of the mixture ?

OPERATION.

7 × 26*c.* = $1.82
4 × 27*c.* = 1.08
10 × 34*c.* = 3.40
21 *lb.* worth $6.30
1 *lb.* " .30

ANALYSIS.—7 *lb.* at 26*c.* are worth $1.82. 4 *lb.* at 27*c.* are worth $1.08. 10 *lb.* at 34*c.* are worth $3.40. Hence the total mixture containing 21 *lb.* is worth $6.30, and 1 *lb.* is worth $6.30 ÷ 21, or 30*c.*

2. A wine merchant mixed together 10 gallons of wine at 40 cents a gallon, 15 gallons at 50 cents, and 25 gallons at 70 cents. What is the value of a gallon of the mixture ?

3. A grocer mixed 60 *lb.* of tea at 25 cents a *lb.*, 75 *lb.* at 30 cents, and 65 *lb.* at 50 cents. What was the value of a pound of the mixture ?

4. A farmer mixes together 20 *bu.* of oats at 40 cents a bushel, 30 *bu.* of corn at 50 cents a bushel, and 50 *bu.* of rye at 70 cents a bushel. What is the value of a bushel of the mixture ?

ALLIGATION ALTERNATE.

691. Alligation Alternate is the process of finding the quantities of different values required to produce a mixture of a given value.

692. The values of several ingredients being given, to produce a mixture of a given value.

EXAMPLES.

693. *1.* How much tea worth 24, 28, 36, and 42 cents a pound must be mixed together, so that the mixture will be worth 30 cents a pound ?

OPERATIONS

		(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)	(*7*)	(*8*)	(*9*)	
30	24	$\frac{1}{6}$	0	1	0	1	2	3	1	3	etc.
	28	0	$\frac{1}{2}$	0	6	6	6	6	12	12	etc.
	36	$\frac{1}{6}$	0	1	0	1	2	3	1	3	etc.
	42	0	$\frac{1}{12}$	0	1	1	1	1	2	2	etc.

ANALYSIS.—If we sell 1 pound for 30 cents, that is worth 24 cents, we gain 6 cents, and to gain 1 cent, we take $\frac{1}{6}$ of a pound. We must now take a kind that is worth more than the average price so as to lose one cent. If we take a pound worth 36 cents and sell it at 30 cents, we will lose 6 cents, and to lose one cent, we must take $\frac{1}{6}$ of a pound. In the same manner, we find that if we take $\frac{1}{2}$ of a pound of the 28-cent tea and mix it with $\frac{1}{12}$ of a pound of the 42-cent tea, there will be no gain nor loss by selling at 30 cents a pound.

$\frac{1}{6}$ is to $\frac{1}{6}$ as 1 is to 1, and $\frac{1}{2}$ ($\frac{6}{12}$) is to $\frac{1}{12}$ as 6 is to 1. Or, columns 3 and 4 may be found by multiplying columns 1 and 2 respectively by the least common denominators of the fractions.

Column 5 is the sum of columns 3 and 4. An unlimited number of answers may be found to examples of this kind by combining 1, 2, or 3, etc., times column 3 with 1, 2, or 3, etc., times column 4.

2. A grocer has sugar at 5¢, 7¢, 12¢, and 13¢ per pound. How much of each kind will form a mixture worth 10 cents per pound ?

3. A jeweler wishes to make a compound of gold that shall be 20 carats fine. He has gold of 15, 19, 23, and 24 carats fine. What quantity of each must he take ?

4. How much tea at 25 cents, 50 cents, 60 cents, and 80 cents per pound must be taken to form a mixture worth 55 cents per pound ?

5. How much wine at 50 cents, 70 cents, 80 cents, $1.00, and $1.20 a gallon must be mixed together that the mixture may be worth 90 cents a gallon ?

694. When the quantity of one ingredient is given.

EXAMPLES.

695. *1.* How much coffee at 30, 34, and 44 cents per pound, must be mixed with 10 pounds at 36 cents a pound, to make a mixture worth 40 cents a pound ?

OPERATION.

		(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8*
40	30	$\frac{1}{10}$	0	0	2	0	0	0	2
	34	0	$\frac{1}{6}$	0	0	2	0	0	2
	44	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	5	3	1	10	18
	36	0	0	$\frac{1}{4}$	0	0	1	10	10

ANALYSIS.—We find the relative quantities (columns 4, 5, and 6) as in Art. **693**. In order to use 10 pounds of the 36-cent coffee, we multiply column 6 by 10, producing column 7. Column 8 is found by adding columns 4, 5, and 7.

Other combinations may be found, by multiplying columns 4 and 5, and adding the results to column 7.

2. How much coffee at 15*c.*, 17*c.*, and 22*c.* a pound must be mixed with 5 *lb.* at 18*c.* per pound to make a mixture worth 20*c.* per pound.

3. How much gold of 21 and 23 carats fine, must be mixed with 30 *oz.* of 20 carats fine, so that the mixture may be 22 carats fine ?

4. How much tea at 20 cents, 25 cents, and 45 cents a pound, must be mixed with 36 *lb.* at 60 cents a pound, so that the mixture will be worth 40 cents a pound ?

5. How much wine at $1.25 and $1.75 a gallon, must be mixed with 15 gallons of water, so that the mixture may be worth $1 a gallon ?

6. How much tea at 30 cents, 46 cents, and 48 cents a pound, must be mixed with 12 pounds at 38 cents, so that the mixture may be worth 40 cents a pound ?

696. When the total quantity of the ingredients is given.

EXAMPLES.

697. *1.* A grocer mixed tea worth 20, 25, and 35 cents a pound. The mixture consisted of 50 pounds, worth 29 cents a pound. How many pounds of each did he take?

		(1)	(2)	(3)	(4)	(5)		(6)
29	20	$\frac{1}{9}$	0	2	0	2	× 5 =	10
	25	0	$\frac{1}{4}$	0	3	3		15
	35	$\frac{1}{6}$	$\frac{1}{6}$	3	2	5		25
				5	5	10	) 50 (5	

ANALYSIS.—We find columns 3 and 4 as in Art. **693**. Column 5 is the sum of columns 3 and 4. The required amount is 50 = 5 times 10, the sum of column 5. Hence the quantity of each may be found by multiplying each number in column 5 by 5.

NOTE.—Many results may be obtained for examples of this kind. Thus, in the above example, 9 times column 3 plus once column 4, 8 times column 3 plus 2 times column 4, etc., would each produce correct results.

2. How much wine worth 50 cents, 60 cents, 90 cents, and $1.20 a gallon, must be mixed together so as to make a hogshead of 110 gallons at 80 cents a gallon?

3. A man bought 20 barrels of flour for $120, paying $4½, $5, $6½, and $7 per barrel. How many barrels of each did he buy?

OPERATION.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
6	4½	$\frac{2}{3}$	0	1	0	1	1	0	2	2	0	3	3
	5	0	1	0	8	0	8	6	0	6	4	0	4
	6½	2	0	3	0	3	3	0	6	6	0	9	9
	7	0	1	0	8	0	8	6	0	6	4	0	4
			2	4			20			20			20

ANALYSIS.—Find columns 2 and 3 as in Art. **693**. Since 6, the sum of columns 2 and 3, is not an exact divisor of 20, the required amount, we must take a certain number of times column 2 and a certain number of times column 3. By trial, we find that 8 times column 2 plus once column 3 equals 20. Therefore multiply column 2 by 8, producing column 4, and column 3 by 1, producing column 5. Column 6 is the sum of columns 4 and 5. In the same manner, we find the results given in columns 9 and 12.

4. A man bought 50 animals for $50, paying for lambs $⅓ each, for sheep $1⅓ each, and for calves $3¼ each. How many of each did he buy?

SQUARE ROOT.

698. The **Square Root** of a number is one of the two equal factors of a number. Thus, the square root of 25 is 5. 5 × 5 = 25.

699. To find the square root of a number.

700. Rule.—*Beginning at units' place, separate the given number into periods of two figures each.*

Find the greatest square in the left-hand period, and write its root at the right in the form of a quotient in division. Subtract this square from the left-hand period, and to the remainder annex the next period to form a dividend.

Double the part of the root already found for a trial divisor. Find how many times this divisor is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root. Annex this quotient to the right of the trial divisor to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

To the remainder annex the next period, and proceed as before.

Note.—When the given number is a decimal, separate the number into periods of two figures each, by proceeding in both directions from the decimal point.

EXAMPLES.

701. *1.* Find the square root of 1156.

Operation.

```
   | 11'56 ( 34
   |  9
   |-----
64 | 256
   | 256
   |-----
   |   0
```

Analysis.—Beginning at units' place, separate the number into periods of two figures each. The greatest square in the left-hand period (11) is 9, and the root is 3, which is written in the quotient. By subtracting this square (9) from the left-hand period (11) and annexing to the remainder (2) the next period (56), we form the dividend, 256. By taking twice the root already found (3), we have 6 as a trial divisor, which is contained in the dividend (25), exclusive of the last figure, 4 times. Write 4 in the quotient, and also to the right of the trial divisor, forming the complete divisor, 64. Multiplying the complete divisor, 64, by 4, the last figure of the root, and subtracting the product (256) from the dividend (256), there is no remainder. 34 is the required root.

Find the square root of

2. 1089.
3. 14641.
4. 18225.
5. 46656.
6. 47524.
7. 65025.
8. 86436.
9. 97344.
10. 119025.
11. 1406.25.
12. 512656.
13. 232.5625.
14. 3976036.
15. 431.8084.
16. 7463824.
17. 387420489.

18. Find the square root of $\frac{256}{625}$. Of $\frac{1369}{1600}$. Of $\frac{5625}{12321}$.

NOTE.—In finding the square root of a fraction, extract the square root of the numerator and denominator separately.

19. Find the side of a square field whose area is 412164 square rods.

20. Find the side (in feet) of a square whose area is one acre.

21. Extract the square root of 2 to 5 decimal places.

CUBE ROOT.

702. The **Cube Root** of a number is one of the three equal factors of that number. Thus, the cube root of 8 is 2. 2 × 2 × 2 = 8.

703. To find the cube root of a number.

704. RULE.—*Beginning at units' place, separate the given number into periods of three figures each.*

Find the greatest cube in the left-hand period, and write its root at the right in the form of a quotient in division. Subtract this cube from the left-hand period, and to the remainder annex the next period to form a dividend.

Multiply the square of the root already found by 300 for a trial divisor. Find how many times this divisor is contained in the dividend, and write the quotient as the next figure of the root. Add to the trial divisor thirty times the product of the last figure of the root and the other figures of the root, and the square of the last figure to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

To the remainder annex the next period, and proceed as before.

EXAMPLES.

705. 1. Find the cube root of 39304.

OPERATION.

		39′304 (34
		27
Trial divisor,	$300 \times 3^2 = 2700$	12 304
	$30 \times 3 \times 4 = 360$	
	$4^2 = 16$	
Complete divisor, ...	3076	12 304
		0

ANALYSIS.—Beginning at units' place, separate the number into periods of three figures each. The greatest cube in the left-hand period (39) is 27, and its root is 3, which is written in the quotient. By subtracting this cube (27) from the left-hand period (39), and annexing to the remainder (12) the next period (304), we form the dividend, 12304. By multiplying the square of the root already found by 300 (multiply by 3 and add two ciphers), we form the trial divisor, 2700 ($300 \times 3^2 = 2700$). The trial divisor, 2700, is contained in the dividend, 12304, 4 times. Write 4 as the next figure of the root. To form the complete divisor, add to the trial divisor (2700) 30 times the product of the last figure (4) of the root and the other figure (3) ($30 \times 3 \times 4 = 360$), and the square of the last figure ($4^2 = 16$). Multiplying the complete divisor, 3076, by 4, the last figure of the root, and subtracting the product (12304) from the dividend (12304), there is no remainder. 34 is the required cube root.

NOTE.—When the given number is a decimal, separate the number into periods of three figures each, by proceeding in both directions from the decimal point.

Extract the cube root of

2. 5832.
3. 10.648.
4. 314432.
5. 474552.
6. 551368.
7. 7529536.
8. 9.663597.
9. 13651.919.
10. 98611128.
11. 279.726264.
12. 435519.512.
13. 676836152.

14. The product of three equal numbers is 551368. What are the numbers?

15. A cubical block of marble contains 103823 cubic inches. What is its length?

16. Find the cube root of [illegible]. Of [illegible]. Of [illegible].

NOTE.—In finding the cube root of a fraction, extract the cube root of the numerator and denominator separately.

17. A cubical box holds 20 bushels. Find the length of its side

MENSURATION.

706. To find the area of a rectangle (319) or parallelogram.

707. Rule.—*Multiply the length by the width.* **(320)**

708. To find the area of a triangle.

709. Rule.—*Multiply the base by half the perpendicular height.*

710. To find the area of a triangle when the three sides are given.

711. Rule.—*From half the sum of the three sides subtract each side separately. Then multiply the half sum and the three remainders together, and extract the square root of the continued product.*

712. To find the area of a trapezoid.

713. Rule.—*Multiply half the sum of its parallel sides by the perpendicular distance between them.*

714. A **Trapezoid** is a four-sided figure having only two of its sides parallel.

715. To find the area of a circle.

716. Rule.—*Multiply the square of the diameter by .7854; or multiply the square of the radius by 3.1416.*

717. To find the surface of a sphere.

718. Rule.—*Multiply the square of the diameter by 3.1416.*

719. To find the solid contents of a prism or cylinder.

720. Rule.—*Multiply the area of the base by the height or length of the prism or cylinder.*

721. To find the solid contents of a sphere.

722. Rule.—*Multiply the cube of the diameter by .5236.*

723. GEOMETRICAL CONSTANTS.

CIRCLES.

Circumference = Diameter × 3.1416.
Circumference = Radius × 6.2832.
Radius = Circumference ÷ 6.2832.
Radius = Circumference × .1592.
Radius = $\sqrt{\text{Area} \div 3.1416}$.
Radius = $\sqrt{\text{Area}} \times .5642$.
Diameter = Circumference ÷ 3.1416.
Diameter = Circumference × .3183.
Diameter = $\sqrt{\text{Area} \div .7854}$.
Diameter = $\sqrt{\text{Area}} \times 1.1284$.
Area = Radius square × 3.1416.
Area = Diameter square × .7854.
Side of inscribed square = Diameter × .7071.
Side of equal square = Diameter × .8862
Side of inscribed equilateral triangle = Diameter × .8603

SPHERES.

Surface = Diameter square × 3.1416.
Surface = Circumference square × .3183.
Volume = Diameter cubed × .5236.
Volume = Circumference cubed × .0169.
Diameter = $\sqrt{\text{Surface}} \times .5642$.
Diameter = $\sqrt[3]{\text{Volume}} \times 1.2407$.
Radius = $\sqrt{\text{Surface}} \times .2821$.
Radius = $\sqrt[3]{\text{Volume}} \times .6204$.
Circumference = $\sqrt{\text{Surface}} \times 1.7725$.
Circumference = $\sqrt[3]{\text{Volume}} \times 3.8978$.
Side of inscribed cube = Diameter × .5774.
Side of inscribed cube = Radius × 1.1547.

ROOTS, ETC.

Diagonal of square = Side × 1.4142.
Square root of 2 = 1.4142.
Side of square = Diagonal × .7071.

EXAMPLES.

[illegible] is the area of a rectangular piece of land [illegible] and 12.50 chains wide?

[illegible] of a triangular piece of land whose base is [illegible] perpendicular width 6.30 chains.

[illegible] area of a triangular piece of ground whose [illegible] and 45 rods respectively?

[illegible] in the form of a trapezoid has two parallel [illegible] and 750 ft. respectively, and the perpendicular [illegible] them is 2500 ft. What is the area?

[illegible] of a circular race-course is one mile. [illegible] that in feet?

[illegible] circumference of a circle whose diameter is 150

[illegible] by a rope 42 feet long. Upon how

[illegible] What is the area of

[illegible] and whose

[illegible] whose diameter is

[illegible] with the [illegible]

[illegible], if a cubic ft. of the

[illegible] cylindrical bin, whose con-

[illegible] What is the length

[illegible] ft. long and 2 ft. in

[illegible] whose side was 204 ft., 10000

[illegible] were taken out. What was its depth?

GENERAL AVERAGE.

725. If, in time of danger or distress, any loss or expense is voluntarily incurred for common safety of vessel, freight, and cargo, such loss or expense is made good by a "**General Average**;" the amount or value of such loss or expense being assessed upon the value of all interests involved and benefited.

All other losses and expenses are of a "**Particular Average**" nature, and are to be borne by the specific interests to which they apply.

726. The losses and expenses constituting general average are as follows:

1. Jettison, or throwing overboard of cargo to lighten the ship; damage to cargo by water going down the hatches during jettison; damage by chafing or breaking after jettison; freight on cargo jettisoned.

2. Sacrificing ship's materials, as the cutting away of masts, spars, etc. One-third of the cost of repairs of ship's materials is a special charge on the ship, as the new work is considered better than the old. No deduction is made for anchors.

3. Expense of floating a stranded ship.

4. Expense of entering a port of refuge, either to repair damage which renders it dangerous to remain at sea, whether such damage were caused by accident or sacrifice; or otherwise to avert a common danger.

5. Expense of discharging cargo for the purpose of making repairs, warehouse rent, reloading cargo, outward expenses, etc.

6. Wages and provisions of crew from the date of bearing up until ready for sea.

727. Contributory Interests and Values.—The ship contributes on its full value at the time which is made the basis of contribution.

The cargo contributes on its net market value at the port of destination, less freight and charges saved.

The freight contributes on the full amount, less ½ for the wages, etc., of crew. In the States of New York, Virginia, California, and some others, ⅓ is deducted.

The underwriters (insurance companies) contribute to the general average such a part of the expense as the insured value is of the market value of the goods [illegible]. For example, a cargo is insured for $2000 and is worth in the market $3000; the underwriters are liable to pay ⅔ of the general average expense.

[illegible] To give rise to general average, it must be shown that there was an imminent common danger, that the sacrifice was voluntary and necessary, and that the act was prudent and successful.

[illegible] An **Average Adjuster** is a person who is familiar with the general average laws of the leading commercial nations, and who adjusts and apportions the losses and expenses of a general average.

The principal difficulty of the adjuster is to determine whether the loss should [illegible] general average [illegible] charge [illegible] After the general average [illegible] the apportionment of [illegible] the several [illegible] problem.

EXAMPLES.

[illegible] 1. The bark [illegible] from New York [illegible] with the following cargo: [illegible] freight. How is the settlement to be made, the value of the ship being $10000 and the adjuster's fee $100 ?

NOTES.—1. In a general average, extracts from the log of the ship, the testimony of its officers, a complete statement of all expenses incurred, with the vouchers for the same, and all papers having any bearing upon the case are presented to the adjuster. The total amount of each item is entered in a column at the left of his statement of charges, and the amount is also entered in its proper column at the right. In addition to the general average column, there are usually columns to the right for the special charges upon the ship, owners, or cargo.

2. After determining the general average loss, divide it among the contributory interests in proportion to their values, by any of the methods given in Ex. 13, page 293.

STATEMENT OF CHARGES.

Total.			General Average.		Ship and Owners.	
1369	43	Expense of entering harbor, landing cargo, etc.	1369	43		
436		Bill of H. Robin & Co., shipwrights, etc.			436	
194	14	" " Joseph Patti, ceiling ship.			194	14
99	98	Agent's commission for advancing funds and paying above bills, 5%.	68	47	31	51
217	35	Agent's commission on value of cargo landed, $17388, 1¼%.	217	35		
630	47	Wages, etc., of seamen.	630	47		
100		Adjuster's fee.	100			
		General average.	2385	72		
3047	37	Ship and owners.			661	65

CONTRIBUTORY INTERESTS AND APPORTIONMENTS IN GENERAL AVERAGE.

		Ship,	value	10000	@	.045	pays			450	
		Freight,	8096								
		Less ½,	4048	4048	@	.045	"			182	16
		Cargo,									
		A,	5600		@	.045	"	252			
		B,	8700		@	.045	"	391	50		
		C,	16308		@	.045	"	733	86		
		D,	8360		@	.045	"	376	20		
				38968	@	.045	"			1753	56
				53016	@	.045	"			2385	72

$2385.72 ÷ $53016 = .045.

SETTLEMENT

	Dr.		Cr.		Balances.			
					To pay.		To receive.	
Vessel and Owners.								
Pay ship's proportion of Gen. Aver.	450							
" freight's "	182	16						
" owner's column.	661	65						
Receive seamen's wages.			630	47	663	34		
Cargo.								
Pay proportion of Gen. Average.	1753	56			1753	56		
Agents of Vessel.								
Receive their disbursements.			1999	57				
" " commission.			317	33			2316	90
Adjusters.								
Receive their fee.			100				100	
	3047	37	3047	37	2416	90	2416	90

2. The general average charges were $4375.86, and the contributory interests $64325. What was the per cent. of loss? What was the loss of Mr. B., whose goods were valued at $7250?

3. Suppose A's goods in Ex. 1 were insured for $5000, how much of the loss would be shared by the insurance company?

4. The ship *Amazon*, from Aspinwall to New York, being in distress, threw overboard part of the cargo, cut away the masts, and finally bore away to a port of refuge to repair in order to complete the voyage. The cost of replacing masts and rigging cut away was $6000 (less $\frac{1}{3}$ new for old); the cargo jettisoned was worth compared with sound cargo delivered at destination $2000; freight on cargo jettisoned, $200; expenses of entering port of refuge, discharging, storing and reloading cargo, $1000; wages of master and crew from time of bearing away until ready for sea, $600; provisions of master and crew for same time, $500; adjuster's fee, $100. The vessel was valued at destination at $20000 (deduct gross repairs and add amount made good); cargo, value on arrival, $40000 (add amount made good); freight collected, $4000 (add amount made good and deduct $\frac{1}{3}$). What was the per cent. of loss, and how was the settlement made?

5. The cargo of the ship *Amazon* was insured for $36000. How much was the claim against the insurance company?

6. The ship *Union,* in her passage from Liverpool to Boston, during a storm threw overboard cargo to the amount of $1580, and cut away masts and rigging. She then entered the port of Halifax for repairs. The cost of replacing the masts and rigging which were voluntarily sacrificed, was $4578 (less ⅓ new for old); cost of repairing accidental damage, $568; freight on cargo jettisoned, $314.75; expense of entering port of refuge, discharging cargo, etc., $716.87; wages and provisions of crew, $608; adjuster's fee, $150. The value of vessel on arrival at Boston was $30000 (deduct gross repairs and add amount made good); value of cargo delivered, less freight and duty, $48475 (add amount jettisoned); total expected earning of freight, $16320 (less ⅓ in Boston. See Art. 727). The cargo was shipped by the following persons: A $8519, B $20376, C $6875, and D $14285. The cargo jettisoned was a part of A's shipment. How ought the settlement to be made?

7. The ship *Ocean Queen,* from Pernambuco to New York, sprang a leak off Cape St. Roque, and for the safety of the vessel and cargo, threw overboard part of the cargo and put into Maranham for repairs. The disbursements at Maranham by the master of the vessel, including commissions, were as follows: Expenses of entering harbor, discharging, storing, and reloading cargo, $648.75; caulking and painting ship, carpenter work, etc., $843. Value of cargo delivered at New York, $34310.24; of cargo jettisoned, $1580.76; freight on cargo jettisoned, $364; wages and provisions of crew, $304; adjuster's fee, $150; agent's commission for collecting amount in general average, 2½%. How shall the settlement be made, if the net value of the ship was $3157 (value on arrival $4000, less repairs $843), and the total expected earning of freight was $2516 (less ½)?

8. A vessel which put into a port of refuge for repairs was without funds. It being very difficult to obtain a loan on bottomry, or to negotiate a draft on the owners of the vessel, the master was obliged to sell part of the cargo to raise funds. Value of cargo sold compared with cargo delivered at destination, $4566.06; produced at sale, $2985.30; freight on cargo sold compared with freight on cargo delivered, $363.93. What was the cost of funds, and how much should be apportioned to each interest, the general average charges being $773.52, the special charges on ship $956.10, and on the owners $1181.06?

REIGN WEIGHTS AND MEASURES.

[illegible]

(See Germany.)

(Metric System.)

[illegible]

(Metric System.)

[illegible]

(Metric System.)

[illegible]

1 Fod (Foot) = 1.03 U. S. ft.
1 Viertel = 2.04 U. S. gal.
1 Pot = .2554 gal.

Coinage laws are metric. The introduction of complete metric system is in prospect.

ECUADOR. (METRIC SYSTEM.)

EGYPT. (METRIC SYSTEM.)

FRANCE. (METRIC SYSTEM.)

The old French aune = 1⅓ yd. is still used to some extent in the silk industries of France and the U. S.

GERMANY.

Metric system with a few changes in subdivisions in general use.

1 Pound (½ kilogram) = 1.1023 lb. av.
1 Centner (100 pounds) = 110.23 lb. av.
1 Tonne (metric ton) = 2204.6 lb. av.

GREAT BRITAIN.

1 Imperial Gallon = 1.2 U. S. gal.
1 Imperial Bushel = 1.03 U. S. bu.
1 Imperial Quarter = 8.25 U. S. bu.
1 Ale or Beer Gallon = 1.22 U. S. gal.
1 Cental = 100 lb.
1 Quarter of Wheat at London = 480 lb.
1 [illegible] = 408 lb.
1 [illegible] = 448 lb.

Metric system permitted by law in 1864.

GREECE.

Metric system with the old denominations in general use.

In the Ionian Islands the English

India.

1 Seer = 16 chattucks.
1 Bombay Maund of 40 seers = 28 *lb.* av.
1 " " 42 " = 29.4 "
1 Surat " 40 " = $31\frac{1}{3}$ "
1 " " 42 " = $39\frac{1}{3}$ "
1 " " 44 " = $41\frac{1}{15}$ "
1 Bengal Factory Maund = $74\frac{2}{3}$ "
1 " Bazaar " = $82\frac{1}{3}$ "
1 Madras Maund = 25 "
1 Bom'y Candy of 20 Maunds = 560 "
1 Surat " " " = $746\frac{2}{3}$ "
1 Madras " " " = 500 "
1 Travancore " " " = 660 "
1 Tola = 180 *gr.*
1 Guz of Bengal = 1 yard.
1 Corge = 20 units.
1 Corge Pound = 20 *lb.*

Metric system permissive.

Italy.

1 Palm = .555 *cu. ft.*

Metric system in general use.

Japan.

1 Picul = $133\frac{1}{3}$ *lb.* av.

For coinage, in part, the metric unit of weight is used.

Java.

1 Amsterdam Pond = 1.09 *lb.* av.
1 Picul = $133\frac{1}{3}$ "
1 Catty = $1\frac{1}{3}$ "
1 Chang = 4 yards.

Mexico.

Weights and measures are legally the metric, but the metric system is not generally in force, the old Spanish weights and measures (see Spain) being still employed.

Netherlands.

Metric system with a change in names in general use.

1 Last (30 hectoliters) = 85.134 *bu.*

Norway and Sweden.

1 Swedish Skalpond = $0.93\frac{1}{3}$ *lb.* av.
1 Swedish Centner = $93\frac{1}{3}$ "
1 Norwegian Pund = 1.1 *lb.* av.
1 Swedish Fot = 11.7 inches.
1 Norwegian Fod = 12.02 "

In Norway the metric system is used to some extent.

In Sweden, the coin weight and the medicinal and apothecary weight are metric. The complete metric system has been obligatory since 1882.

Portugal.

Metric system compulsory since Oct. 1, 1868.

The chief old measures are—

1 Libra = 1.012 *lb.* av.
1 Almunde of Lisbon = 4.42 U. S. *gal.*
1 Alquiere = .3928 U. S. *bu.*

Russia.

1 Pound = 0.9 *lb.* av.
1 Pood (63 to a ton) = 36 "
1 Berkowitz = 360 "
1 Chetvert = 5.956 U. S. *bu.*
1 Vedro = 3.25 U. S. *gal.*
1 Arsheen = 28 inches.
1 Ship Last = 2 tons.

Metric system partially in use.

Spain, (Metric System.)

In many of the South American States and in Cuba, the old Spanish weights and measures, principally Castilian, are used. They are as follows:

1 Libra = 1.014 *lb.* av.
1 Arroba (25 Libras) = 25.36 "
1 Quintal (100 Libras) = 101.44 "
1 Vara = .914 *yd.*

Switzerland.

Metric system used with some changes of names and subdivisions. Pure metric system optional.

Turkey, (Metric System.)

Uruguay, (as Argentine Confederation.)

Venezuela, (Metric System.)

DETECTION OF ERRORS

IN

TRIAL BALANCES.

732. The following hints apply to the detection of errors in trial balances, or in any operation in which errors are made in addition or subtraction, or in transferring numbers from one place to another.

1. Ascertain the exact amount of the error. Much time is sometimes wasted in looking for errors which do not actually exist.

2. Revise carefully the additions of the trial balance before looking for the error in the ledger or other books.

3. If the error is in one figure only (as 2000, 100, 50, etc.), it is probably an error in addition or subtraction.

4. If an amount is entered on the wrong side of an account, or is added when it should be subtracted or *vice versa,* the error will be twice the amount.

5. If the digits of any number are written to the right or left one, two, or three places, and the error be divided by 9, 99, or 999 respectively, the quotient will be the number.

Thus, if $427 be written $4.27, the error will be $422.73; which divided by 99 (by 9 and 11), the quotient will be $4.27.

The number of 9's by which the number can be exactly divided is equal to the number of places which the number has been transferred to the right or the left.

6. If two consecutive digits of any number are transposed, the error will be a multiple of nine; and the quotient obtained by dividing the error by 9 will express the difference between the digits transposed.

Thus, if 437, be written 473, the error will be 36; which divided by 9 produces 4, the difference between 3 and 7. The same error, 36, will arise if the figures transposed are 0 and 4, 1 and 5, 2 and 6, 4 and 8, or 5 and 9.

7. If the error contains a number of figures, it is probable that some account or item has been omitted.

8. Look for the error systematically, and not in certain portions of the work selected at random.

ANSWERS.

Art. 20.

1. 1614.
2. 1654.
3. 19380.
4. 23243.
5. 26162.
6. 35130.
7. 4566.
8. 3722.
9. 53609.
10. 44601.
11. 50480.
12. 34914.

Art. 27.

4. 4915. *9.* 435.
5. 4857. *10.* 508.
6. 394. *11.* 3642.
7. 376. *12.* 3645.
8. 321. *13.* 3755.
14. 54877.
15. 44444.
16. 41568.
17. 36311.
18. 84839.
19. 139059.
20. 10078521.
21. 561. *26.* 3657.
22. 3691. *27.* 6822.
23. 1404. *28.* 1711.
24. 7921. *29.* 1440.
25. 297. *30.* 7529.
31. 14152.
32. 442,254,988.
33. 433.
34. 1771.
35. 4653.
36. 39247.
37. 16098.
38. 813210.
39. 6399.
40. 1,177,761,723.
41. 5,302,516.
42. 324,423,840.
43. $9858.94.
44. $419360.87.

Art. 33.

1. 4337.
2. 907823.
3. 40865.
4. 110547.
5. 8495098.
6. 853759.
7. 999895.
8. 1109975.
9. 6419754.
10. 72540.
11. 57249251.
12. 10648519.
13. 113558829.
14. 15562130.
15. 74,299,273.
16. 5,654,786.
17. 90,119,023.
18. 122,882.
19. 921294.
20. 19,212,939.
21. 4745.
22. 64535.
23. 45009.
24. 27369.
25. 41976.
26. 12464.
27. 62645.
28. 10514.
29. 3211.
30. 5821.
31. 4004.
32. 5038.
33. 1235.
34. 11,594,495.
35. 193,941,760.
36. $93,309,621.
37. 3025.
38. 6558830.
39. 3850814.
40. 388904.
41. 1106.

Art. 35.

1. 1107.90.
2. 317.26.
3. 6622.70.

Art. 41.

1. 164192;
187648.
2. 340236;
226824.
3. 49374; 98748.
4. 110775;
295400.
5. 243580;
438444.
6. 817281;
726472.
7. 130240;
182336.
8. 1,578,246;
2,367,369.
9. 494268;
617835.
10. 4,690,158;
7,035,237.
11. 3,336,072;
2,919,063.
12. 4,072,384;
3,563,336.
13. 3,824,910;
5,737,365.
14. 5,240,172;
3,742,980.
15. 58080.
16. 2016.
17. 24256.
18. 63360.
19. $194.40.
20. 1296.

Art. 44.

1. 63936; 831168.
2. 75218;
1,463,858.
3. 70272;
2,436,096.
4. 209387;
1,915,125.
5. 358661;
1,264,432.
6. 544375;
4,606,875.
7. 720408;
7,213,316.
8. 661982;
6,961,968.
9. 6,896,064;
87,772,352.
10. 5,847,408;
195,035,421.
11. 5,761,888;
195,249,432.
12. 18,413,409;
556,524,675.
13. 8,326,575;
173,434,110.
14. 25,930,788;
317,327,062.
15. 93,309,006;
889,602,580.
16. 4,428,648;
26,888,220.
17. 744.
18. 43200.
19. 2,419,200.
20. 5250.
21. 506880;
1,098,240.
22. 26376.
23. 106515;
153720.
24. 576.
25. 4608.
26. 5016.

Art. 47

1. 144000;
1080000.
2. 138240;
864000.
3. 241920;
1,451,520.
4. 185500;
1,335,600.
5. 120000;
3,200,000.
6. 252000;
1,036,000.
7. 81600;
272000.
8. 84000;
2,100,000.
9. 9,380,000;
67,760,000.
10. 18,500,000;
92,500,000.
11. 7,407,000;
44,442,000.
12. 11,760,000;
181,600,000

13. 6,698,000; 114,260,000.
14. 8,019,200; 98,808,000.
15. 67,200,000; 614,380,000.
16. 86,400,000; 460,800,000.

Art. 50.

3. 264; 176; 198; 352; 473; 363; 792; 891; 407; 484; 1012; 957; 1023; 704; 385; 396; 517; 187; 209; 528; 627.

Art. 53.

2. 2695; 3806; 3575; 4576; 8624; 5687; 9625; 10098; 46398; 80564; 79398; 19008; 48125; 92136.

Art. 56.

2. 52704; 35424.
3. 22227; 50907.
4. 9387; 36207.
5. 283745; 260295.
6. 378216; 600696.
7. 341284; 174804.
8. 112875; 150375.
9. 85425; 42925.
10. 281869; 234969.
11. 338583; 386883.
12. 75576; 338776.
13. 263375; 850875.

Art. 59.

2. 10205; 13345.
3. 5292; 6048.
4. 7830; 9918.
5. 6768; 6016.
6. 17276; 19744.
7. 37520; 42210.
8. 65664; 44928.
9. 134147; 118865.
10. 88375; 94500.
11. 95472; 96408.
12. 91208; 92962.
13. 77280; 80224.
14. 15876; 14847.
15. 40040; 41195.
16. 82215; 80649.
17. 58422; 57876.

Art. 62.

2. 3724; 2964.
3. 2523; 8613.
4. 2655; 3105.
5. 5928; 27768.
6. 16653; 33733.
7. 23925; 56925.

Art. 65.

2. 1206; 1809.
3. 1224; 1530.
4. 4158; 4851.
5. 6048; 6804.
6. 12420; 15525.
7. 10206; 40824.
8. 13986; 32634.
9. 39096; 58644.

Art. 68.

2. 600; 900; 925; 1225; 1550; 9675; 11200; 12800; 18650; 10600; 20425; 23425; 13600; 17925; 7950; 8100; 6400; 13900; 230600; 209450; 132000; 43200; 141200.

Art. 70.

3. 83763; 96929.
4. 126936; 293088.
5. 43344; 42656.
6. 310128; 137592.
7. 47775; 88725.
8. 170556; 863964.
9. 203912; 597376.
10. 288834; 399924.
11. 138446; 394284.
12. 107880; 186180.

Art. 74.

2. 7623; 7161.
3. 8232; 7980.
4. 6768; 6912.
5. 35991; 386613.
6. 39520; 413504.
7. 49104; 523776.

Art. 78.

1. 3136; 2304; 4136.
2. 4875; 3219; 1656.
3. 4851; 6375; 816.
4. 1739; 3819; 3825.
5. 8125; 4536; 12936.
6. 5776; 1296; 12996.

Art. 81.

1. 7134; 7047; 4095.
2. 2068; 2912; 3306.
3. 5548; 5925; 4284.
4. 1892; 2860; 4221.
5. 13572; 11235; 15250.

Art. 84.

2. 624; 7225; 15616.
3. 221; 9024; 13216.
4. 1224; 1221; 11024.
5. 625; 2021; 21021.
6. 1225; 3024; 24016.

Art. 87.

2. 9603; 9118.
3. 8008; 8360.
4. 8277; 8544.
5. 7275; 7350.
6. 9016; 8556.
7. 8084; 8170.
8. 985056; 987042.
9. 981090; 978120.
10. 976108; 977090.
11. 921300; 919450.
12. 868000; 868875.
13. 838530; 836836.

Art. 90.

1. 11872; 12432.
2. 10506; 10608.
3. 13176; 12810.
4. 12412; 12992.
5. 15515; 16240.
6. 19536; 19008.
7. 1,010,024; 1,011,028.
8. 1,134,000; 1,138,500.

Art. 93.

1. 10379; 10165.
2. 10752; 10304.
3. 10904; 11368.
4. 9828; 10692.
5. 91455; 93465.
6. 95665; 97679.
7. 100138; 93104.
8. 95692; 97728.

Art. 99.

1. 39,456,174; 26,304,116.
2. 24,413,116; 16,275,410$\frac{4}{6}$.
3. 3,265,524; 2,721,270.
4. 19,517,701; 11,152,972.
5. 2,057,613; 1,371,742.
6. 197,730,864; 123,581,790.
7. 58,642,209; 26,063,204.
8. 178,606,127; 51,030,322.
9. 49,377,285; 27,431,825.
10. 31,025,988; 24,131,324.
11. 51,525,354$\frac{1}{6}$; 20,610,141$\frac{4}{6}$.
12. 71,387,270; 35,693,685.
13. 106,315,682; 60,751,818$\frac{2}{7}$.
14. 41,152,263; 15,432,098$\frac{5}{8}$.
15. 41,133,539; 22,436,475$\frac{9}{11}$.

16. 112,731,950;
78,912,365.
17. 57,447,290;
23,936,370$\frac{10}{13}$.
18. 29,351,981$\frac{2}{3}$;
16,010,171$\frac{7}{11}$.
19. 144,300,144$\frac{3}{7}$;
112,233,445$\frac{5}{9}$.
20. 25,025,025;
16,683,350.
21. 4628 sq. yds.
22. 10388s.
23. 89 doz.;
$20.47.
24. 5280 ft.
25. 20006 gal.
26. 10908 bu.

Art. 103.

1. 11840$\frac{5}{26}$;
2152$\frac{109}{148}$.
2. 13100$\frac{16}{19}$;
999$\frac{165}{247}$.
3. 9879$\frac{26}{35}$;
1001$\frac{53}{375}$.
4. 10028$\frac{25}{48}$;
4149$\frac{85}{116}$.
5. 16948$\frac{6}{25}$;
2037$\frac{10}{203}$.
6. 170223$\frac{14}{15}$;
7142$\frac{110}{429}$.
7. 20147$\frac{25}{64}$;
2274$\frac{76}{567}$.
8. 97134$\frac{3}{7}$;
16413$\frac{389}{432}$.
9. 72660$\frac{37}{87}$;
10228$\frac{553}{618}$.
10. 25052$\frac{13}{92}$;
3161$\frac{488}{729}$.
11. 119348$\frac{56}{75}$;
10663$\frac{789}{817}$.
12. 17008$\frac{41}{84}$;
3914$\frac{106}{365}$.
13. 41933$\frac{52}{69}$;
20093$\frac{87}{144}$.
14. 526009$\frac{9}{14}$;
57532$\frac{83}{128}$.
15. 30366$\frac{19}{70}$;
6642$\frac{199}{320}$.
16. 58694$\frac{9}{64}$;
16261$\frac{134}{231}$.
17. 116213$\frac{23}{56}$;
11462$\frac{61}{365}$.
18. 39625$\frac{73}{96}$;
5218$\frac{150}{229}$.
19. 50117$\frac{30}{83}$;
2584$\frac{131}{640}$.
20. 29094$\frac{10}{93}$;
17274$\frac{109}{188}$.

21. 365$\frac{5}{24}$.
22. 138$\frac{128}{174}$.
23. 677$\frac{10}{11}$.
24. 356$\frac{11}{56}$.
25. 498299$\frac{53}{63}$.
26. 368$\frac{60}{128}$.
27. 153$\frac{204}{320}$.
28. 786395$\frac{7}{566}$.
29. 2161$\frac{227}{365}$.
30. 151911$\frac{265}{333}$.
31. 3,969,568.
32. 14960.

Art. 108.

47; 66; 68;
115; 153; 198;
152; 69; 71;
73; 80.

Art. 132.

1. $2837.46.
2. $1022.25.
3. $2775.87.
4. $3383.08.
5. $14.91.
6. $18.51.
7. $569.25.
8. $1311.26.

Art. 135.

1. $97.44.
2. $51.
3. $273.53.
4. $677.
5. $3306.
6. $187.60;
$276.48.
7. $86.73;
$939.12.
8. $400.80;
$2164.32.
9. $3168;
$25344.
10. $1305;
$4241.25.
11. $2313.12;
$13878.72.
12. $3756;
$9615.36.

Art. 138.

1. $3.45.
2. 15 lb.
3. $.53; $.24.
4. $19.04;
$13.60.
5. $48; $36.
6. $.87; $.84.
7. $1.48; $.185
8. $25.12;
$1.227.
9. 53; 123.
10. 414; 72.
11. 217; 36.
12. 2304; 256.
13. 1536; 256.
14. 4088; 280.

Art. 139.

1. 4,964,629.
2. 3,204,084.
3. 1,042,916,880.
4. $4799.50.
5. 5989 ft.
6. $18487.
7. $11,014,811.
8. 92250.
9. 456.
11. $458.
12. 3582.
13. 132 acres.
14. $2006.
15. 4664.24.
16. $4.50.
17. $46.
18. $126.28.
19. $17377.65.
20. $30,022,347.95.
21. $34,077,380.
22. A, $12283;
B, $12568;
C, $12371;
D, $12071;
D, lowest.
23. 339 head.
24. $9292.80.
25. 3000 lb.
26. $1044.
27. A, $124;
B, $125.
28. $7.75.
29. 385231 + ;
32102 +.

Art. 158.

1. 3^2, 5, 7, 11.
2. 3, 7, 11, 13.
3. 2, 3^3, 7, 11.
4. 2, 3^2, 5^2, 7.
5. 3, 5^2, 7^2.
6. 2, 3, 5, 7, 11.
7. 2^3, 3^2, 7, 13.
8. 2, 7, 13, 43.
9. 2, 3, 7, 11, 13.
10. 2^3, 11, 61.
11. 3^2, 5^2, 17.
12. 2^2, 11^3
13. 2^{18}.
14. 3^8.
15. 5^5.
16. 2^3, 3^2, 5^2.
17. 3^2, 5, 43.
18. 3^2, 5^2, 11.
19. 2^2, 3^2, 5, 37.
20. 2^4, 3^2, 17.
21. 5^2, 11, 31.
22. 2^4, 3^3, 23.
23. 2^3, 3^2, 7, 19.
24. 5^2, 7, 29.

Art. 165.

1. 30. 12. 360.
2. 48. 13. 288.
3. 120. 14. 2640.
4. 16. 15. 1260.
5. 84. 16. 240.
6. 360. 17. 660.
7. 504. 18. 3024.
8. 60. 19. 360.
9. 120. 20. 840.
10. 480. 21. 9900
11. 480. 22. 2520.

Art. 169.

1. 1. 5. 14$\frac{2}{5}$.
2. 29$\frac{43}{78}$. 6. 4$\frac{23}{35}$.
3. 41$\frac{347}{365}$. 7. 6.
4. 51$\frac{1}{3}$. 8. $\frac{5}{36}$.
9. 2.
10. 1008.
11. $56.
12. $0.51.
13. 9.37\frac{1}{2}$.
14. 150 bu.
15. 750 yd.
16. 133 lb.
17. $31.50.
18. 36 cows.
19. 243 bu.
20. 1290 bu.
21. $16.92.
22. $79.
23. $21.
24. 56 lb.
25. 66. 27. 90.
26. 10. 28. 23$\frac{4}{9}$.

Art. 191.

1. $\frac{2}{5}$. 10. $\frac{11}{14}$.
2. $\frac{3}{4}$. 11. $\frac{2}{3}$.
3. $\frac{2}{3}$. 12. $\frac{1}{5}$.
4. $\frac{7}{8}$. 13. $\frac{14}{15}$.
5. $\frac{9}{16}$. 14. $\frac{11}{16}$.
6. $\frac{17}{21}$. 15. $\frac{9}{10}$.
7. $\frac{2}{3}$. 16. $\frac{11}{12}$.
8. [illegible]. 17. [illegible]
9. [illegible]. 18. [illegible].

[illegible]

9. 116[illegible].
10. 81[illegible].
11. 70[illegible].
12. 72[illegible].
13. 214[illegible].
14. 576[illegible].
15. 483[illegible].
16. 703[illegible].
17. 809[illegible].
18. 1593[illegible].
19. 2143[illegible].
20. 410[illegible].
21. 1329[illegible].
22. 1428[illegible].
23. 576[illegible].
24. 194[illegible].
25. 1460[illegible].
26. 923[illegible].
27. 472[illegible].
28. 1019[illegible].
29. 810[illegible].
30. 396[illegible].

Art. 236.

1. 1[illegible].
2. 20.
3. 37[illegible].
4. 63.
5. 117[illegible].
8. 1[illegible].
9. [illegible].
10. 1[illegible].
11. 6[illegible].
12. 9.
13. 11[illegible].
14. 9.
15. 6[illegible].
16. 2[illegible].
17. 2[illegible].
18. 1[illegible].
19. [illegible].
20. [illegible].
21. 26[illegible].
22. 7[illegible].
23. 15.
24. 21.
25. 10.
26. 22[illegible].
27. 12[illegible].
28. 16[illegible].
29. 18[illegible].
30. 21[illegible].
31. 15.
32. 9[illegible].
33. 13.
34. 46.
35. 13.
36. 23.
37. 83[illegible].
38. 13[illegible].
39. 13[illegible].
40. [illegible]; [illegible]; [illegible].
41. [illegible]; [illegible]; [illegible].
42. [illegible]; [illegible]; 1[illegible].
43. 2; 1[illegible].
44. 2[illegible]; 2.
45. 635[illegible]; [illegible].

Art. 237.

1. [illegible].
2. [illegible].
3. [illegible].
4. 1718[illegible].
5. 193[illegible].
6. 862[illegible].
7. 31[illegible].
8. 302350[illegible].
10. $7352[illegible].
11. 20.
12. 12.
13. 21.
14. 1793[illegible]; 352[illegible].
15. 1800[illegible].
16. 145[illegible].
17. $280.50.
18. $48[illegible]2.
19. 95[illegible] bu.
20. 2280[illegible].
21. [illegible].
22. A. $648;
B. $1080.
23. $4300[illegible].
24. 336.
25. $2475.
26. $262[illegible].
27. 94[illegible].
28. $96[illegible]; 56[illegible].
29. Horse, $705;
Carriage,
$440[illegible].
30. $2003[illegible].
31. $157.67.
32. 50[illegible].
33. $9.46.
34. 123[illegible] gal.
35. Widow,
$2876.12;
Each child,
$1438.06.
36. 1403[illegible].
37. $192.
38. $4600.
39. Lost $0.38[illegible].
40. 20191[illegible];
24348[illegible].
41. 31963[illegible];
185517[illegible].
42. 6224[illegible];
14241[illegible].
43. 19479[illegible];
8426[illegible].
44. $198.31.
45. 19744.
46. $84.24.
47. 27.80.
48. 76.66.
49. 320 rods.
50. 6 days.
51. Gained 2 cts.
52. $629.30.
53. $5487.98.
54. 110 bu.
55. $35.46.
56. $136.99.
57. $115.30.
58. $316.74.
[illegible] 2520.
[illegible] $25,574,327.33.

Art. 256.

1. .50.
2. .875.
3. .75.
4. .375.
5. .4375.
6. .78125.
7. .425.
8. [illegible].
9. .58[illegible].
10. .58[illegible].
11. .714285[illegible].
12. .444[illegible].
13. 16.625.
14. 27.9230769[illegible].
15. 36.9583[illegible].

Art. 259.

1. [illegible].
2. [illegible].
3. [illegible].
4. [illegible].
5. [illegible].
6. [illegible].
7. [illegible].
8. [illegible].
9. [illegible].
10. [illegible].
11. [illegible].
12. [illegible].
13. $\frac{1}{8}$.
14. [illegible].
15. [illegible].
16. [illegible].
17. [illegible].
18. [illegible].
19. [illegible].
20. [illegible].
21. [illegible].
22. [illegible].
23. [illegible].
24. [illegible].
25. 28[illegible].
26. 107[illegible].
27. $175\frac{12}{125}$.
28. 6[illegible].

Art. 262.

1. 492.310787.
2. 7462.31526.
3. 476.3380807.
4. 2.6591587.
5. 9710.27879.
6. 1.83580255.
7. 1764.06.
8. $776.42[illegible].
9. 215.2741[illegible].
10. 21.9026730[illegible].

Art. 265.

1. 3.9803.
2. .26971.
3. 8999.1.
4. .4648.
5. 16.6736.
6. .010102.
7. $86.17.
8. 2.126155.
9. 1.728[illegible].
10. $121.14[illegible].
11. $1727.93[illegible].
12. .924[illegible].
13. 51.0458[illegible].
14. 53.39[illegible].
15. 68.04[illegible].
16. 59.702[illegible].
17. [illegible].
18. 931.057[illegible].
19. .4057[illegible].

Art. 268.

1. [illegible].
2. .8448.
3. [illegible].
4. $43.216;
$182.2875.
5. 45.77125;
55.0229[illegible].
6. .278785;
1.3136.
7. 3.39024;
.409652.
8. .0054062 5;
.845875.
9. 11.208704;
.0100672.
10. 5.705483[illegible];
34.01345[illegible].
11. 28.648083[illegible];
21.984375.
12. .288; 44.0928.
13. 93.05683[illegible];
4.02081[illegible].
14. 115.6660[illegible];
500.4029[illegible].
15. .51153;
3.8583[illegible].
16. 82.0160[illegible];
1061.1796[illegible].
17. 576; 432; 216;
345.6.
18. 170845.80.

Art. 271.

1. .048.
2. 250.
3. 104; 8.625.
4. 1.914; 2.83.
5. .875; 100.8.
6. 481.5; 385.2.
7. 4.25; 6.2.
8. 15.24706;
2.25.
9. .49; 82.6875.
10. .5694; 39.
11. 18.66;
10.39152 +.
12. 2722.02; 42.
13. 86.40; 69.12;
51.84; 138.24;
25.92.

14. 1800.
15. 8720.
16. 12.
17. 2.525.
18. 293.040015.
19. .03$\frac{1}{8}$; .12$\frac{1}{2}$.
20. .16$\frac{2}{3}$; .40.
21. .06$\frac{1}{4}$; .06.
22. .02$\frac{1}{2}$; .03$\frac{1}{3}$.
23. .02$\frac{1}{12}$; .00$\frac{1}{3}$.
24. .14$\frac{2}{7}$; .41$\frac{2}{3}$.
25. $361.60; $452.
26. $10800;
$49.85.
27. $36; $0.82.
28. $21600;
$10285.71.
29. $200;
$153.125.
30. $256000;
$3555.56.
31. 13.569;
1037.647.
32. 4.524; 4.026.
33. 1.403; 6.247.
34. 1.728; 1.89.
35. 9.681; 414.881.
36. 142.105;
98.743.

Art. 274.

1. $6. 2. $34.
3. $48.13.
4. $92.96.
5. $312.
6. $5.44.
7. $114.58.
8. $102.24.
9. $280.64.
10. $164.64.
11. $16.32.
12. $12.40.
13. $15.75.
14. $38.75
15. $242.10.
16. $16.50.
17. $198.40.

Art. 275.

1. .1651386.
2. 289.3624$\frac{8}{9}$.
3. .8125.
4. 158.916$\frac{2}{3}$.
5. 117.16\frac{2}{3}$.
6. 61.875.
7. 8300
8. .04.
9. $\frac{11}{16}$.
10. 868.68064.

11. $\frac{1}{12}$.
12. 176.27$\frac{1}{4}$.
13. 2.
14. 576.
15. 5.
16. 3$\frac{1}{3}$.
17. 331.2.
18. $56.16.
19. $1575.
20. $108.99.
21. $143.06.
23. $42.07;
$61.91.
24. $16.84;
$11.17.
25. $28.98;
$32.30.
26. $103.22;
$47.41.
27. $67.76;
$79.20.
28. $25.09;
$62.56.
29. $33107.12.
30. B, $10542.48;
C, D, E, each
$7028.32.
31. $31444.87.
32. 203$\frac{29}{80}$ A.;
B, 36$\frac{39}{40}$ A.;
C, 67$\frac{63}{80}$ A.;
$4575.66.
33. 7040 ft.
34. Wife, $48500;
Son, $36375;
Daughter,
$24250;
Total, $145500.
35. $1065.
36. $4.63.
37. 1100 bu.

Art. 282.

1. 2326d.
2. 220 gills.
3. 108404 f.
4. 13265 lb.
5. 256 days.
6. 18211d.
7. 235923 f.
8. 1566 qt.
9. 14948 lbs.
10. 4480 pwt.
11. 27005 ft.
12. 2407680 ft.
13. 572 pts.
14. 111 pts.
15. 69816 in.
16. 7895$\frac{1}{4}$ sq. ft.

17. 2934$\frac{1}{4}$ sq. yd.
18. 3200 A.
19. 354 da.
20. 2160 cu. ft.
21. 42885d.
22. 982 da.
23. 27782 lbs.
24. 7583d.

Art. 285.

1. £35 6s. 3d.
2. 75 cd. 83 cu. ft.
3. 117 bu. 2 pk.
7 qt.
4. 1227 gal. 1 pt.
5. 27 yd. 1 ft. 3 in.
6. 7 mo. 6 da.
7. 2 mi. 235 rd.
8. 37 wk. 6 da. 15
hr.
9. 2 hr. 38 m. 57 s.
10. 173 yd. 11 in.
11. £55 15s. 7d.
12. 329 yd.
13. 8 T. 14 cwt.
16 lb.
14. 50 yd. 9 in.
15. 73 bu. 2 pk.
3 qt. 1 pt.
16. £34 17s. 6d.
17. 33 A. 36 sq. rd.
18. 4 cwt. 63 lb.
10 oz.
19. 68 cd. 12 cu. ft.
20. £20 2s. 5d.

Art. 288.

1. 135d.
2. 13s. 9d.
3. 1980 ft.
4. 496 lbs.
5. 15s. 5d.
6. 13 pts.
7. 4785 ft.
8. 87 lbs. 8 oz.
9. 5$\frac{1}{4}$ pts.
10. 104 cu. ft.

Art. 290.

1. 150d.
2. 17s. 6d.
3. 153d.
4. 13s. 8d.
5. 2 y. 4 m.
6. £16 9s. 4d.
7. £205 6s. 9d.
8. 2 y. 5 m.
9. £15 6s. 9d.
10. 3 m. 3 d.

Art. 292.

1. £$\frac{1}{112}$.
2. $\frac{7}{8}$ oz.
3. $\frac{5}{56}$ m.
4. $\frac{11}{1760}$ m.
5. £$\frac{121}{180}$.
6. £$\frac{7}{40}$.
7. $\frac{5}{8}$ y.
8. $\frac{3}{80}$ T.
9. $\frac{7}{64}$ bu.
10. £$\frac{1}{4}$.

Art. 294.

1. £.04375.
2. .6 T.
3. £.925.
4. £14.7875.
5. .90625 cd.
6. £247.7375.
7. .4 A.
8. £27.525.
9. $166.56.
10. $604.02.
11. 15s. 4.5d.

Art. 296.

1. £85 7s. 10d.
2. 69 T. 14 cwt.
66 lb.
3. £460 7s. 6d.
4. 12 y. 2 m. 16 d.
5. 15 cd. 48 cu. ft.
6. 26 h. 46 m.
7. 37 bu. 5 qt.
8. 27 yd. 4 in.
9. 10 lb. 2 pwt.
20 gr.
10. 124 gal. 2 qt.
11. £205 12s. 5d.
12. £546 1s. 7d.

Art. 298.

1. £88 4s. 2d.
2. £21 4s. 4d.
3. £3 14s. 3d.
4. £23 13s. 10d.
5. 4 y. 3 m. 2 d.
6. 4 y. 4 m. 22 d.
7. 6 m. 10 d.
8. 2 y. 6 m. 24 d.

Art. 300.

1. £122 14s. 8d.;
£157 16s.;
£192 17s. 4d.;
£263.
2. 34 cd. 16 cu. ft.
3. £28 2s. 6d.
4. 3 lb. 3 oz. 12
pwt.

5. 16 h. 23 m. 40 s.; 19 h. 40 m. 24 s.; 24 h. 35 m. 30 s.; 29 h. 30 m. 36 s.
6. 55 da. 13 h.
7. £20 12s. 6d.
8. £6 7s. 10d.; £7 5s. 9d.
9. £3 9s. 3d.; £5 10s. 9d.; £8 6s. 1d.; £9 13s. 10d.

Art. 302.

1. £1 4s. 9d.; £1 10s. 3d.; 8s. 3d.
2. 192 yd.; 324 yd.; 352 yd.
3. 44 yd.
4. 7° 6′ 32$\frac{1}{4}$″; 5° 41′ 14″; 4° 44′ 21$\frac{3}{5}$″; 3° 9′ 34$\frac{1}{5}$″.
5. 40 a. 51$\frac{1}{4}$ sq. rd.; 32 a. 41 sq. rd.; 24 a. 130 sq. rd.; 20 a. 25$\frac{5}{8}$ sq. rd.
6. 9s. 11d.; £5 19s.; £9 18s. 4d.; £18 6s. 11d.
7. £34 10s.; £21 11s. 3d.; £17 5s.; £9 11s. 8d.; £5 15s.

Art. 308.

1. 26920 min.
2. 232 da.
3. 2 yr. 4 mo. 15 da.
4. $0.59.
5. 2 yr. 2 mo. 16 da.
6. 5[illegible] min.
7. 3; 7; 3.
8. [illegible] 22 [illegible] 50 [illegible]
9. 1 [illegible] 3 [illegible] 40 [illegible]
10. $33.47

Art. 311.

1. 7 mo. 18 da.
2. 8 mo. 18 da.
3. 1 yr. 8 mo. 11 da.
4. 2 yr. 5 mo. 7 da.
5. 10 mo. 8 da.
6. 9 mo. 12 da.
7. 9 mo. 12 da.
8. 4 mo. 15 da.
1. 230 days.
2. 263 days.
3. 618 days.
4. 888 days.
5. 312 days.
6. 286 days.
7. 286 days.
8. 138 days.
9. 6574 days.
11. Dec. 6.
12. June 22.

Art. 315.

1. 93 ft. 1 in.
2. 22$\frac{1}{2}$ in.
3. 89760 ft.; 577$\frac{1}{2}$ ft.
4. 1$\frac{1}{4}$ in.
5. 9 mi. 100 rd. 1 yd. 2 ft.
6. 6514 in.
7. 4620 ft.
8. 66 ch. 20 ft.; 52512 in.
9. 99 ch.; 6534 ft.
10. 2478.96 ft.; 150.24 rd.
11. 6$\frac{1}{2}$ in.; 8$\frac{1}{2}$ in.; 11$\frac{1}{3}$ in.
12. 287 fathoms; 522$\frac{3}{4}$ fathoms.
13. 9 mi. 45 rd. 4 yd. 1 ft. 6 in.
14. 117.46 ch.
15. $7[illegible].50.
16. 1[illegible]80 posts; 73[illegible]200 ft.

Art. 321.

1. 177 a. 140 sq. [illegible]
2. [illegible] sq. ft.
3. 22[illegible] sq. [illegible]
4. [illegible]
5. [illegible]
6. 2[illegible] sq. ft.
7. 900 sq. ft.
8. 1388 sq. ft.; 154 sq. yd.
9. 960 sq. ft.
10. 55.
11. 588 a.
12. 132.183 a.
13. 1.00352 a.
14. 23$\frac{3}{4}$ sq. yd.
15. 17640 sq. ft.
16. 24 ft.
17. $7200.
18. 990 ft.
19. $\frac{1}{4}$ sq. ft.; $\frac{1}{4}$ sq. ft.; $\frac{3}{4}$ sq. ft.
20. 22$\frac{1}{2}$ sq. yd.
21. $140.
22. 300 sq. ft.
23. 8000 sq. yd.
24. 19800 sq. ft.
25. $\frac{1}{27}$ sq. yd.; [illegible] sq. yd.; [illegible] sq. yd.
26. 108.
27. 10368.
28. 11$\frac{7}{8}$ sq. ft.
29. 1512.
30. 17820.
31. 187200.
32. $1040.
33. $153600.
34. $1600; 32 a.
35. 612 sq. ft.
36. 122 sq. in.
37. 62 sq. yd.; 26$\frac{2}{3}$ sq. yd.
38. 128 sq. yd.
39. 772 sq. ft.
40. 3631[illegible] sq. ft.
41. 4 yd.
42. 24; 28.8.
43. 576; 900; 720.
44. 51[illegible]; 102[illegible]; 140.
45. 24000; 33600; 38400.
46. 750; 930.
47. $\frac{3}{4}$ sq. yd.
48. $64.
49. 5 [illegible] and [illegible]; 35 yd.
50. 25 [illegible]
51. 1[illegible] yd.
52. 4 sq. [illegible]; 36 sq. [illegible]
53. 15 [illegible]
54. [illegible]
55. 2[illegible]5
56. [illegible]

Art. 325.

1. 123 cd.
2. 302400 cu. in.
3. 1384 cu. yd.
4. 67 cds. 119 cu. ft.
5. 560 cu. yd.
6. $72.
7. $26.95.
8. 8 ft.
9. 3$\frac{3}{4}$ cd.
10. 135 cu. ft.
11. 233280 cu. in.
12. 168 tons.
13. $70; £21 17s. 6d.
14. 218500 cu. ft.
15. 21$\frac{3}{4}$ perches.
16. 27.
17. 250 cu. ft.
18. 1200 cu. ft.; 20400; $238.14.
19. 34944.
20. 2112 cu. ft.
21. 2240; 1120.
22. 3686.4 cu. ft.; 36 bbl.; 28.8 cu. yd.
23. 102375.

Art. 328.

1. 12 sq. ft.
2. 48 sq. ft.
3. 480 sq. ft.; $6.72.
4. 5700 ft.
5. 12 ft.
6. 9$\frac{1}{2}$ ft. 7. 8 ft.
8. 21 ft.
9. 16 ft.
10. 23$\frac{1}{4}$ ft.
11. 24 ft.
12. 54 ft.
13. 480 sq. ft.
14. 12066 ft.; $1[illegible].14.
15. 240 ft.
16. 1[illegible]0 ft.
17. [illegible]
18. 66[illegible] posts.
19. 240 ft.
20. [illegible]
21. 2[illegible] ft.

Art. 331.

1. [illegible]
2. [illegible]

3. 28 gal. 3 qt. 1 pt.
4. 3174 bbl. 19 gal.
5. 576 gal.
6. 133.68 + cu. ft.
7. 1300 gal.
9. 538.56 gal.
10. 2393.6 gal.
11. 38297.6 gal.
13. 1504 gal.
14. 56400 gal.
15. 169200 gal.
16. *f.* ℥ 2901.
17. 7 gal. 3 qt.
18. 72.

Art. 333.

1. 187 pt.
2. 156 bu. 1 pk.
3. $25.60.
4. 161281.5 cu. in.
5. 150 bu.
6. 321.4, ex.; 320, app.
7. 160 bu., app.
8. 235.2 bu., app.
9. 640 bu., app.
10. 576 bu.

Art. 340.

1. 1 lb. 8 oz. 16 pwt. 16 gr.
2. 15744 gr.
3. 859 oz. 7 pwt. 12 gr.; 1100 oz.
4. 53 oz. 15 pwt.
5. 803 oz. 15 pwt.
6. $42.98.
7. $1\frac{3}{4}$ oz.
8. $68.75.
9. 1,546,875 oz.; 171875 oz.
10. 10480 gr.
11. 1152; 1400.
12. 51 lb. 12 oz.
13. 112000 gr.
14. 85 lb. 16 pwt. 16 gr.
15. 192 oz. av.
16. $16.31.
17. $10.06.
18. $321.66.
20. $91.46.
21. $94.76.
22. $73.92.
23. $70.73.
24. 139^{11} bu.
25. 162^{14} bu.
26. 18^{45} bu.
27. 86^{12} bu.
28. 105^{15} bu.
29. 324^{32} bu.
30. 221^{15} bu.
31. 416^{32} bu.
32. 190^{12} bu.
33. 279^{7} bu.
35. $153.54.
36. $142.92.
37. $167.13.
38. $233.40.
39. $237.45.
40. $119.02.
41. $295.38.
42. $200.67.
43. $94.75.
44. $84.29.

Art. 342.

1. £135 1s. 10d.
2. £3 15s. 8d.
3. £42 10s.
4. £34 17s. 11d.
5. 3080 d.
6. £18 8s. 11d.
7. £901 5s.
8. £49 7s.
9. £1 16s. 2d.; £2 6s. 6d.; 10s. 6d.
10. $39\frac{8}{43}$; $51\frac{38}{43}$.
11. £10 10s.
13. £.808.
14. £.921.
15. £.533.
16. £.867.
17. £.363.
18. £.696.
20. 8s. 6d.; 12s. 9d.
21. 16s. 4d.; 4s. 11d.
22. 5s. 1d.; 3s. 8d.
23. 7s. 6d.; 9s. 11d.
24. £158 6s. 8d.; £79 3s. 4d.; £63 6s. 8d.; £39 11s. 8d.
25. 16s. 8d.; £1: 13s. 4d.
26. $1100.25; $683.91.
27. £204 18s. 4d.; £332 19s. 10d.

Art. 345.

1. $28.80.
2. $2.94.
3. 288; 960.
4. 792.
5. $22\frac{1}{2}$¢.
6. $5.61.
7. $345.60.
8. $9.18.
9. 50.

Art. 347.

1. 151° 50′ 1″.
2. 134° 53′ 59″.
3. 7° 28′ 46″.
4. 65° 56′ 20″; 197° 49′.
5. 5° 4′ 47″; 6° 20′ $58\frac{1}{4}$″.
6. 11° 58′ 49″; 11° 13′ $53\frac{7}{18}$.″
7. 277° 46′ 40″.
8. 159400″.
9. 180°.

Art. 353.

1. 73° 54′ 25″.
2. 73° 23′ 52″.
3. 34° 49′.
4. 178° 34′ 17″.
5. 88° 33′ 45″.
6. 149° 14′ 13″.
7. 4 hr. 56 min. $\frac{1}{5}$ sec.
8. 54 min. $30\frac{4}{5}$ sec.
9. 11 hr. 2 min. $59\frac{2}{15}$ sec.
10. 50 min. $11\frac{7}{15}$ sec.
11. 5 hr. 4 min. $20\frac{1}{5}$ sec.
12. 11 hr. 58 min. $11\frac{13}{15}$ sec.
13. 2 hr.
14. 1 hr.
15. 3 hr.
16. 1 hr.
17. 2 hr.
18. 0 hr.
19. 15 min. 46 sec.
20. 40 sec.
21. 9 min. 47 sec.
22. 1 min. 1 sec.
23. 9 min. 29 sec.
24. 20 min. $19\frac{3}{5}$ sec.
25. 154° 8′ 30″ W.
26. 5 P. M.
27. 19° 31′.
28. 11 hr. 20 min., A. M.
29. 4 A. M. Tuesday.
30. 122° 26′ 45″ W.
31. 87° 37′ 45″ W.

Art. 359.

3. 38364 cm.; .38364 Km.
4. .0017516 Km.; 1.7516 m.
5. 8742.57 m.
6. 119 Km.
7. $98.70.
8. 2306.8 m.
9. 910 m.
10. 650.
11. $2400.
12. 10240 m.
13. 24 hr.

Art. 362.

2. 83.09 sq. m.
3. 4700 sq. m.
4. 602500 ca.
5. 256 sq. m.
6. 400.
7. 160.4 H.
8. $12.44.
9. 40.
10. 96 H.

Art. 365.

2. 17.218027 cu. m.
3. 28 cu. m.
4. 3.3 cu. m.
5. 4.81208 cu. m.
6. 2051.28 cu. m.
7. $109.69.
8. 82.5 sters; 43.2 m.

Art. 367.

1. 1000 l.
2. $1.28.
3. 72 Hl.
4. 168 Hl.
5. 635 l.; 8375 l.
6. $1498.
7. 160 Hl.
8. 180 bags.
9. 10080 Dl.
10. $14.

Art. 369.

1. 1,000,000 g.; 1000 g.
2. 16.816 T.
3. 80.
4. 30.
5. 750 g.
6. $12.608
7. 62.
8. 9300 Kg

Art. 371.

1. 246.06 yd.; 8858.25 in.
2. 9.6558 Km.
3. 259.008 H.; 25900.8 A.
4. 32808.3 ft.; 6.2137 mi.
5. 828.04776 lb.
6. 204.12.
7. 26.73 grams; 27.216 grams.
8. 1762 Hl.
9. 668.9375 cu. m.
10. 1308 cu. yd.
11. 6540.48 l.
12. 291.824 sq. yd. 2626.416 sq. ft.
13. 6237 g.; 6.237 Kg.

Art. 387.

1. $35\frac{49}{128}$.
2. 1135.94.
3. 5 mo. 24 da.; 178 da.
4. 8.
5. 32 ft.
6. 702 sq. ft.; 210 sq. ft.; 1320 sq. ft.
7. $172.36.
8. $127.49.
9. 43200 sq. rd.
10. 480 sq. ft.
11. 3110 ft.
12. $128.
13. 628 ft.
14. 15.708 ft.
15. 28.27 sq. ft. 78.54 sq. ft.
16. 429 ft.
17. 35942 lb.
18. 474 qt.
19. $487\frac{44}{}$ bu.; $237.81.
20. $237.94.
21. 16500 ft.
22. $327.78.
23. $1634.09.
24. $808.65.
25. £19.8375.
26. $1100.25.
27. £15 16s. 1d.
28. £10 10s.
29. 55° 48′.
30. 165000 l.
31. $0.526.
32. $0.055.

Art. 389.

1. $18.99; $20.34; $16.55.
2. $2.06.
3. $12.16.
4. $13.50.
5. $5.42.
6. $12.16.
7. $16.39.
8. $32.89.
9. $3.44.
10. $40.25.
11. $54.07.
12. $76.48.
13. $245.80.

Art. 391.

2. $179.80.
3. $49.50.
4. $193.45.
5. $168.
6. $277.53.
7. $4.
8. $289.39.
9. $96.25.
10. $21.62.
11. $7.56.
12. $36.60.
13. $85; $107.
14. $18.75.
15. $81; $90.
16. $274.81.

Art. 402.

1. 51.84.
2. 60.
3. 17.92.
4. 23.22.
5. 56.7.
6. $33.12.
7. $57.28.
8. $644.
9. $411.95.
10. $41.04.
11. $60.10.
12. $33.17.
13. $52.55.
14. $1031.97.
15. $155.36.
16. $23.75.
17. $112.68.
18. $143.75 $2731.25.
19. $16121.
20. $225.
21. $823.82.
22. $1612.50; $2700.40.
23. $2325.38.
24. $84.
26. $0.94.
27. $6.56.
28. $14.71.
29. $3.96.
30. $79.13.
31. $53.83.
32. $1476.40.

Art. 404.

1. 432.
2. 868.
3. 1604.5.
4. 1816.
5. $125.
6. $106.
7. $162.
8. $1975.
9. $243.60.
10. $487.10.
11. $176.43.
12. $328.35.
14. $2434.79.
15. $184.38.
16. $64.94.
17. $4100.
18. $4500; $16.88.

Art. 407.

1. 8.
2. 800.
3. 5400.
4. 5600.
5. 9375.
6. 22.8.
7. 1800.
8. 800.
9. 2500.
10. $34.56.
11. $4050.
12. $900.
13. $228.80.
14. $3500.
15. $720.
16. $59.38.
17. $2500.
18. $1944.
19. 2500.
20. 2400.
21. $361.60.
22. $3240.
23. $1502.40.
24. $69.12.
25. 495 ft.
26. £7200.
27. $196.
28. $15360.
29. $91500.
30. $65500.
31. $564.
32. 400.
33. $930.
34. $444.
35. $324.
36. $6210.
37. $456.80.
38. $505.
39. 1425 boxes.
40. $59062.50.
41. $6464.

Art. 410.

1. $.01\frac{1}{4}$
2. 20%.
3. $.08\frac{1}{3}$.
4. $.02\frac{1}{12}$.
5. $3\frac{1}{3}\%$.
6. $1\frac{1}{4}\%$.
7. $6\frac{1}{4}\%$.
8. $2\frac{1}{2}\%$.
9. $1\frac{3}{7}\%$.
10. $33\frac{1}{3}\%$.
11. 25%.
12. $\frac{3}{20}\%$.
13. 2200.
14. $12\frac{1}{2}\%$.
15. 45%.
16. 44%.
17. $\frac{1}{8}\%$; $42.50.
18. $4\frac{1}{8}\%$.
19. 16%.
20. $56\frac{1}{4}\%$.
21. $4\frac{1}{6}\%$.
22. $18\frac{3}{4}\%$.
23. $6\frac{1}{3}\%$.
24. $\frac{3}{4}\%$.
25. $9\frac{2}{25}\%$.
26. 45%.
27. $21\frac{13}{15}\%$
28. 20%.

29. $320;
$280
30. 12$\frac{1}{2}$%.
31. 20%
32. [illegible]%.
33. [illegible]%.

Art. 411.

1. 1392.
2. 120.
3. 125%.
4. $1197.
5. 275.
6. $7500.
7. $16.88.
8. 30¢.
9. $157790.70.
10. $28546.56.
11. $5.57.
12. $1150.
13. $164.
14. $12960.
15. $9363.44.
16. 2$\frac{2}{3}$%.
17. $6907.51.
18. $123450;
$1313.28.
19. $6058.95;
95%; 20%.
20. $1052.63;
$1422.37.
21. $1079.12.
22. 7$\frac{1}{2}$%; 29%.
24. £11 10s. 5d.
25. £26 9s.
26. £18 15s.
27. £1 12s. 5d.
28. £3 0s. 6d.
29. £3 14s. 11d.
30. £32.
31. £35 8s. 4d.

Art. 414.

1. $40.
2. $364.
5. $4726.
7. $3.
8. $10179.
10. $1048.20.
11. 72¢.
12. $2425.
14. 16%.
15. 13[illegible]%.
16. 12$\frac{1}{2}$%.
17. $1775.
18. $608.
19. $11225.
20. $520.
21. 44¢.

22. $72.
23. 23$\frac{1}{3}$%.
24. 150%.
25. 57$\frac{1}{2}$¢.
26. 12%.
27. 10%.
28. 4%.
29. $210.
30. 36¢.
31. 96%.
32. Loss, 4%.
33. 38$\frac{2}{3}$%.
34. $432.
35. $4.25, each.
36. 12%.
37. $6.40.
38. $7.50.
39. 6$\frac{2}{3}$%; $3.80.
40. $2532.96;
$3102.88;
22$\frac{1}{2}$%.
41. $400, loss.

Art. 417.

1. $78.55.
2. $31.50.
3. $206.55.
4. $76.58.
5. $40.30.
6. $18.24.
8. 23$\frac{1}{3}$%; 50$\frac{1}{3}$%;
30%; 64%;
78$\frac{1}{6}$%; 42$\frac{2}{3}$%;
74$\frac{2}{3}$%.
9. $106.
10. $40.83.
11. 200 yd.
12. $354.06.
13. $102.82.
14. $420.26.
15. $535.42.
16. $147.67.
17. $26.25.
18. $168.93.
19. $905.23;
$896.08.
20. $450.96.
21. 42$\frac{6}{7}$%.
22. 33$\frac{1}{3}$%.
23. 40$\frac{5}{8}$%.
24. 100%.
25. 33$\frac{1}{3}$%; 50%;
53[illegible]%;
40[illegible]%; 42$\frac{6}{7}$%;
30[illegible]%.
26. 31$\frac{1}{4}$%.
27. 11$\frac{1}{9}$%.
28. 58[illegible]%.
30. $2.50.

31. $40.
32. $250.
33. $50.
34. $15.36.
36. 50%.
37. 31[illegible]%.
38. 13[illegible]%.
39. 1.26\frac{2}{3}$;
30$\frac{1}{3}$%.

Art. 419.

1. $44.10.
2. $926.12.
3. $72.
4. $533.22.
5. $135.73.
6. $118.73.
7. $360.07.
8. $900.43;
$855.41.
9. $149.05.
10. $340.64;
$333.83;
$337.23.
11. $1082.55.
12. $145.
13. $65.64.
14. $54.71.
15. $258.32;
$245.40.
16. $74.31;
$72.82.
17. $27.43.
18. $49.76.
19. $1692.46.
20. $93.36.
21. $50.12.
22. $359.58.
23. $1563.87.
24. $155.40.

Art. 427.

1. $21.60.
2. $200000.
3. $212.50.
4. $2256.25.
5. $2295.
6. 2%.
7. 480.
8. $3099.37.
9. $12.61.
10. $81.75.
11. $2161.17 net amount.
12. $65.63.
13. $869.60, com.
14. $125.
15. 5%.
16. $5091.

17. $2554.75.
18. $43500.
19. $1773.50.
20. $8450.89.
21. $2438.90.
22. $7872.07.
23. $2175.44.
24. $50000.
25. $2751.14.
26. $852.91.
27. $185.40.
28. $1388.63.
29. $988.
30. $5136.
31. $61200.
32. $128.31.
33. $495.19.
34. $1234.
35. $875.52.
36. $900.
37. $288.39.
38. $18909.18.

Art. 441.

1. $122.50.
2. $115.65.
3. $525.25.
4. $702.51.
5. $917.72.
6. $214.31.
7. $535.50.
8. $397.49.
9. $106.58.
10. $3393.33.
11. $1059.71.
12. $470.63.
13. $18.91,
14. $1791.94.

Art. 445.

1. $20.
2. $21.82.
3. $1.79.
4. $2.92.
5. $47.11.
6. $0.91.
7. $6.48.
8. $6.81.
9. $1.75.
10. $16.16.
11. $6.91;
$5.76.
12. $127.58;
$148.85.
13. $101.23;
$88.57.
14. $53.23;
$97.70.

15. $50.69;
$33.79.
16. $663.33;
$1105.56.
17. $33.25; $24.94.
$33.98; $25.48.
18. $23.16; $28.95.
$23.44; $29.30.
19. $92.67;
$185.33.
$93.33;
$186.67.
20. $90.35;
$150.58.
$92.37;
$153.95.
21. $654.50;
$872.67.
$656.83;
$875.88.

Art. 459.

1. $8.64.
2. $1.32.
3. $3.48.
4. $8.64.
5. $6.57.
6. $6.
7. $5.34.
8. $1.
9. $7.89.
10. $0.85.
11. $10.63.
12. $11.73.
13. $18.14.
14. $0.31.
15. $3.73.
16. $10.07.
17. $35.63.
18. $9.50.
19. $26.04.
20. $46.67.
21. $16.28; $18.57.
22. $64.76;
$32.38.
23. $36.85; $42.99.
24. $24.70; $20.58.
25. $56.68; $75.57.
26. $180.10;
$120.07.
27. $11.43; $17.14.
28. $39.45; $46.03.
29. $19.79; $23.09.
30. $2.85; $2.37.
31. $13.75; $16.04.
32. $106.66;
$142.22.
33. $137.72;
$114.77
34. $107.36;
$80.52.
35. $93.53;
$140.30.
36. $19.27;
$14.46.
37. $494.15;
$247.07.
38. $418.64;
$837.29.
39. $434.60;
$437.58.
40. $1393.29;
$1446.05.
41. $623.85;
$615.81.
42. $2267.51;
$2249.67.
43. $6503.40;
$6597.60.
44. $4416.93;
$4390.69.
45. $389.94;
$387.12.
46. $1450.87;
$1605.74.
47. $5092.50;
$5032.71.
48. $2.
49. $10.66.
50. $53.44.
51. $14.19.
52. $0.50.
53. $33.11.
54. $61.20.
55. $8.38.
56. $11.33.
57. $9.33.
58. $34.96; $33.21.
$35.65; $33.87.
59. $14.27; $18.73.
$14.47; $18.99.
60. $26.73; $19.09.
$27.29; $19.49.
61. $193.96;
$113.14.
$195.15;
$113.83.
62. $309.07;
$347.70.
$310.14;
$348.90.
63. $57.27; $81.81.
$57.82; $82.60.
64. $7.59; $10.12.
65. $25.95; $11.53
66. $111; $27.75.
67. $76.50; $60.56.
$77.40; $61.28.
68. $16.28; $14.65.
$16.57; $14.92.
69. $53; $59.63.
70. $46;
$48.30.
$47;
$49.35.
71. $9.80;
$8.98.
$10; $9.17.
72. $10.27;
$9.54.
$10.50;
$9.75.
73. $18.18;
$7.93.
$18.56;
$8.12.
74. $14.04;
$7.41.
$13.86;
$7.32.
75. $51.33;
$38.50;
$51.91;
$38.93.
76. $10658.20.
77. $1050.
78. $1556.66.
79. $27.84.
80. $28.93.
81. $1356.18.
82. $1562.50.
83. $86.07.
84. $5643.
85. $601.39.

Art. 463.

1. $1.79.
2. $116.47.
3. $11.54.
4. $5.29.
5. $3.92.
6. $15.12.
7. $5.75.
8. $6.16.
9. $42.18.
10. $14.83.
11. $4.44.
12. $10.44.
13. $39.35.
14. $246.89.
15. $58.97.
16. $27.74.
17. $41.64.
18. $7 58.
19. $2.14.
20. $30.21
21. $5.

Art. 466.

1. 7%.
2. 7%.
3. 6%.
4. 5%.
5. 7%.
6. 8⅓%.
7. 8%.
8. 4⅓%.
9. 4%.
10. 6%.
11. 8⅓%.
12. 9%.
13. 10⅓%.
14. 30%.
15. 8%.
16. 6%.
17. 7%.

Art. 469.

1. 6 m. 6 d.
2. 1 y. 4 m. 20 d.
3. 2 y. 2 m. 2 d.
4. 2 m. 15 d.
5. 6 m. 14 d.
6. 1 y. 10 m. 22 d.
7. 4 y. 9 m. 15 d.
8. 4 y. 8 m. 24 d.
9. 9 m. 20 d.
10. 11 m. 21 d.
11. 1 y. 5 m. 15 d.
12. 1 y. 5 m. 18 d.
13. 16 y. 8 m.
14. 28 y. 6 m. 26 d.

Art. 472.

1. $12107.84.
2. $871.31.
3. $2241.
4. $7719.16.
5. $1997.87.
6. $3000.
7. $3228.38.
8. $29419.35.
9. $30612.25.
10. $31746.03.
11. $11973.33.
12. $14370.69.
13. $3436.99.

Art. 475.

1. $1234.
2. $5280.
3. $3456.
4. $375.60.
5. $12375.
6. $1728.
7. $728.01.
8. $879.54.
9. $1511.67.
10. $2309.28.
11. $3770.52.
12. $5307.72.
13. $1642.31.
14. $2138.94
15. $5063.11

16. $2863.86.
17. $3590.09.

Art. 479.

1. $892.86;
$107.14.
2. $438.60;
$61.40.
3. $547 95;
$52.05.
4. $762.63;
$37.37.
5. $283.35;
$41.65.
6. $161.64;
$13.36.
7. $595.39;
$204.61.
8. $641.79;
$258.21.
9. $0.68.
10. $3629.03.
11. $204.29.

Art. 480.

1. $42.82.
2. $1843.93.
3. 5%.
4. 1 y. 10 m. 28 d.
5. $1722.02.
7. $14\frac{2}{7}\%$.
8. Latter $1\frac{1}{2}\%$
better.
9. Oct. 3.
10. 7%.
11. $19230.77.
12. $4298.04;
$4342.65.
13. June 11, 1874.
14. $129;
$131.50;
$129.70.
15. $650.
16. $1483.98.
17. $89.17.
18. $606.60.
19. £199 3s. 8d.
20. £8 10s.
21. £5 8s. 4d.
22. £9 5s. 10d.
23. £2 12s. 11d.
24. £3 17s. 4d.
25. £10 9s. 8d.

Art. 485.

1. $526.44;
$506.48.
2. $45.18; $37 37
3. $83.26.
4. $1909.63;
$2104.72.
5. $1211.
6. $220.80;
$268.51.
7. $8583.80.
8. $1811.44;
$2564.94.
9. $2794.32;
$7798.54.
10. $993.03.
11. $4445.17.
12. $2450.13.
13. -$747.27.
14. $4172.57.
15. $405.34.
16. $13363.84.
17. $4659.94.
18. $343.90.

Art. 495.

1. $1022.
2. $910.46;
$919.21;
$917.46.
3. Mar. 7, 1890;
$6022.10.
4. $431.10;
$424.65.
6. Aug. 18, 1887.
7. Friday.
8. Wednesday;
Wednesday.
9. Oct. 23.

Art. 500.

2. $7941.33, or
$7942 14.
3. Apr. 24;
$1182, or
$1182.25.
4. Aug. 18;
$5199.04, or
$5200.15.
5. Oct. 30:
$2525.87, or
$2526.33.
6. Oct. 4;
$3666.90, or
$3668.12.
7. Nov. 15;
$6398.23, or
$6398.60.
8. Sept. 13;
$8139.57, or
$8142.79.
9. Aug. 29;
$4433.75, or
$4434.66.
10. Sept. 25:
$8208.41, or
$8208.85.
11. June 26;
$4277.97, or
$4278.60.
12. Nov. 27;
$4772.10, or
$4773.65.
13. May 3;
$8904, or
$8905.32.
14. Jan. 12;
$4912.50, or
$4913.70.
15. Jan. 30;
$5939.10, or
$5939.93.
16. $5949.
17. Oct. 2;
$4949.17.
18. Apr. 1;
$3711.87.
19. Aug. 3;
$6886.25.
20. July 1;
$8913.50.
21. Dec. 18;
$4841.94.
22. Sept. 26;
$4453.50.
23. Dec. 2;
$8874.50.
24. Dec. 5;
$2972 25.
25. June 21;
$9816.67.
26. Oct. 8;
$5896.50.
27. May 10;
$5900.
28. Sept. 3;
$8607.18.
29. $7837.33.
30. $8812.50.
31. $5\frac{1}{2}\%$.
32. $14.90.
33. $3012.38.
34. Aug. 27;
$3737.62.
35. May 16;
$1641.34.
36. Apr. 12;
$882.49.
38. $1522.84.
39. $1522.50.
40. $3079.52.

Art. 505.

1. $678 54.
2. $242.17;
$148.16.
3. $1102.69;
$1184.37.
4. $1327.21;
$1410.94.
5. $885.74;
$924.38.
6. $898.88.
7. $3073;
$3363.56.
8. $517.82;
$716.62.
9. $3260.28;
$4559.19.

Art. 509.

2. $440;
$447.16.
3. $228.31;
$214.37.
4. $651.97;
$753.30.

Art. 513.

2. $262.24;
$161.49.
3. $1108.57;
$1192.53.
4. $1324.75;
$1406.47.
5. $833.87;
$920 94.
6. $897.77.
7. $3067.14;
$3347 31.
8. $549 89;
$764.05.
9. $3260 51;
$4594.82.

Art. 523.

1. 12.
3. 160.
4. $36.
5. $50.
6. 240 lb.
7. $111.
8. $21.875.
9. $55.50.
10. $85\frac{1}{3}$ yd.
11. 396 ft.
12. £25 11s. 9d.
13. $960.

14. $2410.71.
15. A, $1875.90;
B, $1598.40.
16. 15.9883 : 1.
17. $179.56.
18. $126.
19. $3.79.
20. $137.03.
21. $153600.
22. 2355⅝ ft.
23. 25.215 fr.
24. 252 8⁄9 yd.
25. 20 hr. 45 min.
9 sec.
26. $11.10.
27. 69 da.
28. $3000.
29. $912.23.
30. 47⅓ yd.
31. 4449 1⁄15 bu.
32. $84.18.
33. $0.73.
34. $3.12.
35. $17.60.
36. $22.67; $25.33.
37. 45 days.
38. $1.09⅜.
39. 1492.26 ft.
40. 18¾ mo.
41. $5328, assets;
$11100, debts.

Art. 526.

1. $105.63.
2. 6 hr.
3. 126 A.
4. $4665.60.
5. 4 hr.
6. 125.
7. ½ day.
8. 4 days.
9. 15 hr.
10. 9 days.
11. $64.
12. 80 days.
13. $66.13.
14. $1493.33.
15. 32 days.
16. 16320 lb.

Art. 544.

1. $93.75.
2. $68.75.
3. $354.50
4. 80¢.
5. $3600.
6. $281.25.
.. $22.50.
8. $56.88.
9. $71.43.
10. $57.60.
11. $4880.
12. $28.56.
13. $42.
14. $9000.
15. $9000.
16. 40%.
17. $16.20;
$27.
18. $1440.
19. $61.25.
20. $792.
21. M, $1761.36;
P, $1409.09;
T, $880.68.
22. A, $151.54;
C, $568.18.
23. 2.767%.
24. $2627.78.
25. $315.33.
26. $28.03.
27. $155.70.

Art. 553.

1. $8782.81.
2. $8395.94.
3. $5006.25.
4. $4358.59.
5. $8427.52.
6. $9922.37.
7. $5270.79.
8. $4287.11.
9. $3417.18.
10. $2821.96.
11. $9898.30.
12. $8406.44.
14. $1871.48.
15. $2443.86.
16. $3346.56.
17. $2227.28.
18. $8144.96.
19. $7373.16.
20. $9222.61.
21. $6431.57.
22. $9231.46.
23. $5283.96.
24. $23.29;
$9310.10.
25. $2973.75.
26. $11839.
27. $7858.67.
28. 1% premium.
29. $3120.05.
30. $10113.78.

Art. 567.

1. $1083.94.
2. $1583.40.
4. $2497.50.
5. $3760.69.
6. $4050.03.
7. $409.34.
8. $2483.15.
9. $4076.72.
10. $3290.93.
11. $1087.98.
12. $4261.23.
13. $4593.93.
14. $2611.06.
15. $8495.46.
16. $2373.24.
18. £1225 18s. 6d.
19. 4.83⅓.
20. £1864 6s. 4d.
21. $1341.32.
22. $1162.79.
23. $965.02.
24. $767.20.
25. $1631.38.
26. $1393.72.
27. $189.60.
28. $1856.08.
29. $2023.10.
30. $1534.69.
31. $2386.62.
32. $1688.75.
33. $1411.11.
34. 80318.70 fr.
35. 17972.04 fr.
36. 5.18⅔.
38. $1592.
39. $1967.24.
40. $822.96.
41. $289.54.
42. $2359.10.
43. $1543.24.
44. $3867.75.
45. $3102.24.
46. $1547.25.
47. $2061.40.
48. $2359.33.
49. 3467.73 marks.
50. 13824 marks.
51. 94¼.
52. $2905.05.
53. $4976.40.
54. 4562 guilders.
55. 7128 guilders.
56. 40¾.
57. 4.885.
58. $17366.99.
59. $3.60.
60. 118.4; 117.1.
61. 85.98.
62. 147.
63. 13950.75 fr.
64. £391 4s.
65. 7343.75 fl.
66. £1009 7s. 6d.
67. £512 3s. 2d.

Art. 574.

1. Oct. 10.
2. May 11.
3. Feb. 11.
4. Aug. 4.
5. June 5, 1882.
6. June 20.
7. July 18;
$1694.90;
$1686.16.
8. Feb. 15, 1882.
9. Oct. 6, 1881;
$2403.88;
$2367.55.
10. May 31;
$2480.32;
$2492.72.
11. July 14.
12. Nov. 15.
13. Sept. 5.
15. Oct. 26, 1882.
16. Sept. 25;
$2425.16;
$2437.33.
17. July 8.
18. Mar. 21, 1882.
19. Feb. 13, 1882.
20. April 12, 1883.

Art. 582.

1. Jan. 17, 1889.
2. May 25, 1888.
3. Dec. 3, 1882.
4. May 13, 1882.
5. Sept. 6, 1882;
$276.53.
6. Feb. 15, 1883;
$1053.23.
7. Feb. 8, 1881.
8. Aug. 25, 1882.

Art. 586.

1. $2074.06;
Aug. 28;
Sept. 2;
Sept. 2.
2. $5324.48;
Nov. 28.
3. $2751.14;
Sept. 27.
4. $12505.70;
Apr. 26.

33. A, $3847.56;
B, $4902.44.
34. A, $12346.82;
B, $5877.67.
35. A, $2505.74;
B, $17307.58;
C, $16723.84.
36. A, $17527.74;
B, $20323.76;
C, $6310.43;
Loss, $144.24.
37. A, $7331.20;
B, $4950.50.
38. A, $4932.38;
B, $3497.25;
C, $6570.37.

Art. 645.

1. 72%;
$5760.
2. $702000;
$35100.
3. $35625.
4. $77400;
$64000;
$86000.
5. $21375;
$475000.
6. $3,338,100.
7. 26.26%;
$23,944,096.50;
$1,213,591.50.
8. $1,433,831.70.
9. $1047.54;
$427.84.
10. $2313.65.
11. 12%;
10%;
19.56%.
12. $4\frac{64}{105}$%.

Art. 650.

1. $374.60;
$374.93.
2. $669.35;
$671.70.
3. $25.14.
4. $755.28;
$757.38.
5. $462.10;
$464.38.
6. $190.91;
$191.04.
7. $96.15.
8. $328.60;
$329.37.
9. $557.31;
$557.86.
10. $636.58;
$637.45.

Art. 662.

1. $131.90.
2. $137.20.
3. $116.36.
4. $1638.40.
5. $0.054;
$0.072;
$0.104.
6. $0.655.
7. $356.
8. $13472.
9. $10469.12.
10. $3700;
$13700.
11. $9080;
$29080.
12. $996.80.
13. $717.68.
14. $12379.12.
15. $342.67;
$26.98;
$296.57.
16. $42919;
$29169.33.
17. $4190.03;
$3084.60;
$933.35;
54 yr.
18. $152.79,
$77.86.
19. $13.63.
20. $2655.65.

Art. 663.

1. $104\frac{1}{2}$.
2. 21659.6.
3. 40320.
4. $164.52.
5. 16° 48′ 15″.
6. $16666\frac{2}{3}$ sq. yd.
7. £69 0s. 6d.
8. $140.25.
9. $38\frac{1}{3}$ sq. yd.;
$24.15.
10. £19 3s. 6d.
11. $5567.50;
$4.443, wood;
$.486, grain.
12. $1803.07.
13. $ 8392.
14. £219375;
$23.72;
$1067588.44.
15. $45.73.
16. $430.72.
17. £10. 0s. 9d.
18. $102.
19. $433.93.
20. $889.36.
21. $261.83.
22. 120 bbl.
23. 135458 lb.
24. $676.77.
25. 0.
26. Loss, $500.
27. $30\frac{10}{13}$%.
28. $500.
29. $35.58.
30. $137.26.
31. £3 19s. 10d.
32. Sept. 19;
$4950.
33. $5076.14.
34. $851.96.
35. $925.62;
$1074.38.
36. $912.83;
$1087.17.
37. $72.22;
$73.89;
$72.88.
38. $4166.30;
$4160.47.
39. $5171.73;
$5181.19.
40. $4165.03.
41. $630.97.
42. $2474.07.
43. $9825.03.
44. 17651.29 fr.
45. $1889.22.
46. $7.18; $2.39.
47. $114.75.
48. Aug. 29;
$5359.35.
49. Jan. 14, 1883;
$1409.40.
50. $1409.37.
51. $320.31.
52. $1625.
53. $425.
54. $37000.
55. $81.25.
56. $4224.
57. $128.21.
58. $295.75.
59. $208.50.
60. B. $5011.83;
C. $1794.53;
Dr. Mdse., etc.,
$12410;
Dr. Sundry Debtors,
$17030;
Cr. B, $12695;
Cr. C, $12695;
Cr. Sundry Creditors,
$4050.

Art. 667.

1. 36.
2. 48.
3. 55.
4. 72.
5. 81.
6. 126.
7. 125.
8. 144.

Art. 671.

1. 29.
2. 37.
3. 31.
4. 43.
5. 47.
6. 6.
7. 23.
8. 1.

Art. 674.

1. $9715.20.
2. $5187.21.
3. $13648.
4. $5631.50.
5. $4051.44.
6. $1474.27.
7. $3426.67.

Art. 677.

2. $261.58.
4. $1327.61.
8. $549.51.
9. $3260.51.

Art. 682.

2. $120.
3. $72.
4. $6.20.
5. $27.

Art. 687.

1. $50.17.
2. $36.60.
3. $159.

Art. 690.

2. 58c.
3. 35c.
4. 58c.

Art. 693.

2. 2, 1, 5, 1;
3, 2, 3, 5.
3. 3, 4, 5, 1;
4, 3, 1, 5
4. 1, 5, 6, 1
5, 1, 1, 6.
5. 1, 1, 3, 6, 1;
8, 3, 1, 1, 6.

Art. 695.

2. 10, 10, 45.
3. 30, 90.
4. 12, 48, 48;
36, 36, 108.
5. 15, 15.
6. 9, 15, 3;
16, 4, 20.

Art. 697.

2. 5, 60, 15, 30;
8, 52, 24, 26.
4. 33, 12, 5.

Art. 701.

2. 33.
3. 121.
4. 135.
5. 216.
6. 218.
7. 255.
8. 294.
9. 312.
10. 345.
11. 375.
12. 716.
13. 15.25.
14. 1994.
15. 20.78.
16. 2732.
17. 19683.
18. [illegible]; $\frac{47}{40}$; $\frac{78}{111}$.
19. 642 rods.
20. 208.71 ft.
21. 1.41421.

Art. 703.

2. 18.
3. 2.2.
4. 68.
5. 78.
6. 82.
7. 196.
8. 2.13
9. 23.9.
10. 462.
11. 6.54.
12. 75.8.
13. 878.
14. 82.
15. 47 in.
16. [illegible]; [illegible]; [illegible].
17. 35.03 in.

Art. 724.

1. 18.9875 A.
2. 1.625 A.
3. 290.47 sq. rd.
4. 34 A. 145 sq. rd. 25 sq. yd. 8 sq. ft. 108 sq. in.
5. 1080.6 ft.
6. 471.24 ft.
7. 5541.78 sq. ft.
8. 14.7 sq. ft.
9. 3525 gal.
10. 5.32 cu. ft.
11. 15053⅓ tons.
12. 59.9 in.
13. 466.69 ft.
14. 75.3984 cu. ft.
15. 6 ft. 6 in.
16. 32 yd.
17. 45.14 rd.;
141.8 rd.
18. 502.656 A.

Art. 730.

2. 6.8027%;
$493.20.
3. $225.
4. V r. $3581.16;
C p. $3681.16;
Adj. rec. $100.
5. $4869.57.
6. Ship receives
$1888.08;
A r. $964.23·
B p. $1472.82;
C p. $496.94;
D p. $1032.55;
Adj. rec. $150.
7. C p. $1200.79;
S rec. $974.60;
Adj. rec. $150;
Agent receives
$76.19.
8. G. A., $516.81;
S, $638.79;
O, $789.09.

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